

1)

$$\sec x - \tan x \sin x = \frac{1}{\sec x}$$

$$\frac{1}{\cos x} - \frac{\sin x}{\cos x} \cdot \sin x$$

$$\frac{1}{\cos x} - \frac{\sin^2 x}{\cos x} = \frac{1 - \sin^2 x}{\cos x}$$

$$\rightarrow \frac{\cos^2 x}{\cos x} = \boxed{\cos x = \frac{1}{\sec x}}$$

2)

$$\frac{\sec \theta}{\cos \theta} - \frac{\tan \theta}{\cot \theta} = 1$$

$$\frac{\sec \theta}{\frac{1}{\sec \theta}} - \frac{\tan \theta}{\frac{1}{\tan \theta}}$$

$$(\sec \theta) \left(\frac{\sec \theta}{1} \right) - (\tan \theta) \left(\frac{\tan \theta}{1} \right)$$

$$\sec^2 \theta - \tan^2 \theta = \boxed{1}$$

* Recall identity
 $1 + \tan^2 \theta = \sec^2 \theta$

3)

$$\cos^2 y - \sin^2 y = 1 - 2\sin^2 y$$

↓

$$(1 - \sin^2 y) - \sin^2 y$$

$$1 - \sin^2 y - \sin^2 y \rightarrow \boxed{1 - 2\sin^2 y}$$

4)

$$\csc^2 \theta \tan^2 \theta - 1 = \tan^2 \theta$$

$$\frac{1}{\cancel{\sin^2 \theta}} \cdot \frac{\cancel{\sin^2 \theta}}{\cos^2 \theta} - 1$$

$$\frac{1}{\cos^2 \theta} - \frac{1}{1} \rightarrow \frac{1}{\cos^2 \theta} - \frac{\cos^2 \theta}{\cos^2 \theta}$$

$$\frac{1 - \cos^2 \theta}{\cos^2 \theta} \rightarrow \frac{\sin^2 \theta}{\cos^2 \theta} = \boxed{\tan^2 \theta}$$

5)

$$(\sin \theta + \cos \theta)^2 + (\sin \theta - \cos \theta)^2 = 2$$

$$(\sin \theta + \cos \theta)(\sin \theta + \cos \theta) + (\sin \theta - \cos \theta)(\sin \theta - \cos \theta)$$

$$\sin^2 \theta + \cancel{\cos \theta \sin \theta} + \cancel{\cos \theta \sin \theta} + \cos^2 \theta + \sin^2 \theta - \cancel{\cos \theta \sin \theta} - \cancel{\cos \theta \sin \theta} + \cos^2 \theta$$

$$1 + 1 = \boxed{2}$$

6)

$$\frac{\cos x + 1}{\sin^3 x} = \frac{\csc x}{1 - \cos x}$$

$$\frac{\cos x + 1}{(\sin^2 x)(\sin x)} \rightarrow \frac{(\cos x + 1)}{(1 - \cos^2 x)(\sin x)}$$

$$\frac{(\cancel{\cos x + 1})}{(1 - \cancel{\cos x})(\cancel{1 + \cos x})(\sin x)}$$

$$\frac{1}{(1 - \cos x)} \cdot \frac{1}{\sin x}$$

$$\frac{1}{1 - \cos x} \cdot \csc x \rightarrow$$

$$\boxed{\frac{\csc x}{1 - \cos x}}$$

7)

$$\sec 2x = \frac{1}{1 - 2\sin^2 x}$$

$$\frac{1}{\cos 2x} \rightarrow \boxed{\frac{1}{1 - 2\sin^2 x}}$$

8)

$$\frac{\cot^2 x - 1}{\csc^2 x} = \cos 2x$$

$$\frac{\cot^2 x}{\csc^2 x} - \frac{1}{\csc^2 x}$$

$$\frac{\cos^2 x}{\sin^2 x} - \sin^2 x$$

$$\frac{1}{\sin^2 x}$$

$$\frac{\cos^2 x}{\cancel{\sin^2 x}} \cdot \frac{\cancel{\sin^2 x}}{1} - \sin^2 x$$

$$\cos^2 x - \sin^2 x = \boxed{\cos 2x}$$