Analytic Geometry Unit 5B Practice

1. $y = x^2 - 4x - 12$

Form: ____________________ a = ______ Opens: ______

Vertex: ____________ AOS: ___________

x-int: ________________ y-int: ________________

Domain: _______________ Range: ________________

End Behavior: As $x \to \infty, f(x) \to$ ________

As $x \to -\infty, f(x) \to$ ________

Interval of Increase: ______________________________

Interval of Decrease: ______________________________

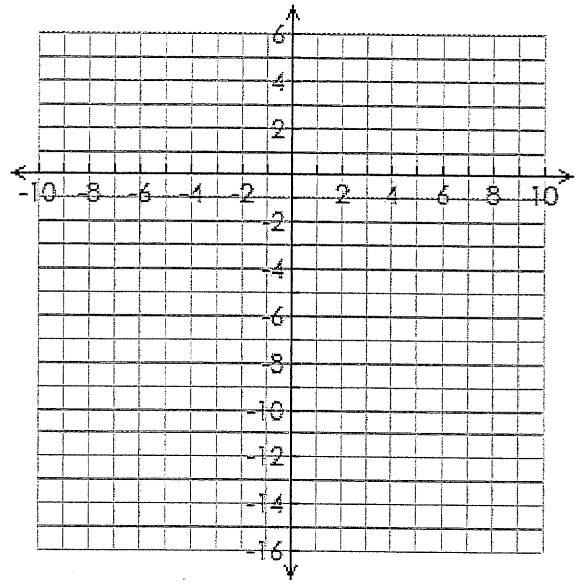
Interval(s) of Positive: ______________________________

Interval(s) of Negative: ______________________________

Min: ____

Max: ____

Average Rate of Change from [4, 6] = ______________



2. $y = 2(x + 3)(x - 1)$

Form: ____________________ a = ______ Opens: ______

Vertex: ____________ AOS: ___________

x-int: ________________ y-int: ________________

Domain: _______________ Range: ________________

End Behavior: As $x \to \infty, f(x) \to$ ________

As $x \to -\infty, f(x) \to$ ________

Interval of Increase: ______________________________

Interval of Decrease: ______________________________

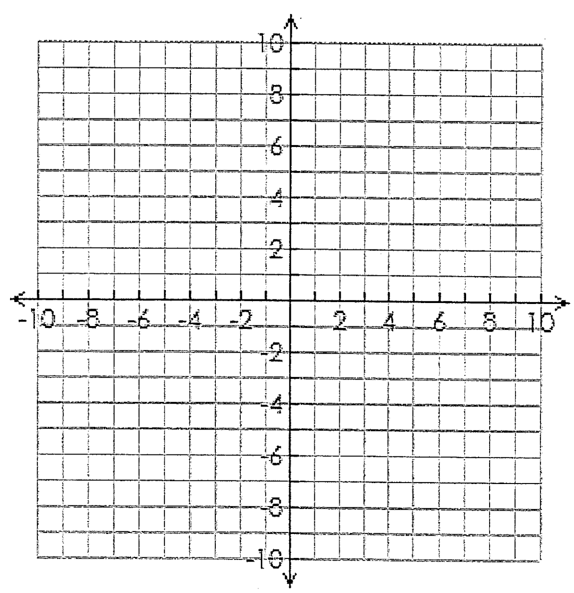
Interval(s) of Positive: ______________________________

Interval(s) of Negative: ______________________________

Min: ____

Max: ____

Average Rate of Change from [-4, 0] = ______________



3. $y = -(x-4)^2 + 16$

Form: ____________________ a = _______ Opens: ______

Vertex: _____________ AOS: ____________

x-int: _________________ y-int: _________________

Domain: ______________ Range: ________________

End Behavior: As $x \to \infty, f(x) \to$ _________

As $x \to -\infty, f(x) \to$ _________

Interval of Increase: ______________________________

Interval of Decrease: ______________________________

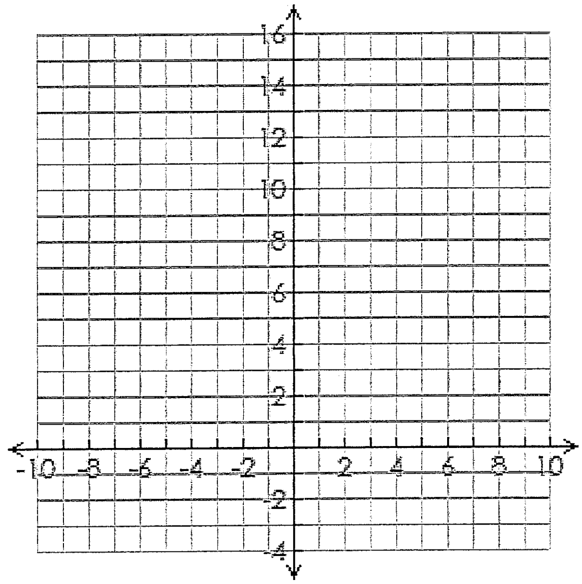
Interval(s) of Positive: ______________________________

Interval(s) of Negative: ______________________________

Min: ____

Max: ____

Average Rate of Change from [0, 2] = _______________



4. $y = -x^2 - 10x - 21$

Form: ____________________ a = _______ Opens: ______

Vertex: _____________ AOS: ____________

x-int: _________________ y-int: _________________

Domain: ______________ Range: ________________

End Behavior: As $x \to \infty, f(x) \to$ _________

As $x \to -\infty, f(x) \to$ _________

Interval of Increase: ______________________________

Interval of Decrease: ______________________________

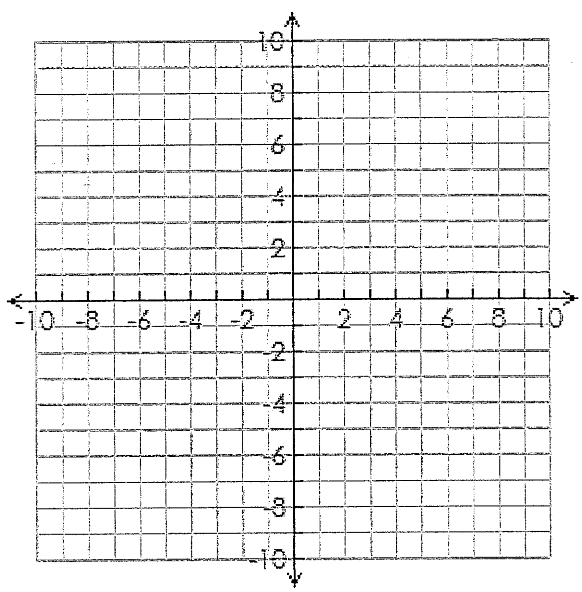
Interval(s) of Positive: ______________________________

Interval(s) of Negative: ______________________________

Min: ____

Max: ____

Average Rate of Change from [-6, -2] = _______________



For the following problems, either state the transformation that are occurring or write the equation based on the ransformations given.

5. $y = -2(x-4)^2 + 7$

Vertex: ________________ Horizontal translation: ________________________________

Vertical translation: ________________ Stretch/compress/reflect: ________________________________

6. $y = -\frac{1}{2}(x-3)^2 - 9$

Vertex: ________________ Horizontal translation: ________________________________

Vertical translation: ________________ Stretch/compress/reflect: ________________________________

7. $y = -(x+5)^2$

Vertex: ________________ Horizontal translation: ________________________________

Vertical translation: ________________ Stretch/compress/reflect: ________________________________

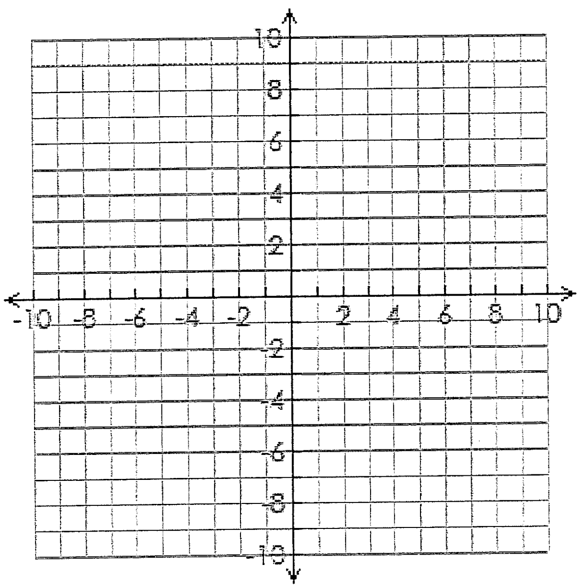
8. The function $y = x^2$, is translated 3 to the right and 4 down.

9. The function $y = x^2$ is reflected over the $x-$ axis and translated 8 up.

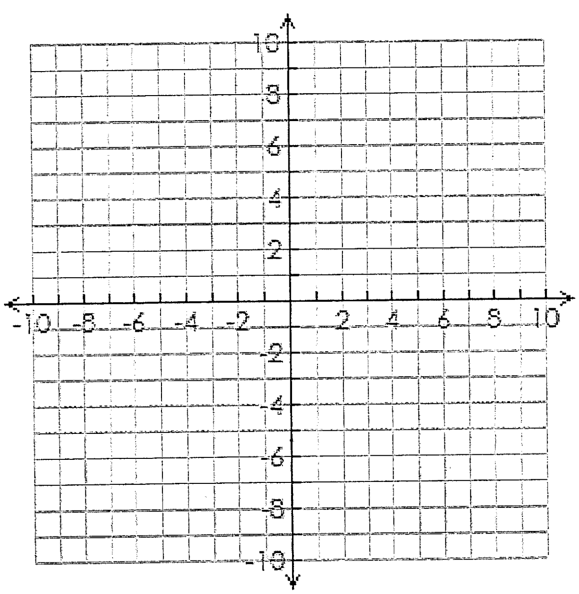
10. The function $y = x^2$ is stretched by a factor of 3 and translated 3 to the left.

For the following problems, sketch a graph of the inequality and write the solutions in interval notation.

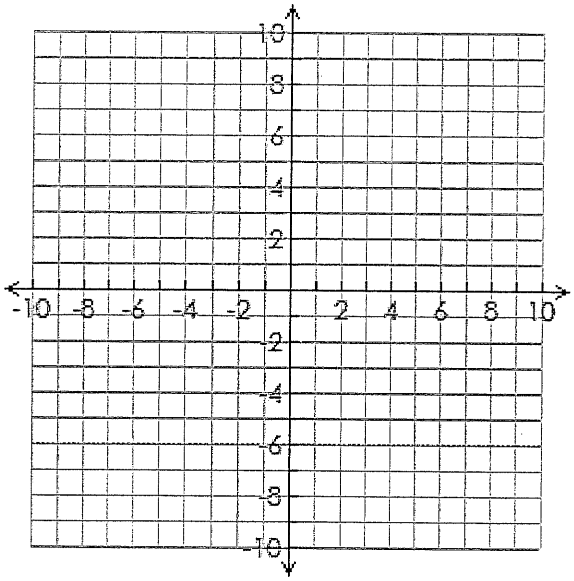
11. $y \geq -4x^2 - 24x - 32$ Solutions: ______________________



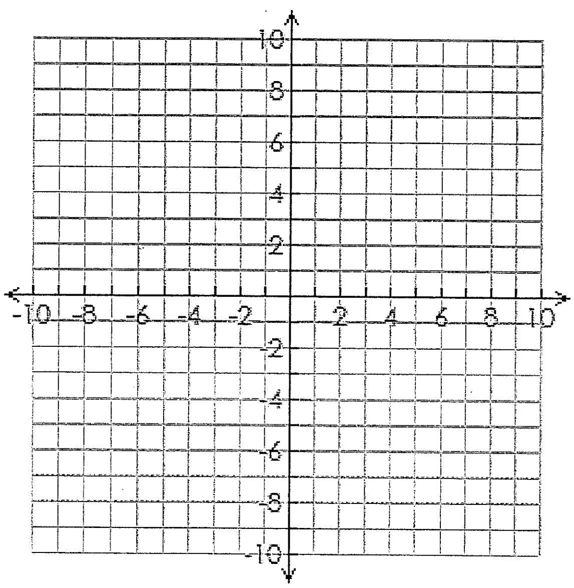
12. $y < -2x^2 + 4x$ Solutions: ______________________



13. $y \geq x^2 + 4x + 3$ Solutions: ____________________



14. $y < 2(x-7)(x+1)$ Solutions: ____________________



15. The product of two numbers is 640. Their difference is 12. Find the two numbers.

16. Jason jumped off of a cliff that is 480 feet high, into the ocean in Acapulco with initial velocity of 16 ft/s, while vacationing with some friends. His height as a function of time could be modeled by the function $h(t) = -16t^2 + v_i t + h_i$, where t is the time in seconds and h is the height in feet.

a. How long did it take for Jason to reach the water?

b. What was the highest point that Jason reached? (think about how to find the vertex of this parabola)

17. The length of a rectangle is 6 less than 3 times the width. The area of the rectangle is 144 square inches. Find the length and the width.

Key

1. **$y = x^2 - 4x - 12$**

$x = \frac{-b}{2a} = \frac{+4}{2(1)} = 2$

x	y
0	-12
1	-15
2	-16
3	-15
4	-12

Form: standard a = 1 Opens: up

Vertex: (2, -16) AOS: x=2

x-int: (-2,0), (6,0) y-int: (0,-12)

Domain: $(-\infty, \infty)$ Range: $[-16, \infty)$

End Behavior: As $x \to \infty, f(x) \to$ $+\infty$

As $x \to -\infty, f(x) \to$ $+\infty$

Interval of Increase: $(2, \infty)$

Interval of Decrease: $(-\infty, 2)$

Interval(s) of Positive: $(-\infty, -2) \cup (6, \infty)$

Interval(s) of Negative: $(-2, 6)$

Min: (2, -16) or -16

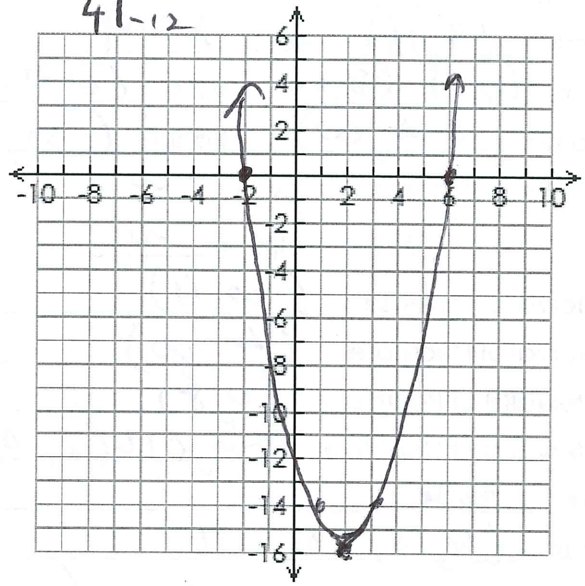
Max: none

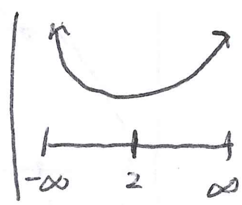
Average Rate of Change from [4, 6] = 6

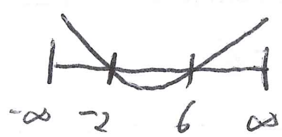
(4, -12)

(6, 0)

$\frac{-12-0}{4-6} = \frac{-12}{-2} = 6$







2. **$y = 2(x + 3)(x - 1)$**

Form: intercept / factored a = 2 Opens: up

Vertex: (-1, -8) AOS: x = -1

x-int: (-3,0) (1,0) y-int: (0,-6)

Domain: $(-\infty, \infty)$ Range: $[-8, \infty)$

End Behavior: As $x \to \infty, f(x) \to$ $+\infty$

As $x \to -\infty, f(x) \to$ $+\infty$

Interval of Increase: $(-1, \infty)$

Interval of Decrease: $(-\infty, -1)$

Interval(s) of Positive: $(-\infty, -3) \cup (1, \infty)$

Interval(s) of Negative: $(-3, 1)$

Min: (-1, -8) or -8

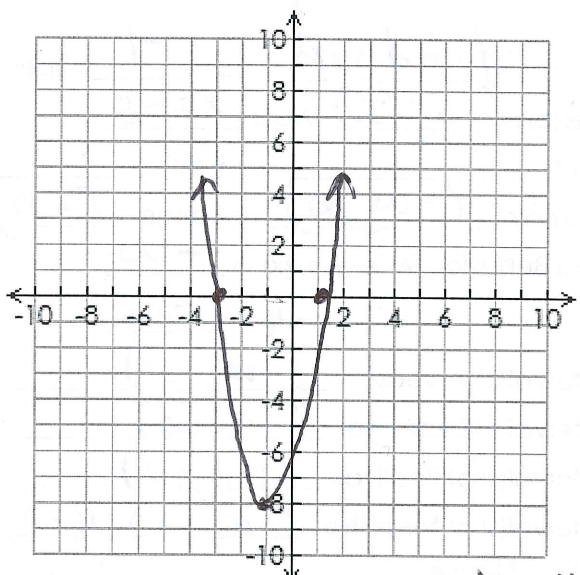
Max: none

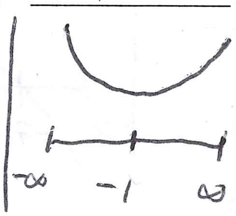
Average Rate of Change from [-4, 0] = -1

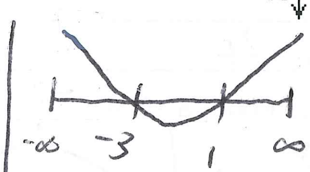
(-4, 10)

(0, -6)

$\frac{10-6}{-4-0} = \frac{4}{-4}$







x	y
-3	0
-2	-6
-1	-8
0	-6
1	0

Analytic Geometry Unit 5B Practice

x	y
2	12
3	15
4	16
5	15
6	12

3. $y = -(x-4)^2 + 16$

Form: vertex a = -1 Opens: down

Vertex: $(4,16)$ AOS: $x=4$

x-int: $(0,0)$ $(8,0)$ y-int: $(0,0)$

Domain: $(-\infty,\infty)$ Range: $(-\infty,16]$

End Behavior: As $x\to\infty, f(x)\to$ $-\infty$

As $x\to-\infty, f(x)\to$ $-\infty$

Interval of Increase: $(-\infty,4)$

Interval of Decrease: $(4,\infty)$

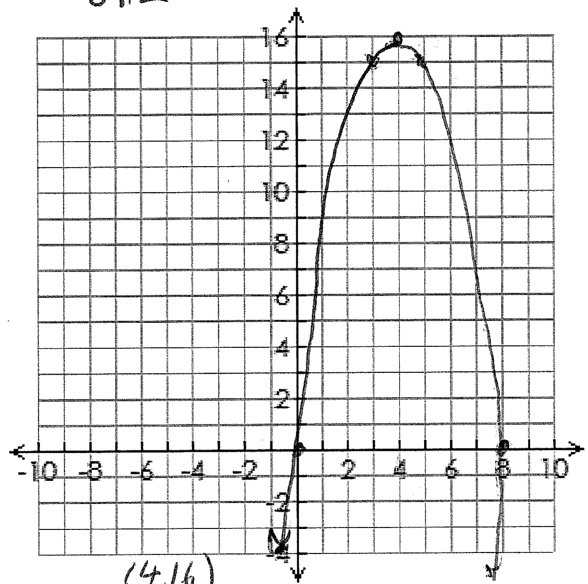
Interval(s) of Positive: $(0,8)$

Interval(s) of Negative: $(-\infty,0)\cup(8,\infty)$

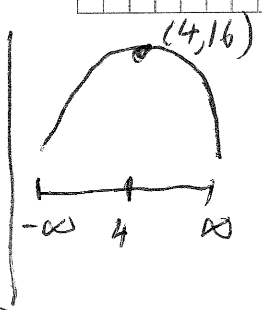
Min: none

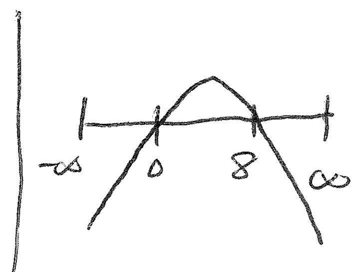
Max: $(4,16)$ or 16

Average Rate of Change from [0, 2] = 6



$(0,0)$ $(2,12)$ $\frac{12-0}{2-0}=6$





$x=\frac{-b}{2a}=\frac{10}{2(-1)}=-5$

4. $y = -x^2 - 10x - 21$

Form: standard a = -1 Opens: down

Vertex: $(-5,4)$ AOS: $x=-5$

x-int: y-int:

Domain: $(-\infty,\infty)$ Range: $(-\infty,-5]$

End Behavior: As $x\to\infty, f(x)\to$ $-\infty$

As $x\to-\infty, f(x)\to$ $-\infty$

Interval of Increase: $(-\infty,-5)$

Interval of Decrease: $(-5,\infty)$

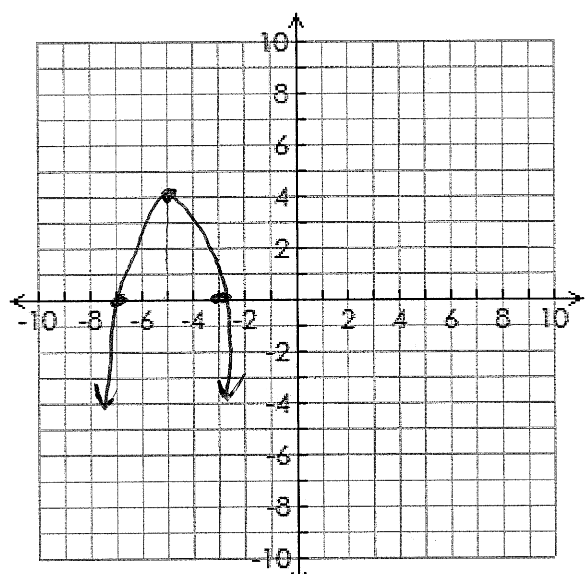
Interval(s) of Positive: $(-7,-3)$

Interval(s) of Negative: $(-\infty,-7)\cup(-3,\infty)$

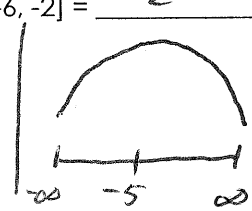
Min: none

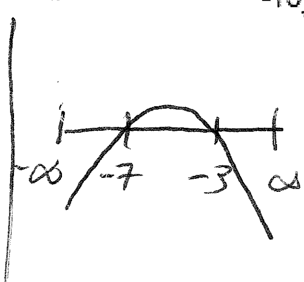
Max: $(-5,4)$ or 4

Average Rate of Change from [-6, -2] = -2



$(-6,3)$ $(-2,-5)$ $\frac{3+5}{-6+2}=\frac{8}{-4}=-2$





x	y
-7	0
-6	3
-5	4
-4	3
-3	0

Analytic Geometry Unit 5B Practice

For the following problems, either state the transformation that are occurring or write the equation based on the transformations given.

5. $y = -2(x-4)^2 + 7$

Vertex: $(4, 7)$ Horizontal translation: right 4 units

Vertical translation: up 7 units Stretch/compress/reflect: ① reflection ② stretch by factor of 2

6. $y = -\frac{1}{2}(x-3)^2 - 9$

Vertex: $(3, -9)$ Horizontal translation: right 3 units

Vertical translation: down 9 units Stretch/compress/reflect: ① reflection ② compress by factor of 1/2

7. $y = -(x+5)^2 + 0$

Vertex: $(-5, 0)$ Horizontal translation: left 5 units

Vertical translation: none Stretch/compress/reflect: ① reflection ② no stretch/compression

8. The function $y = x^2$, is translated 3 to the right and 4 down.

$$y = (x-3)^2 - 4$$

9. The function $y = x^2$ is reflected over the x – axis and translated 8 up.

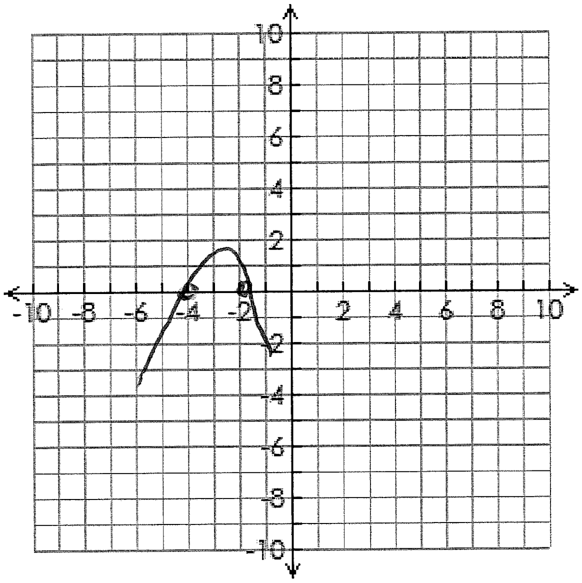
$$\boxed{y = -(x-0)^2 + 8 \text{ or } y = -(x)^2 + 8}$$

10. The function $y = x^2$ is stretched by a factor of 3 and translated 3 to the left.

$y = 3(x+3)^2 + 0$

For the following problems, sketch a graph of the inequality and write the solutions in interval notation.

11. $y \geq -4x^2 - 24x - 32$ Solutions: $[-4, -2]$

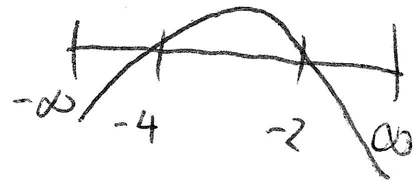


$-4x^2 - 24x - 32$

$-4(x^2 + 6x + 8)$

$-4(x+2)(x+4)$

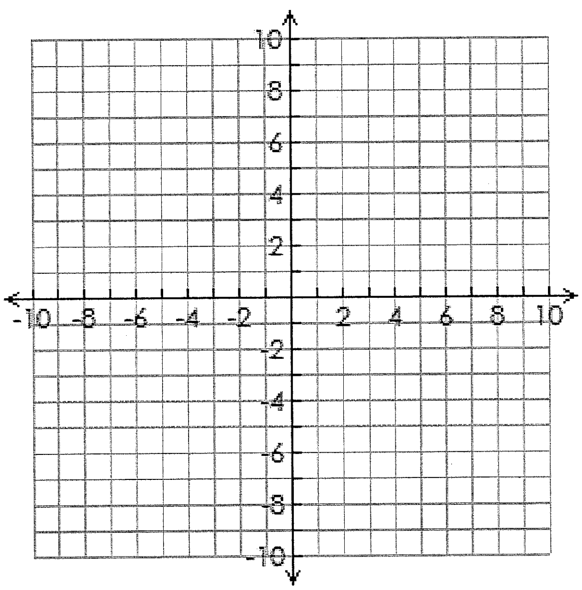
$x = -2, -4$



pos: $[-4, -2]$

neg: $(-\infty, -4] \cup [-2, \infty)$

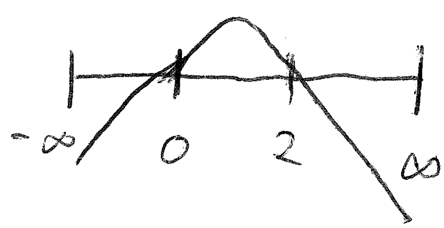
12. $y < -2x^2 + 4x$ Solutions: $(-\infty, 0) \cup (2, \infty)$



$-2x^2 + 4x$

$-2x(x-2)$

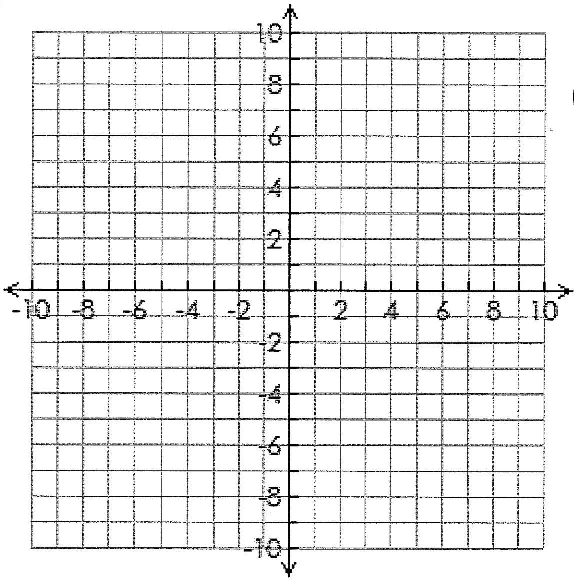
$x = 0, x = 2$



pos: $(0, 2)$

neg: $(-\infty, 0) \cup (2, \infty)$

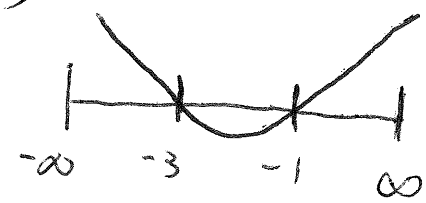
13. $y \geq x^2 + 4x + 3$ Solutions: $(-\infty, -3] \cup [-1, \infty)$



$x^2+4x+3=0$

$(x+3)(x+1)=0$

$x=-1, -3$

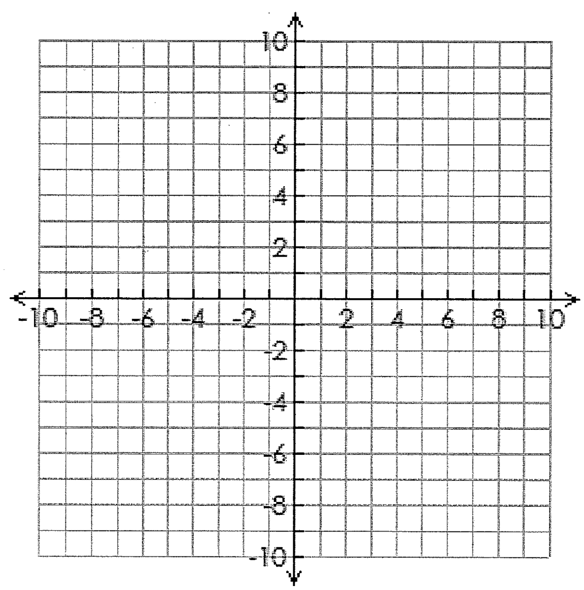


opens up

pos: $(-\infty, -3) \cup (-1, \infty)$

neg: $(-3, -1)$

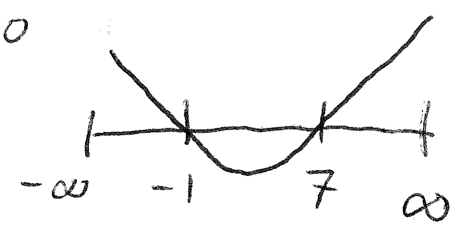
14. $y < 2(x-7)(x+1)$ Solutions: $(-1, 7)$



$2(x-7)(x+1)=0$

$x=7, x=-1$

opens up



pos: $(-\infty, -1) \cup (7, \infty)$

neg: $(-1, 7)$

15. The product of two numbers is 640. Their difference is 12. Find the two numbers.

$x \cdot y = 640$ $x - y = 12$

first #: x

2nd #: $x-12$

$x(x-12)=640$

$x^2-12x=640$

$x^2-12x-640=0$

$\frac{12 \pm \sqrt{12^2-4(1)(-640)}}{2(1)} = \frac{12 \pm \sqrt{2704}}{2}$

$\frac{12 \pm 52}{2} = \frac{12+52}{2}$ and $\frac{12-52}{2}$

$x=32$ $x=-20$

$\boxed{32 \text{ and } 20}$ $\boxed{-20 \text{ and } -32}$

16. Jason jumped off of a cliff that is 480 feet high, into the ocean in Acapulco with initial velocity of 16 ft/s, while vacationing with some friends. His height as a function of time could be modeled by the function $h(t) = -16t^2 + v_i t + h_i$, where t is the time in seconds and h is the height in feet.

a. How long did it take for Jason to reach the water?

$h(t) = 0$ $\quad 0 = -16t^2 + 16t + 480$

$V_i = 16$ $\quad 0 = -16(t^2 - t - 30)$

$h_i = 480$ $\quad 0 = -16(t+5)(t-6)$

extraneous solution $t = -5$, $\boxed{t = 6}$

$\boxed{t = 6 \text{ seconds}}$

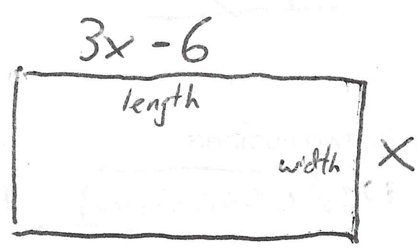
b. What was the highest point that Jason reached? (think about how to find the vertex of this parabola)

$\frac{-b}{2a} = \frac{-16}{2(-16)} = \frac{1}{2}$

$h(\frac{1}{2}) = -16(\frac{1}{2})^2 + 16(\frac{1}{2}) + 480$

$\boxed{h(\frac{1}{2}) = 484 \text{ ft.}}$

17. The length of a rectangle is 6 less than 3 times the width. The area of the rectangle is 144 square inches. Find the length and the width.



length × width = Area

$(3x - 6)(x) = 144$

$3x^2 - 6x = 144$

$3x^2 - 6x - 144 = 0$

$3(x^2 - 2x - 48) = 0$

$3(x - 8)(x + 6) = 0$

$x = 8, \ x = -6$

$\boxed{\text{width} = x = 8 \text{ in.}}$

$\boxed{\text{length} = 3x - 6 = 3(8) - 6 = 18 \text{ in.}}$