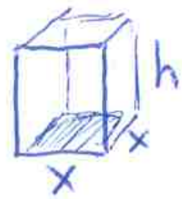
Rectangular box with open top has volume of $30 m^3$. ~~Eea~~ Box has a square base. Material for base is $\$15/m^2$. Material for sides are $\$7/m^2$. Find cost, surface area, and dimensions of cheapest container.



$V = x^2 h$

$30 = x^2 h$

$\boxed{\frac{30}{x^2}} = h$

$S = \underbrace{x^2}_{\$15} + \underbrace{xh + xh + xh + xh}_{\$7}$ $7(4xh)$

$S = x^2 + 4xh$

* $\text{Cost} = 15x^2 + 28x\boxed{h}$

$\text{Cost} = 15x^2 + 28x\left(\frac{30}{x^2}\right)$

$\text{Cost} = 15x^2 + \frac{840}{x}$

$C(x) = 15x^2 + 840x^{-1}$

$C'(x) = 30x - 840x^{-2}$

$0 = 30x - \frac{840}{x^2}$

$\frac{840}{x^2} = \frac{30x}{1}$

$30x^3 = 840$

$x^3 = 28$

$\boxed{x = 3.037}$

$h = \frac{30}{(3.037)^2} = 3.253$

Dimensions: 3.037m x 3.037m x 3.253m

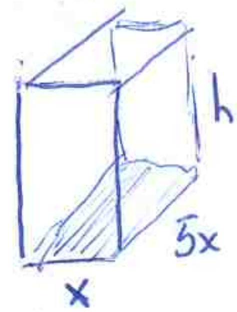
Surface Area = $3.037^2 + 4(3.037)(3.253)$

$\boxed{S = 48.741 m^2}$

$\boxed{\text{Cost} = \$414.94}$

Ex. **optimization**

Rectangular box with closed top has volume of $20\ m^3$. length of base is 5 times the width. Material for base is $\$20/m^2$. Material for the sides and top is $\$4/m^2$. Find cost of the cheapest container.



$$V = 5x^2h$$
$$20 = 5x^2h \rightarrow \frac{20}{5x^2} = h \qquad \boxed{\frac{4}{x^2} = h}$$

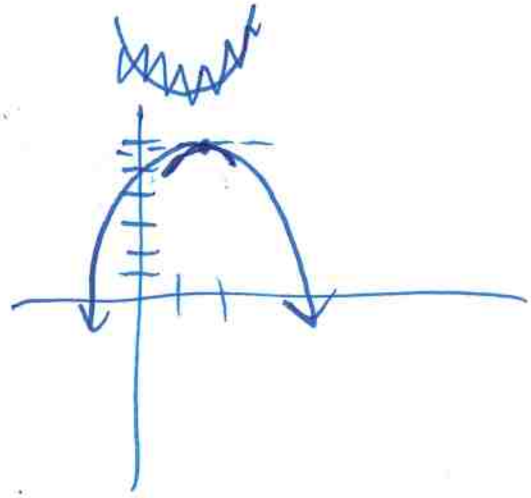
$$S = \underbrace{5x^2}_{20} + \underbrace{5x^2 + 5xh + 5xh + xh + xh}_{4}$$

$$\cancel{S}\ C = 20(5x^2) + 4(5x^2) + 4(12xh)$$
$$C = 100x^2 + 20x^2 + 48xh$$
$$* \quad C = 120x^2 + 48x\boxed{h}$$
$$C = 120x^2 + 48x\left(\frac{4}{x^2}\right)$$
$$C = 120x^2 + \frac{192}{x}$$
$$C = 120x^2 + 192x^{-1}$$

$$C'(x) = 240x - 192x^{-2}$$
$$0 = 240x - \frac{192}{x^2}$$
$$\frac{192}{x^2} = \frac{240x}{1} \qquad 240x^3 = 192$$
$$x^3 = 0.8$$
$$\boxed{x = 0.928}$$
$$\text{Cost} = 120(0.928)^2 + \frac{192}{0.928} \qquad \boxed{\$310.24}$$

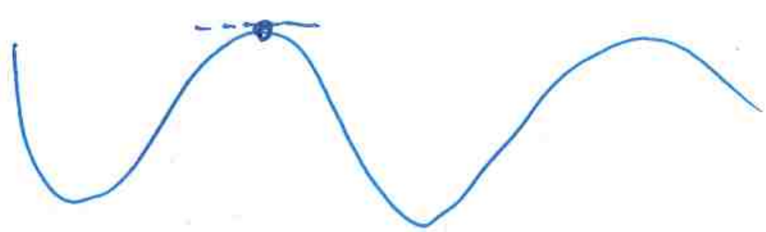
If $f(2)=6$, $f'(2)=0$, $f''(2)=-67$

what can we conclude about the graph at $x=2$?

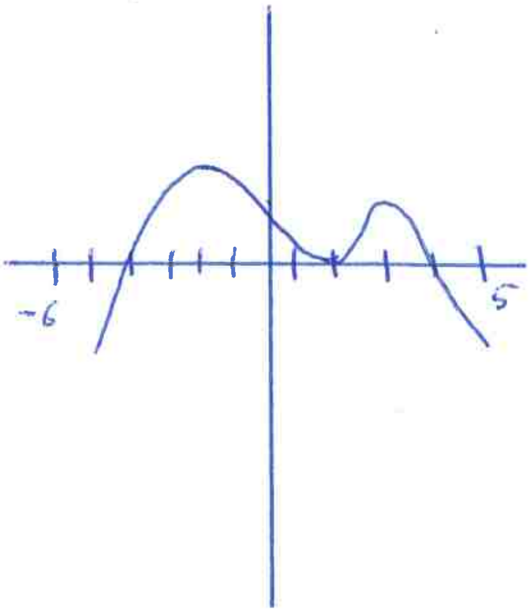


$f''(2)<0$

Rel. max at $(2,6)$



Derivative Graph Practice:



$f(-6)=3$

$f(5)=8$

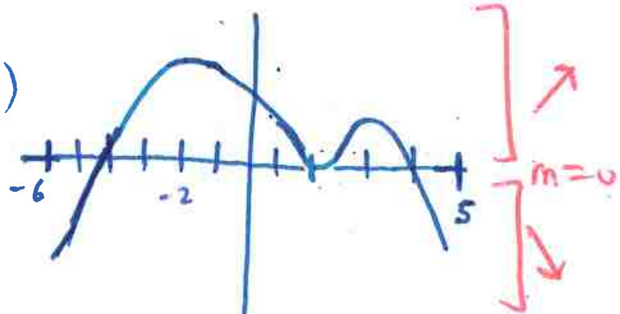
Range $[1,10]$

(26)

11) ~~We want to construct a box whose base length is 3 times the base width. The material used to build the top and bottom cost $10 per square foot and the material used to build the sides cost $6 per square foot. If the box must have a volume of 50 cubic feet, determine the dimensions that will minimize the cost to build the box.~~

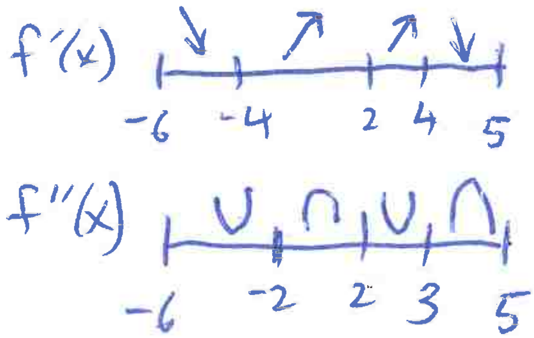
· Derivative Graph

$f'(x)$

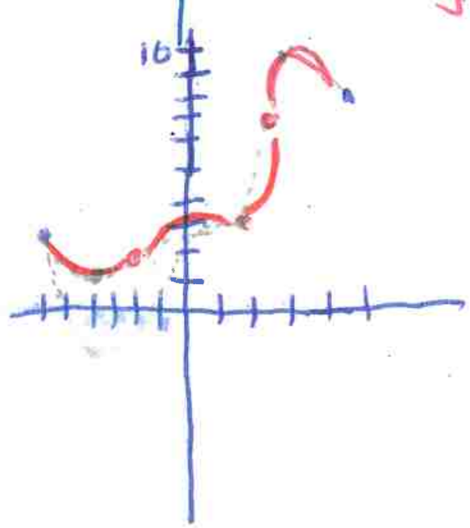


$f(-6)=3$ Range [1, 10]

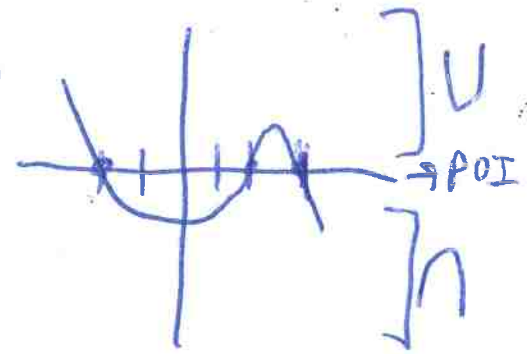
$f(5)=8$



$f(x)$

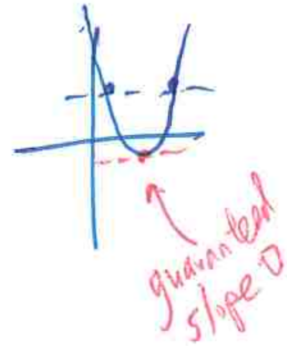


$f''(x)$



12) There are 50 apple trees in an orchard. Each tree produces 800 apples. For each additional tree planted in the orchard, the output per tree drops by 10 apples. How many trees should be added to the existing orchard in order to maximize the total output of trees?

2) Rolle's Theorem $f(x) = (x-3)(x+2)^2$ $[-2,3]$



$f(x)$ continuous $[-2,3]$

$f(x)$ differentiable $(-2,3)$

$f(-2) = 0$

$f(3) = 0$

$m = \frac{0-0}{3-(-2)} = \frac{0}{5} = 0$

Avg. RoC = 0

Instantaneous RoC = Avg. RoC

$f(x) = (x-3)(x^2+4x+4)$

$f(x) = x^3 + 4x^2 + 4x - 3x^2 - 12x - 12$

$f(x) = x^3 + x^2 - 8x - 12$

$f'(x) = \boxed{3x^2 + 2x - 8 = 0}$

$(3x-4)(x+2) = 0$

$x = 4/3$, $\cancel{x = -2}$

$\boxed{c = 4/3}$

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3) MVT $f(x) = x(x^2-x-2)$ on $[-1,1]$

$f(x)$ continuous $[-1,1]$, $f(x)$ differentiable $(-1,1)$

$f(-1) = -1(1+1-2) = 0$

$f(1) = 1(1-1-2) = -2$

$\rightarrow$ Avg. RoC $= \frac{-2-0}{1-(-1)} = \frac{-2}{2} = \boxed{-1}$

$f(x) = x^3 - x^2 - 2x$

$\boxed{f'(x) = 3x^2 - 2x - 2}$

$3x^2 - 2x - 2 = -1$

$3x^2 - 2x - 1 = 0$

$(3x+1)(x-1) = 0$

$\cancel{3x^2 - 2x = 1}$

$\cancel{x(3x-2) = 1}$

$3x+1=0$	$x-1=0$
$\boxed{x = -1/3}$	$x = 1$

$\boxed{c = -1/3}$