Unit 4A Test Review WS #2

Verify the following identities

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$$\frac{1}{\tan\beta} + \tan\beta = \frac{\sec^2\beta}{\tan\beta}$$

$$\frac{1}{\tan\beta} + \frac{\tan\beta}{1}(\tan\beta)$$

$$\frac{1}{\tan\beta} + \frac{\tan^2\beta}{\tan\beta} = \frac{1+\tan^2\beta}{\tan\beta}$$

$$= \boxed{\frac{\sec^2\beta}{\tan\beta}}$$

2)

$$\frac{1-\cos^2 x}{\sin 2x} = \frac{1}{2}\tan x$$

$$\frac{\sin^{\cancel{2}} x}{2\cancel{\sin x}\cos x} \rightarrow \frac{\sin x}{2\cos x} \rightarrow \frac{1}{2}\cdot\frac{\sin x}{\cos x}$$

$$\rightarrow \boxed{\frac{1}{2}\tan x}$$

*use $\cos 2x = \cos^2 x - \sin^2 x$

* rewrite in terms sin, cos

3)

$$\frac{\cos 2x}{1-\tan^2 x} = \cos^2 x$$

$$\frac{\cos^2 x - \sin^2 x}{\frac{1}{1} - \frac{\sin^2 x}{\cos^2 x}} \rightarrow \frac{\cos^2 x}{\cos^2 x} - \frac{\sin^2 x}{\cos^2 x}$$

$$\frac{\cos^2 x - \sin^2 x}{\frac{\cos^2 x - \sin^2 x}{\cos^2 x}}$$

$$\frac{\cancel{(\cos^2 x - \sin^2 x)}}{1} \cdot \frac{\cos^2 x}{\cancel{(\cos^2 x - \sin^2 x)}} = \boxed{\cos^2 x}$$

4)

$$\sec y + \tan y = \frac{\cos y}{1-\sin y}$$

$$\frac{1}{\cos y} + \frac{\sin y}{\cos y} \rightarrow \frac{1+\sin y}{\cos y} \cdot \frac{\cos y}{\cos y}$$

$$\frac{(1+\sin y)\cos y}{\cos^2 y} \rightarrow \frac{(1+\sin y)\cos y}{1-\sin^2 y}$$

factor

$$\frac{\cancel{(1+\sin y)}\cos y}{(1-\sin y)\cancel{(1+\sin y)}} = \boxed{\frac{\cos y}{1-\sin y}}$$

5)

$$\cos 2x = \frac{1-\tan^2 x}{\underline{1+\tan^2 x}} \rightarrow \sec^2 x$$

$$\frac{1-\tan^2 x}{\sec^2 x} \rightarrow \frac{1-\frac{\sin^2 x}{\cos^2 x}}{\frac{1}{\cos^2 x}}$$

$$\left(1-\frac{\sin^2 x}{\cos^2 x}\right)\cdot\frac{\cos^2 x}{1}$$

$$\cos^2 x - \sin^2 x = \boxed{\cos 2x}$$

6)

$$\underline{\sin 2x}\cot x = 2-2\sin^2 x$$

$$\frac{2\cancel{\sin x}\cos x}{1}\cdot\frac{\cos x}{\cancel{\sin x}} \rightarrow 2\underline{\cos^2} x$$

$$2(\underline{1-\sin^2 x}) \rightarrow \boxed{2-2\sin^2 x}$$

7)

$$* \tan 2x = \frac{2\tan x}{1-\tan^2 x}$$

$$\tan x = \csc 2x - \cot 2x$$

$$\frac{1}{\sin 2x} - \frac{\cos 2x}{\sin 2x} \rightarrow \frac{1-\cos 2x}{\sin 2x}$$

$$\frac{1-(1-2\sin^2 x)}{2\sin x\cos x} = \frac{1-1+2\sin^2 x}{2\sin x\cos x}$$

$$\frac{2\sin^2 x}{2\sin x\cos x} = \frac{\sin x}{\cos x} = \boxed{\tan x}$$

8)

$$\frac{\overbrace{\sin^2\theta+\cos^2\theta}^{=1}+\cot^2\theta}{\underline{1+\tan^2\theta}} = \cot^2\theta \qquad \sec^2\theta$$

$$\frac{1+\cot^2\theta}{\sec^2\theta} \rightarrow \frac{\csc^2\theta}{\sec^2\theta}$$

$$\frac{\frac{1}{\sin^2\theta}}{\frac{1}{\cos^2\theta}} \rightarrow \frac{1}{\sin^2\theta}\cdot\frac{\cos^2\theta}{1}$$

$$= \frac{\cos^2\theta}{\sin^2\theta} \rightarrow \boxed{\cot^2\theta}$$

9)

$$\cos x + \sin x\tan x = \sec x$$

$$\cos x + \sin x\left(\frac{\sin x}{\cos x}\right)$$

$$(\cos x)\ \frac{\cos x}{1} + \frac{\sin^2 x}{\cos x} \rightarrow \frac{\cos^2 x}{\cos x} + \frac{\sin^2 x}{\cos x}$$

$$\frac{\cos^2 x+\sin^2 x}{\cos x} \rightarrow \frac{1}{\cos x} \rightarrow \boxed{\sec x}$$

10)

$$(1+\sin\theta)\ \frac{1+\sin\theta}{\cos\theta} + \frac{\cos\theta}{1+\sin\theta}(\cos\theta) = 2\sec\theta$$

$$\frac{1+2\sin\theta+\sin^2\theta}{\cos\theta(1+\sin\theta)} + \frac{\cos^2\theta}{\cos\theta(1+\sin\theta)}$$

$$\frac{1+2\sin\theta+\overbrace{\sin^2\theta+\cos^2\theta}^{1}}{\cos\theta(1+\sin\theta)}$$

$$\frac{2+2\sin\theta}{\cos\theta(1+\sin\theta)} \rightarrow \frac{2\cancel{(1+\sin\theta)}}{\cos\theta\cancel{(1+\sin\theta)}}$$

$$\frac{2}{\cos\theta} = \boxed{2\sec\theta}$$