Key

Ch. 6 Unit Review AP Practice Problems (p.459-460)

1. The closed interval $[a, b]$ is partitioned into n subintervals each of width $\Delta x = \frac{b-a}{n}$. If u_i is any number in the ith subinterval, what is $\lim_{n\to\infty} \sum_{i=1}^{n} \sqrt[3]{u_i}\,\Delta x$?

(A) $\frac{3}{2}[b^{2/3} - a^{2/3}]$ $\boxed{\text{(B)}}$ $\frac{3}{4}[b^{4/3} - a^{4/3}]$

(C) $\frac{4}{3}[b^{4/3} - a^{4/3}]$ (D) $\frac{1}{3}\left[\frac{1}{b^{2/3}} - \frac{1}{a^{2/3}}\right]$

$\int_a^b \sqrt[3]{x}\,dx = \int x^{1/3}dx$

$\frac{x^{4/3}}{4/3} \to \frac{3}{4}x^{4/3}\Big]_a^b$

$\boxed{\frac{3}{4}b^{4/3} - \frac{3}{4}a^{4/3}}$

2. If $\int_1^8 f(x)\,dx = 3$, find $\int_1^8 f(9-x)\,dx$.

(A) -3 (B) 6 $\boxed{\text{(C) } 3}$ (D) 9

* u-substitution

$\int_1^8 f(9-x)dx$

$u = 9-x$

$\frac{du}{dx} = -1$

$dx = -1du$

$\int f(u)\cdot -1du$

$-\int f(u)du$

*convert bounds

If $x=1$, $u=9-x \to u=9-1=8$

If $x=8$, $u=9-x \to u=9-8=1$

$-\int_8^1 f(u)du =$

$-\int_1^8 -f(u)du =$

$\int_1^8 f(u)du =$ $\boxed{3}$

3. The area under the graph of the function $f(x) = \frac{2}{x}$, from $x = k$ to $x = 4k$, $k > 0$ is

(A) $\ln 8$ (B) $2\frac{\ln(4k)}{\ln k}$ $\boxed{\text{(C)}}$ $2\ln 4$ (D) $2\ln(4k)$

$\int_k^{4k} \frac{2}{x}dx \to 2\ln|x|\Big]_k^{4k} = 2\ln|4k| - 2\ln|k| \to 2\Big[\ln|4k| - \ln|k|\Big] = 2\ln\left|\frac{4k}{k}\right| = \boxed{2\ln 4}$

4. If $\int_{-3}^{4} f(x)\,dx = 8$ and $\int_6^4 f(x)\,dx = -6$, what is $\int_{-3}^{6} f(x)\,dx$?

(A) -14 (B) 2 $\boxed{\text{(C)}}$ 14 (D) 16

$\int_4^6 f(x)dx = 6$

$\int_{-3}^{6} f(x)dx = \int_{-3}^{4} f(x)dx + \int_4^6 f(x)dx$

$= 8 + (6) = \boxed{14}$

5. Find $\int_{-2}^{6} f(x)\,dx$ when $f(x) = \begin{cases} -x & \text{if } -2 \le x < 2 \\ x+3 & \text{if } 2 \le x \le 6 \end{cases}$.

(A) 24 $\quad$ $\boxed{\text{(B)}}$ 28 $\quad$ (C) 32 $\quad$ (D) 44

$\int_{-2}^{2} -x\,dx + \int_{2}^{6} x+3\,dx$

$\rightarrow -\frac{x^2}{2}\Big]_{-2}^{2} = -\frac{4}{2} - \left(-\frac{4}{2}\right) =$

$\rightarrow -2 + 2 = 0$

$\int_2^6 x+3\,dx \rightarrow \frac{x^2}{2} + 3x\Big]_2^6$

$\frac{36}{2} + 3(6) - \left(\frac{4}{2} + 3(2)\right)$

$18 + 18 - 8$

$36 - 8 = \boxed{28}$

6. An object is moving along the x-axis. If its velocity v at time t (in minutes) is $v(t) = t^2 - 2t$ (in feet per minute), what is the total distance the object travels between $t = 0$ and $t = 4$ minutes?

(A) 0 ft $\quad$ (B) $\frac{16}{3}$ ft $\quad$ $\boxed{\text{(C)}}$ 8 ft $\quad$ (D) $\frac{40}{3}$ ft

* total Distance is $\int_0^4 |v(t)|\,dt$

$v(t) = t^2 - 2t$

$v(t) = t(t-2)$

2 4

$\rightarrow \int_0^2 (t^2-2t)\,dt + \int_2^4 t^2-2t\,dt$

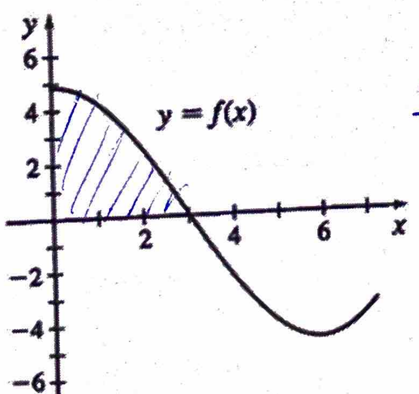
$\frac{t^3}{3} - \frac{2t^2}{2}\Big]_0^2 = \frac{8}{3} - 4 \rightarrow \left|\frac{8}{3} - \frac{12}{3}\right| = \boxed{\frac{4}{3}}$

$\frac{t^3}{3} - \frac{2t^2}{2}\Big]_2^4 = \frac{64}{3} - 16 - \left(\frac{8}{3} - 4\right)$

$= \frac{56}{3} - 12 \rightarrow \frac{56}{3} - \frac{36}{3} = \boxed{\frac{20}{3}}$

$\frac{4}{3} + \frac{20}{3} = \frac{24}{3} = \boxed{8}$

7. The graph of a function f that is differentiable is shown below. If $F(x) = \int_0^x f(t)\,dt$, which of the following is true?



$F'(x) = \frac{d}{dx}\int_0^x f(t)\,dt = f(x)$

$F''(x) = f'(x)$

$F(3) = \int_0^3 f(t)\,dt \approx 8 > 0$

$F'(3) = f(3) = 0$

$F''(3) = f'(3) \approx -3 < 0$

(A) $F(3) < F'(3) < F''(3)$ $\quad$ (B) $F'(3) < F''(3) < F(3)$

(C) $F''(3) < F(3) < F'(3)$ $\quad$ $\boxed{\text{(D)}}$ $F''(3) < F'(3) < F(3)$

8. If $\int_0^6 f(3x+1)\,dx = 9$, then

(A) $\int_0^6 f(u)\,du = 3$ $\quad$ (B) $\int_1^{19} f(u)\,du = 9$

(C) $\int_0^{18} f(u)\,du = 27$ $\quad$ $\boxed{\text{(D)}}$ $\int_1^{19} f(u)\,du = 27$

$\int_0^6 f(3x+1)\,dx$

$u = 3x+1$

$\frac{du}{dx} = 3$

$dx = \frac{du}{3}$

$\int f(u)\frac{du}{3} \rightarrow \frac{1}{3}\int f(u)\,du$

* convert bounds:

If $x = 0$, $u = 3(0)+1 = 1$

If $x = 6$, $u = 3(6)+1 = 19$

$\frac{1}{3}\int_1^{19} f(u)\,du = \int_0^6 f(3x+1)\,dx$

$\int_1^{19} f(u)\,du = 3 \int_0^6 f(3x+1)\,dx$

$3 \cdot (9) = \boxed{27}$

19. $\int x^3 e^{-x^4}\,dx =$

$\boxed{\text{(A)}}$ $-\frac{1}{4}e^{-x^4}+C$ (B) $-\frac{1}{4}e^{x}+C$

(C) $\frac{x^4}{4}e^{-4x^3}+C$ (D) $\frac{x^4}{4}e^{-x^4}+C$

$\int x^3 \cdot e^{-x^4}\,dx$

$u=-x^4$

$\frac{du}{dx}=-4x^3$

$dx=\frac{du}{-4x^3}$

$\int \cancel{x^3}\cdot e^u \cdot \frac{du}{-4\cancel{x^3}}$

$-\frac{1}{4}\int e^u\,du$

$-\frac{1}{4}e^u+C$

$\boxed{-\frac{1}{4}e^{-x^4}+C}$

---

10. If at every point (x, y) on the graph of a function f, the slope of the tangent line is given by $y = 3-4x$ and if the point $(2, 3)$ is on the graph of f, then

(A) $f(x)=-5x+7$ (B) $f(x)=-2x^2+3x-11$

(C) $f(x)=-2x^2+3x$ $\boxed{\text{(D)}}$ $f(x)=-2x^2+3x+5$

* this is describing the derivative

$f'(x)=3-4x$

$f(x)=\int 3-4x\,dx \rightarrow 3x-\frac{4x^2}{2}+C$

$f(x)=3x-2x^2+C$ ← plug in point $(2,3)$

$f(2)=3(2)-2(2)^2+C$

$3=6-4+c$

$3=2+C$

$\boxed{1=C}$

$\boxed{f(x)=3x-2x^2+1}$

---

11. $\int \frac{x^2-3x+2\sqrt{x}-1}{x}\,dx =$

$\boxed{\text{(A)}}$ $\frac{1}{2}x^2-3x+4x^{1/2}-\ln|x|+C$

(B) $\frac{1}{3}x^3-\frac{3}{2}x^2+\frac{4}{3}x^{3/2}-\ln|x|+C$

(C) $\frac{1}{2}x^2-3x+\frac{6}{5}x^{5/2}-\ln|x|+C$

(D) $\frac{1}{2}x^2-3x+x^{1/2}-\ln|x|+C$

* split into individual fractions

$\int \frac{x^2}{x}-\frac{3x}{x}+\frac{2\sqrt{x}}{x}-\frac{1}{x}\,dx$

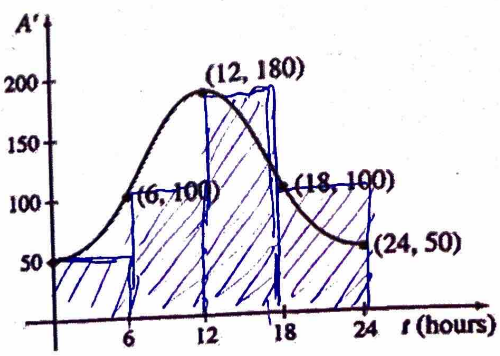
$\int x-3+2x^{-1/2}-\frac{1}{x}\,dx$

$\frac{x^2}{2}-3x+\frac{2x^{1/2}}{1/2}-\ln|x|+c$

$\boxed{\frac{x^2}{2}-3x+4x^{1/2}-\ln|x|+C}$

---

12. On a typical day, a dam releases water at a rate of $\frac{dA}{dt}$ (hundred thousand gallons per hour) as shown in the graph. Use a Left Riemann sum with four equal subintervals to approximate the total amount A (in hundreds of thousands of gallons) of water released in a day.



(A) 1720 $\boxed{\text{(B)}}$ 2580 (C) 2760 (D) 3660

$\int_0^{24} A'(t)\,dt \approx 6\cdot A'(0)+6\cdot A'(6)+6\cdot A'(12)+6A'(18)$

$=6(50)+6(100)+6(180)+6(100)$

$=300+600+1080+600$

$=900+1680$

$=\boxed{2580}$

Free Response Questions

13. For the function $f(x) = \dfrac{\sin x}{x^2 + 1}$

(a) Find the derivative of f when $x = 1$.

(b) Find the area under the graph of f from 0 to π.

(c) What is the average value of f over the closed interval $[0, \pi]$?

*quotient Rule

a) $f'(x) = \dfrac{\overbrace{\cos x}^{f'}\,\overbrace{(x^2+1)}^{g} - \overbrace{\sin x}^{f}\,\overbrace{(2x)}^{g'}}{\underbrace{(x^2+1)^2}_{g^2}}$

$f'(x) = \dfrac{x^2\cos x + \cos x - 2x\sin x}{(x^2+1)^2}$

$f'(1) = \dfrac{1\cos 1 + \cos 1 - 2\sin 1}{(1+1)^2}$

$f'(1) = \dfrac{2\cos(1) - 2\sin(1)}{2^2}$

$f'(1) = \dfrac{2(\cos(1) - \sin(1))}{4} \rightarrow$ a) $\boxed{\dfrac{\cos(1) - \sin(1)}{2}}$

a) $f'(1) = \dfrac{\cos 1 - \sin 1}{2}$

b) $\int_0^{\pi} f(x)dx \approx 0.716$

c) $\frac{1}{\pi}\int_0^{\pi} f(x)dx = 0.228$

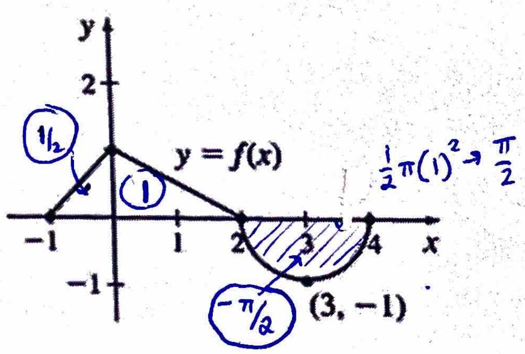
use calculator

b) $\int_0^{\pi} f(x)dx \approx \boxed{0.716}$

c) $\frac{1}{\pi - 0}\int_0^{\pi} f(x)dx$

$= \frac{1}{\pi}(0.716) = \boxed{0.228}$

14. Use the graph of the function f shown below to answer the questions.



(a) Use a Right Riemann sum with five subintervals of equal width to approximate $\int_{-1}^{4} f(x)\, dx$.

(b) Write $\int_{-1}^{4} f(x)\, dx$ as a sum of three integrals using properties of definite integrals.

(c) Find $\int_{-1}^{4} f(x)\, dx$ using geometry.

~~(d)~~ Find $\int_{-1}^{4} f(x)\, dx$ using technology.

(e) Explain why $\int_{-1}^{2} f(x)\, dx > \int_{-1}^{4} f(x)\, dx$.

e) Since $\int_2^4 f(x)dx < 0$, then $\int_{-1}^{2} f(x)dx > \int_{-1}^{4} f(x)dx$

a) Right Riemann Sum: $\int_{-1}^{4} f(x)dx = (1)(1) + 1(\tfrac{1}{2}) + 1(0) + 1(-1) + 1(0) = \boxed{\tfrac{1}{2}}$

b) $\int_{-1}^{4} f(x)dx = \int_{-1}^{0} x+1\,dx + \int_{0}^{2} -\tfrac{1}{2}x + 1\,dx + \int_{2}^{4} f(x)dx$

c) $\int_{-1}^{4} f(x)dx = \frac{1}{2} + 1 - \frac{\pi}{2} = \boxed{\frac{3}{2} - \frac{\pi}{2}}$