AP Calculus Ch. 6 Test Review WS 1 (Non-Calculator)

1. $\int \frac{x^2+x+1}{\sqrt{x}}\,dx$

2. $\int 2x\sqrt{1-3x^2}\,dx$

3. $\int 5\sqrt{x}\,(4-3x^2)\,dx$

4. $\int 5x\sec^2(3x^2)\,dx$

5. $\int x^2\sqrt{7-x}\,dx$

6. $\int_1^2 x(1-2x^2)^3\,dx$

7. Find $f'(x)$ if $f(x) = \int_{2x^3}^{\sqrt{x}} \sqrt{1-t^2}\,dt$

8. Find $f'(x)$ if $f(x) = \int_{3x^2}^{\pi} \sqrt{1-t^2}\,dt$

9. $\int_{-5}^{6} |x+2|\,dx$

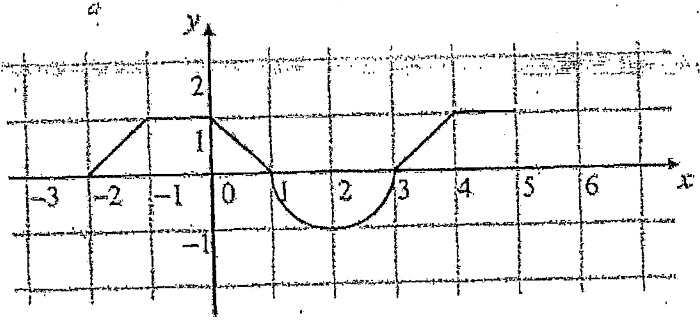
10. $\int_{-2}^{7} |x-4|\,dx$

11. If $a(t) = 12t^2 + 18t - 4$ and $x(1) = 3$ and $v(-1) = 9$, find the below:

a) Find the specific function for v(t)

b) Find the specific function for x(t)

12. The graph of f below consists of a semicircle, triangles, and squares. Find the average value of f on the interval [-2, 5]



AP Calculus Ch 6 Test Review WS (Non-Calculator)

Solution Key

1. $\int \frac{x^2+x+1}{\sqrt{x}}\,dx$ $\quad \int \frac{x^2}{x^{1/2}}+\frac{x}{x^{1/2}}+\frac{1}{x^{1/2}}\,dx$

$\int x^{3/2}+x^{1/2}+x^{-1/2}\,dx$

$=\frac{x^{5/2}}{5/2}+\frac{x^{3/2}}{3/2}+\frac{x^{1/2}}{1/2}+C$

$=\boxed{\frac{2}{5}x^{5/2}+\frac{2}{3}x^{3/2}+2x^{1/2}+C}$

2. $\int 2x\sqrt{1-3x^2}\,dx = \int 2x(1-3x^2)^{1/2}dx$

$u=1-3x^2$

$\frac{du}{dx}=-6x$

$dx=\frac{du}{-6x}$

$\int 2\cancel{x}\cdot u^{1/2}\cdot\frac{du}{-6\cancel{x}}$

$-\frac{1}{3}\int u^{1/2}du$

$=-\frac{1}{3}\left(\frac{u^{3/2}}{3/2}\right)=-\frac{1}{3}\left(\frac{2}{3}\right)u^{3/2}+C$

$=\boxed{-\frac{2}{9}(1-3x^2)^{3/2}+C}$

3. $\int 5\sqrt{x}\,(4-3x^2)\,dx = \int 5x^{1/2}(4-3x^2)dx$

$=\int 20x^{1/2}-15x^{5/2}dx$

$=20\left(\frac{x^{3/2}}{3/2}\right)-15\left(\frac{x^{7/2}}{7/2}\right)+C$

$=20\cdot\frac{2}{3}x^{3/2}-15\cdot\frac{2}{7}x^{7/2}+C$

$=\boxed{\frac{40}{3}x^{3/2}-\frac{30}{7}x^{7/2}+C}$

4. $\int 5x\sec^2(3x^2)\,dx$

$u=3x^2$

$\frac{du}{dx}=6x$

$dx=\frac{du}{6x}$

$\int 5\cancel{x}\cdot\sec^2(u)\cdot\frac{du}{6\cancel{x}}$

$\frac{5}{6}\int\sec^2u\,du$

$\frac{5}{6}\tan u+C$

$=\boxed{\frac{5}{6}\tan(3x^2)+C}$

5. $\int x^2\sqrt{7-x}\,dx = \int x^2(7-x)^{1/2}dx$

$u=7-x \rightarrow x=7-u$

$\frac{du}{dx}=-1$

$dx=-du$

$\int x^2\cdot u^{1/2}(-du)$

$\int (7-u)^2u^{1/2}(-du)$

$\int -u^{1/2}(49-14u+u^2)du$

$\int -49u^{1/2}+14u^{3/2}-u^{5/2}du$

$-49\left(\frac{u^{3/2}}{3/2}\right)+14\left(\frac{u^{5/2}}{5/2}\right)-\frac{u^{7/2}}{7/2}+C$

$-49\cdot\frac{2}{3}u^{3/2}+14\cdot\frac{2}{5}u^{5/2}-\frac{2}{7}u^{7/2}+C$

$\boxed{-\frac{98}{3}(7-x)^{3/2}+\frac{28}{5}(7-x)^{5/2}-\frac{2}{7}(7-x)^{7/2}+C}$

6. $\int_1^2 x(1-2x^2)^3dx$ $\quad \int \cancel{x}\cdot u^3\cdot\frac{du}{-4\cancel{x}}=-\frac{1}{4}\int u^3du$

$u=1-2x^2$

$\frac{du}{dx}=-4x$

$dx=\frac{du}{-4x}$

if $x=1$, $u=1-2(1)^2=-1$

if $x=2$, $u=1-2(2)^3=-7$

$=-\frac{1}{4}\frac{u^4}{4}\Big]_{-1}^{-7}=-\frac{1}{16}(-7)^4-\left(-\frac{1}{16}(1)\right)$

$=\frac{-2401}{16}+\frac{1}{16}$

$=\frac{-2400}{16}=\boxed{-150}$

$\frac{d}{dx}\int_{g(x)}^{p(x)} f(t)dt = f(p(x))\cdot p'(x) - f(g(x))\cdot g'(x)$

Ch. 4 Review WS #2 (continued)

2/2

$\boxed{SFTC}$

$\sqrt{x} \rightarrow x^{1/2}$

7. Find $f'(x)$ if $f(x) = \int_{2x^3}^{\sqrt{x}} \sqrt{1-t^2}\,dt$

$f'(x) = \sqrt{1-(\sqrt{x})^2}\cdot\frac{1}{2}x^{-1/2} - \sqrt{1-(2x^3)^2}\cdot 6x^2$

$= \boxed{\frac{\sqrt{1-x}}{2\sqrt{x}} - 6x^2\sqrt{1-4x^6}}$

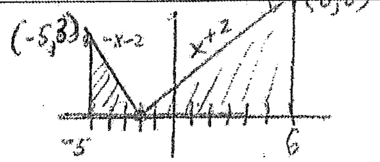
$\boxed{SFTC}$

$\frac{d}{dx}\int_{a}^{p(x)} f(t)dt = f(p(x))\cdot p'(x)$

8. Find $f'(x)$ if $f(x) = \int_{3x^2}^{\pi} \sqrt{1-t^2}\,dt = \int_{\pi}^{3x^2} -\sqrt{1-t^2}\,dt$

$f'(x) = -\sqrt{1-(3x^2)^2}\cdot 6x = \boxed{-6x\sqrt{1-9x^4}}$

9. $\int_{-5}^{6} |x+2|\,dx$



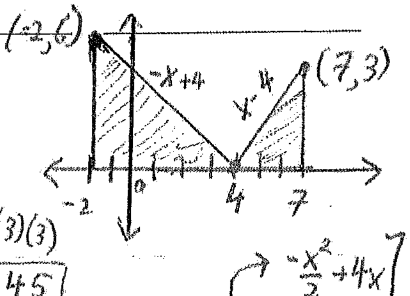
$\frac{1}{2}bh = \frac{1}{2}(3)(3) + \frac{1}{2}(8)(8) = \frac{9}{2} + \frac{64}{2} = \boxed{\frac{73}{2}}$

OR

$\int_{-5}^{-2} -x-2\,dx + \int_{-2}^{6} x+2\,dx = -\frac{x^2}{2} - 2x\Big]_{-5}^{-2} = \frac{9}{2}$

$+ \frac{x^2}{2} + 2x\Big]_{-2}^{6} = 64/2 \rightarrow \frac{9}{2} + \frac{64}{2} = \boxed{\frac{73}{2}}$

10. $\int_{-2}^{7} |x-4|\,dx$



$\frac{1}{2}bh = \frac{1}{2}(6)(6) + \frac{1}{2}(3)(3)$

$= \frac{36}{2} + \frac{9}{2} = \boxed{\frac{45}{2}}$

OR

$\int_{-2}^{4} -x+4\,dx + \int_{4}^{7} x-4\,dx$

$-\frac{x^2}{2} + 4x\Big]_{-2}^{4} = 18$

$\frac{x^2}{2} - 4x\Big]_{4}^{7} = 9/2$

$18 + 9/2 = \boxed{\frac{45}{2}}$

11. If a(t) = 12t^2 + 18t – 4 and x(1) = 3 and v(-1) = 9, find the below:

$) = \int a(t)\,dt = \int 12t^2 + 18t - 4\,dt = \frac{12t^3}{3} + \frac{18t^2}{2} - 4t + C$

$v(t) = 4t^3 + 9t^2 - 4t + C$

$9 = 4(-1)^3 + 9(-1)^2 - 4(-1) + C$

$9 = -4 + 9 + 4 + C$

$0 = C$

$v(t) = 4t^3 + 9t^2 - 4t$

$x(t) = \int v(t)\,dt = \int 4t^3 + 9t^2 - 4t\,dt$

$x(t) = \frac{4t^4}{4} + \frac{9t^3}{3} - \frac{4t^2}{2} + K$

$x(t) = t^4 + 3t^3 - 2t^2 + K$

a) Find the specific function for v(t)

$\boxed{v(t) = 4t^3 + 9t^2 - 4t}$

b) Find the specific function for x(t)

$3 = 1^4 + 3(1)^3 - 2(1)^2 + k$

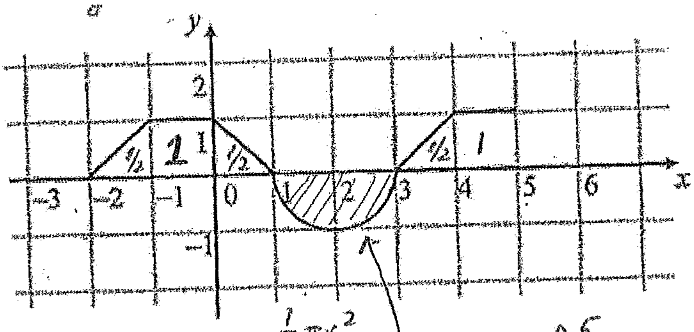
$3 = 1 + 3 - 2 + k$

$3 = 4 - 2 + k$

$1 = k$

$\boxed{x(t) = t^4 + 3t^3 - 2t^2 + 1}$

12. The graph of f below consists of a semicircle, triangles, and squares. Find the average value of f on the interval [-2, 5]



$= \frac{1}{2}\pi r^2$

$= \frac{1}{2}\pi(1) = \boxed{-\pi/2}$

$\text{Avg. value} = \frac{1}{b-a}\int_a^b f(x)\,dx = \frac{1}{5-{-2}}\int_{-2}^{5} f(x)\,dx$

$= \frac{1}{7}\left(\frac{7-\pi}{2}\right)$

$\boxed{\text{Avg. value} = \frac{7-\pi}{14}}$

or $\frac{1}{2} - \frac{\pi}{14}$

$\int_{-2}^{5} f(x)\,dx = \frac{1}{2} + 1 + \frac{1}{2} - \pi/2 + \frac{1}{2} + 1 = \frac{7}{2} - \frac{\pi}{2} = \frac{7-\pi}{2}$

A.P. Calculus WS # 2

Review Integral Rules:

$\int u^n \, du =$

$\int \frac{1}{u} du =$

$\int e^u \, du =$

$\int \tan u \, du =$

$\int \cot u \, du =$

$\int \sec u \, du =$

$\int \csc u \, du =$

$\int a^u \, du =$

1. $\int \frac{dx}{2x+3}$

2. $\int \frac{x}{4x^2+1} dx$

3. $\int \frac{2x-5}{x} dx$

4. $\int \frac{x}{x+1} dx$

5. $\int \frac{(\ln x)^2 \, dx}{x}$

6. $\int \frac{dx}{\sqrt{x}\left(1+\sqrt{x}\right)}$

7. $\int \frac{x^2-4x+8}{x-3}dx$

8. $\int \frac{x^3-5x^2+x-2}{x+1}dx$

9. $\int \frac{dx}{(2x+3)^2}$

10. $\int 5^{\sec x}\sec x\tan x dx$

11. $\int_0^{\sqrt{2}} xe^{-\frac{1}{2}x^2}\,dx$

12. $\int_3^4 e^{3-x}dx$

13. $\int (\tan x+\sec x+\cot x+\csc x)dx$

14. $\int (5-2x)^2\, 7^{(5\text{-}2x)^3}\,dx$

A.P. Calculus **Review #2 key...**

Key

Review Integral Rules:

$\int u^n du = \frac{u^{n+1}}{n+1} + C$	$\int \frac{1}{u} du = \ln\lvert u \rvert + c$	$\int e^u du = e^u + C$
$\int \tan u \, du = -\ln\lvert \cos u \rvert + c$	$\int \cot u \, du = \ln\lvert \sin u \rvert + c$	$\int \sec u \, du = \ln\lvert \sec u + \tan u \rvert + C$
$\int \csc u \, du = -\ln\lvert \csc u + \cot u \rvert + c$	$\int a^u du = \frac{1}{\ln a} \cdot a^u + C$	

1. $\int \frac{dx}{2x+3}$

$u = 2x+3$

$\frac{du}{dx} = 2$

$dx = \frac{du}{2}$

$\int \frac{1}{u} \cdot \frac{du}{2} = \frac{1}{2}\int \frac{1}{u} du$

$\frac{1}{2}\ln\lvert u \rvert + c$

$\boxed{\frac{1}{2}\ln\lvert 2x+3 \rvert + C}$

2. $\int \frac{x}{4x^2+1} dx$

$u = 4x^2+1$

$\frac{du}{dx} = 8x$

$dx = \frac{du}{8x}$

$\int \frac{\cancel{x}}{u} \cdot \frac{du}{8\cancel{x}} = \frac{1}{8}\int \frac{1}{u} du$

$\frac{1}{8}\ln\lvert u \rvert + c$

$\boxed{\frac{1}{8}\ln\lvert 4x^2+1 \rvert + c}$

3. $\int \frac{2x-5}{x} dx$

$\int (2x-5)x^{-1} dx$

$\int 2 - 5x^{-1} dx = \int 2dx - 5\int \frac{1}{x} dx$

$\boxed{2x - 5\ln\lvert x \rvert + c}$

4. $\int \frac{x}{x+1} dx$

$u = x+1$

$\frac{du}{dx} = 1$

$dx = du$

$\int \frac{x}{u} \cdot du$

$x = u-1$

$\int \frac{u-1}{u} du$

$\int 1 - \frac{1}{u} du$

$u - \ln\lvert u \rvert + c$

$x+1 - \ln\lvert x+1 \rvert + c$

$x - \ln\lvert x+1 \rvert + 1 + c$

$\boxed{x - \ln\lvert x+1 \rvert + C}$

5. $\int \frac{(\ln x)^2 dx}{x}$

$u = \ln x$

$\frac{du}{dx} = \frac{1}{x}$

$dx = x du$

$\int \frac{u^2}{\cancel{x}} \cdot \cancel{x} du = \int u^2 du$

$\frac{u^3}{3} + C = \boxed{\frac{(\ln x)^3}{3} + C}$

6. $\int \frac{dx}{\sqrt{x}(1+\sqrt{x})}$

$u = 1+\sqrt{x}$

$u = 1 + x^{1/2}$

$\frac{du}{dx} = \frac{1}{2}x^{-1/2}$

$\frac{du}{dx} = \frac{1}{2\sqrt{x}}$

$dx = 2\sqrt{x} du$

$\int \frac{2\cancel{\sqrt{x}} du}{\cancel{\sqrt{x}} \cdot u}$

$2\int \frac{1}{u} du$

$2\ln\lvert u \rvert + c$

$\boxed{2\ln\lvert 1+\sqrt{x} \rvert + C}$

7. $\int \frac{x^2-4x+8}{x-3}dx$

$x-1+\frac{5}{x-3}$

$x-3\,\overline{)\,x^2-4x+8}$

$\ominus x^2 \oplus 3x$

$-1x+8$

$\oplus 1x \mp 3$

5

OR

$$3 \;\Big|\; \begin{array}{ccc} 1 & -4 & 8 \\ & 3 & -3 \\ \hline 1 & -1 & 5 \end{array}$$

$x-1+\frac{5}{x-3}$

$\int x-1+\frac{5}{x-3}dx$

$u=x-3$

$\frac{du}{dx}=1$

$dx=du$

$5\int\frac{1}{u}du$

$=5\ln|u|$

$$\boxed{\frac{x^2}{2}-x+5\ln|x-3|+C}$$

8. $\int \frac{x^3-5x^2+x-2}{x+1}dx$

$x^2-6x+7-\frac{9}{x+1}$

$x+1\,\overline{)\,x^3-5x^2+x-2}$

$\ominus x^3 \overset{\ominus}{+} x^2$

$-6x^2+x-2$

$\oplus 6x^2 \oplus 6x$

$7x-2$

$\ominus 7x \overset{\ominus}{+} 7$

-9

OR

$$-1 \;\Big|\; \begin{array}{cccc} 1 & -5 & 1 & -2 \\ \downarrow & -1 & 6 & -7 \\ \hline 1 & -6 & 7 & \boxed{-9} \end{array}$$

$\int x^2-6x+7-\frac{9}{x+1}dx$

$u=x+1$

$\frac{du}{dx}=1$

$dx=du$

$-9\int\frac{1}{u}du$

$\frac{x^3}{3}-\frac{6x^2}{2}+7x-9\ln|u|$

$$\boxed{\frac{x^3}{3}-3x^2+7x-9\ln|x+1|+C}$$

9. $\int \frac{dx}{(2x+3)^2}$

$\int \frac{1}{(2x+3)^2}dx$

$u=2x+3$

$\frac{du}{dx}=2$

$dx=\frac{du}{2}$

$\int\frac{1}{u^2}\cdot\frac{du}{2}$

$\frac{1}{2}\int\frac{1}{u^2}du$

$\frac{1}{2}\int u^{-2}du$

$\frac{1}{2}\frac{u^{-1}}{-1}+C$

$\frac{-1}{2u}+C=\boxed{\frac{-1}{2(2x+3)}+C}$

10. $\int 5^{\sec x}\sec x\tan x dx$

$u=\sec x$

$\frac{du}{dx}=\sec x\tan x$

$dx=\frac{du}{\sec x\tan x}$

$\int 5^u\cdot\cancel{\sec x\tan x}\cdot\frac{du}{\cancel{\sec x\tan x}}$

$\int 5^u du=\frac{5^u}{\ln 5}=\boxed{\frac{5^{\sec x}}{\ln 5}+C}$

11. $\int_0^{\sqrt{2}} xe^{-\frac{1}{2}x^2}dx$

If $x=0$, $u=-\frac{1}{2}(0)=0$

If $x=\sqrt{2}$, $u=-\frac{1}{2}(\sqrt{2})^2=-1$

$u=-\frac{1}{2}x^2$

$\frac{du}{dx}=-x$

$dx=\frac{du}{-x}$

$\int\cancel{x}e^u\cdot\frac{du}{-\cancel{x}}$

$-\int e^u du$

$=-e^u\Big]_0^{-1}=-e^{-1}-(-e^0)$

$=\boxed{-\frac{1}{e}+1}$

12. $\int_3^4 e^{3-x}dx$

If $x=3$, $u=3-3=0$

If $x=4$, $u=3-4=-1$

$u=3-x$

$\frac{du}{dx}=-1$

$dx=-du$

$\int e^u(-du)=-e^u\Big]_0^{-1}=-e^{-1}-(-e^0)$

$=\boxed{-\frac{1}{e}+1}$

13. $\int(\tan x+\sec x+\cot x+\csc x)dx$

$$\boxed{-\ln|\cos x|+\ln|\sec x+\tan x|+\ln|\sin x|-\ln|\csc x+\cot x|+C}$$

14. $\int(5-2x)^2\,7^{(5-2x)^3}dx$

$u=(5-2x)^3$

$\frac{du}{dx}=3(5-2x)^2(-2)$

$dx=\frac{du}{-6(5-2x)^2}$

$\int\cancel{(5-2x)^2}\cdot 7^u\cdot\frac{du}{-6\cancel{(5-2x)^2}}$

$-\frac{1}{6}\int 7^u du$

$-\frac{1}{6}\cdot\frac{1}{\ln 7}\cdot 7^u+C$

$$\boxed{\frac{-7^{(5-2x)^3}}{6\ln 7}+C}$$

A.P. Calculus ___ **Review #3**

Review Integral Rules:

$\int u^n du =$	$\int \frac{1}{u} du =$	$\int e^u du =$
$\int \tan u \, du =$	$\int \cot u \, du =$	$\int \sec u \, du =$
$\int \csc u \, du =$	$\int a^u du =$	

1. $\int \frac{dx}{x^{2/3}(1+x^{1/3})}$

2. $\int \frac{x^4+x-4}{x^2+2} dx$

3. $\int \frac{\sec x \tan x}{\sec x - 1} dx$

4. $\int \frac{e^{2x}+2e^x+1}{e^x} dx$

5. $\int_1^3 \frac{e^{3/x}}{x^2}dx$

6. $\int \frac{2x}{1-3x}dx$

7. $\int \sec\left(\frac{x}{2}\right)dx$

8. $\int \frac{4^{3x}}{1-4^{3x}}dx$

9. $\int \frac{\cos x}{2^{\sin x}}dx$

A.P. Calculus **Review#3**

Key

Review Integral Rules:

$\int u^n du = \frac{u^{n+1}}{n+1} + C$	$\int \frac{1}{u} du = \ln\lvert u\rvert + C$	$\int e^u du = e^u + C$
$\int \tan u\, du = -\ln\lvert\cos u\rvert + C$	$\int \cot u\, du = \ln\lvert\sin u\rvert + C$	$\int \sec u\, du = \ln\lvert\sec u + \tan u\rvert + C$
$\int \csc u\, du = -\ln\lvert\csc u + \cot u\rvert + C$	$\int a^u du = \frac{1}{\ln a}\cdot a^u + C$ or $\frac{a^u}{\ln a} + C$	

1. $\int \frac{dx}{x^{2/3}(1+x^{1/3})}$

$u = 1 + x^{1/3}$

$\frac{du}{dx} = \frac{1}{3}x^{-2/3}$

$\frac{du}{dx} = \frac{1}{3x^{2/3}}$

$dx = 3x^{2/3}du$

$\int \frac{3\cancel{x^{2/3}}\,du}{\cancel{x^{2/3}}\cdot u}$

$3\int \frac{1}{u} du$

$3\ln\lvert u\rvert + C$

$\boxed{3\ln\lvert 1+x^{1/3}\rvert + C}$

2. $\int \frac{x^4+x-4}{x^2+2}dx$

$$\begin{array}{r} x^2 - 2 + \frac{x}{x^2+2} \\ x^2+2\,\overline{)\,x^4 + x - 4} \\ -x^4 - 2x^2 \\ \hline -2x^2 + x - 4 \\ +2x^2 + 4 \\ \hline x \end{array}$$

$\int x^2 - 2 + \frac{x}{x^2+2} dx$

$u = x^2+2$

$\frac{du}{dx} = 2x$

$dx = \frac{du}{2x}$

$\int \frac{\cancel{x}}{u}\cdot\frac{du}{2\cancel{x}}$

$\frac{1}{2}\int \frac{1}{u} du$

$\boxed{\frac{x^3}{3} - 2x + \frac{1}{2}\ln\lvert x^2+2\rvert + C}$

3. $\int \frac{\sec x \tan x}{\sec x - 1}dx$

$u = \sec x - 1$

$\frac{du}{dx} = \sec x \tan x$

$dx = \frac{du}{\sec x \tan x}$

$\int \frac{\sec x \cancel{\tan x}}{u}\cdot\frac{du}{\sec x \cancel{\tan x}}$

$\int \frac{1}{u} du$

$\ln\lvert u\rvert + C$

$\boxed{\ln\lvert \sec x - 1\rvert + C}$

4. $\int \frac{e^{2x}+2e^x+1}{e^x}dx$

$\int (e^{2x} + 2e^x + 1)e^{-x} dx$

$\int e^x + 2 + e^{-x} dx$

$u = -x$ $\frac{du}{dx} = -1$

$dx = -du$

$-\int e^u du$

$\boxed{e^x + 2x - e^{-x} + C}$

5. $\int_1^3 \frac{e^{3/x}}{x^2}dx$

If $x=1,\ u=3/1=3$

If $x=3,\ u=3/3=1$

$u=3/x$

$u=3x^{-1}$

$\frac{du}{dx}=-3x^{-2}$

$\frac{du}{dx}=\frac{-3}{x^2}$

$-3dx=x^2du$

$dx=-\frac{x^2}{3}du$

$\int \frac{e^u}{\cancel{x^2}}\cdot\frac{-\cancel{x^2}}{3}du$

$-\frac{1}{3}\int e^u du$

$-\frac{1}{3}e^u\Big]_3^1$

$-\frac{1}{3}e^1-\left(-\frac{1}{3}e^3\right)$

$\boxed{-\frac{1}{3}e+\frac{1}{3}e^3}$

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6. $\int \frac{2x}{1-3x}dx$

$u=1-3x$

$\frac{du}{dx}=-3$

$dx=\frac{du}{-3}$

$\int \frac{2x}{u}\cdot\frac{du}{-3}$

$u=1-3x$

$3x=1-u$

$x=\frac{1-u}{3}$

$\int \frac{2\left(\frac{1-u}{3}\right)}{u}\cdot\frac{du}{-3}$

$\int \frac{2}{3}\left(\frac{1-u}{u}\right)\frac{du}{-3}=-\frac{2}{9}\int\frac{1-u}{u}du$

$-\frac{2}{9}\int\frac{1}{u}-1\,du$

$-\frac{2}{9}\ln|u|-u+c$

$\boxed{-\frac{2}{9}\ln|1-3x|-(1-3x)+c}$

7. $\int \sec\left(\frac{x}{2}\right)dx$

$u=x/2$

$\frac{du}{dx}=\frac{1}{2}$

$dx=2du$

$\int \sec u\cdot 2du$

$2\int \sec u\,du$

$2\ln|\sec u+\tan u|+c$

$\boxed{2\ln|\sec(x/2)+\tan(x/2)|+C}$

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8. $\int \frac{4^{3x}}{1-4^{3x}}dx$

$u=1-4^{3x}$

$\frac{du}{dx}=-\ln 4\cdot 4^{3x}\cdot 3$

$dx=\frac{du}{-3\ln 4\cdot 4^{3x}}$

$\int \frac{\cancel{4^{3x}}}{u}\cdot\frac{du}{-3\ln 4\cdot \cancel{4^{3x}}}$

$\frac{-1}{3\ln 4}\int\frac{1}{u}du$

$\frac{-1}{3\ln 4}\ln|u|+c$

$\boxed{\frac{-1}{3\ln 4}\ln|1-4^{3x}|+c}$

9. $\int \frac{\cos x}{2^{\sin x}}dx$

$\int 2^{-\sin x}\cos x\,dx$

$u=-\sin x$

$\frac{du}{dx}=-\cos x$

$dx=\frac{du}{-\cos x}$

$\int 2^u\cdot\cancel{\cos x}\cdot\frac{du}{-\cancel{\cos x}}$

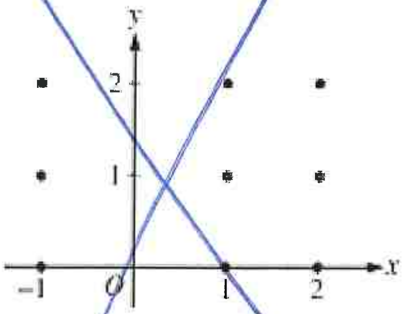
$-\int 2^u du$

$-\frac{1}{\ln 2}\cdot 2^u+c=\boxed{\frac{-2^{-\sin x}}{\ln 2}+c}$

Ch. 5&6 Test Review WS #4

~~1)Consider the differential equation~~ $\frac{dy}{dx} = (x^3 - 3)(2y - 1)$

~~a) On the axes below, sketch a slope field for the given differential equation at the nine points indicated~~



~~b) Find the particular solution y = f (x) to the given differential equation with the initial condition~~ $f(0) = 1$.

~~c) Find the equation of the line tangent to y = f (x) at the point where x = 0 and use it to approximate f (0.2).~~

2) $\int_0^3 \frac{3e^x}{\sqrt{1+2e^x}}\,dx =$

3) Word Problem: The rise in population of a town is directly proprtional to the number present at any given time t.

a) Write the differential equation and the general solution

b) Population doubled in the 50 years between 1920 and 1970. In 1998, the population was 75,500. What was the population in 1920?

4) ____________ Match the differential equation with the slope field graphed to the right.

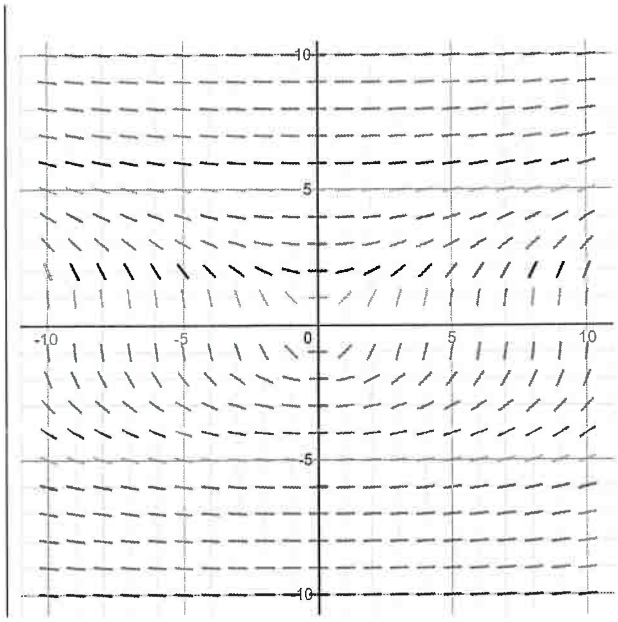
A) $\frac{dy}{dx} = \frac{x^2}{-y}$

B) $\frac{dy}{dx} = \frac{x}{y^2}$

C) $\frac{dy}{dx} = \frac{x^2}{y^2}$

D) $\frac{dy}{dx} = \frac{x}{y^3}$

E) $\frac{dy}{dx} = \frac{y^2}{x}$



5) $\int \sin\left(\frac{\pi x}{3}\right) - csc(5x)\, dx =$

6) $\int \frac{3x}{(4x^2)\sqrt{16x^4-7}} dx =$

7) $\int \frac{7}{x^2+16x+67} dx =$

8)

$$\int_1^3 \frac{2x^3-5}{x+1} dx =$$

9)

$$\int \frac{5}{2x \ln x^3} \, dx$$

Derivative & Integral Rules to memorize for Test

Derivative Rules:

Power Rule:

$$\frac{d}{dx}x^n = nx^{n-1}$$

Trig Derivatives:

$$\frac{d}{dx}\sin u = \cos u * u'$$

$$\frac{d}{dx}\cos u = -\sin u * u'$$

$$\frac{d}{dx}\tan u = \sec^2 u * u'$$

$$\frac{d}{dx}\cot u = -csc^2 u * u'$$

$$\frac{d}{dx}\sec u = \sec u \tan u * u'$$

$$\frac{d}{dx}\csc u = -\csc u \cot u * u'$$

$$\frac{d}{dx}e^u = e^u * u'$$

$$\frac{d}{dx}\ln u = \frac{u'}{u}$$

$$\frac{d}{dx}a^u = \ln a * \ a^u * u'$$

$$\frac{d}{dx}log_a u = \frac{1}{\ln a} * \frac{u'}{u}$$

Integral Rules:

Power Rule:

$$\int u^n du = \frac{u^{n+1}}{n+1} + C$$

Trig Integrals:

$$\int sin\, u\, du = -cos\, u + C$$

$$\int cos\, u\, du = sin\, u + C$$

$$\int sec^2 u\, du = tan\, u + C$$

$$\int sec\, u\, tan\, u\, du = sec\, u + C$$

$$\int csc^2 u\, du = -cot\, u + C$$

$$\int csc\, u\, cot\, u\, du = -csc\, u + C$$

$$\int \frac{1}{u} du = \ln|u| + C$$

$$\int e^u du = e^u + C$$

$$\int a^u du = \left(\frac{1}{\ln a}\right) a^u + C$$

More Trig Integral Rules:

$$\int \tan u\, du = -\ln|\cos u| + C$$

$$\int \sec u\, du = \ln|\sec u + \tan u| + C$$

$$\int \cot u\, du = \ln|\sin u| + C$$

$$\int \csc u\, du = -\ln|\csc u + \cot u| + C$$

Arc-Trig Integral Rules

16. $\displaystyle\int \frac{du}{\sqrt{a^2 - u^2}} = \arcsin \frac{u}{a} + C$

17. $\displaystyle\int \frac{du}{a^2 + u^2} = \frac{1}{a} \arctan \frac{u}{a} + C$

18. $\displaystyle\int \frac{du}{u\sqrt{u^2 - a^2}} = \frac{1}{a} \operatorname{arcsec} \frac{|u|}{a} + C$

Arc-Trig derivative Rules

19. $\displaystyle\frac{d}{dx}[\arcsin u] = \frac{u'}{\sqrt{1 - u^2}}$

20. $\displaystyle\frac{d}{dx}[\arccos u] = \frac{-u'}{\sqrt{1 - u^2}}$

21. $\displaystyle\frac{d}{dx}[\arctan u] = \frac{u'}{1 + u^2}$

22. $\displaystyle\frac{d}{dx}[\operatorname{arccot} u] = \frac{-u'}{1 + u^2}$

23. $\displaystyle\frac{d}{dx}[\operatorname{arcsec} u] = \frac{u'}{|u|\sqrt{u^2 - 1}}$

24. $\displaystyle\frac{d}{dx}[\operatorname{arccsc} u] = \frac{-u'}{|u|\sqrt{u^2 - 1}}$

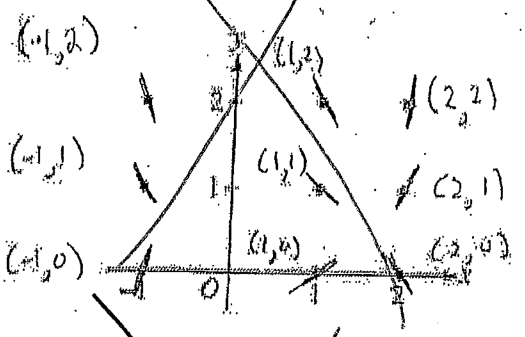
23

Key

Ch. 5&6 Test Review WS #4

1)Consider the differential equation $\frac{dy}{dx} = (x^3 - 3)(2y - 1)$

a) On the axes below, sketch a slope field for the given differential equation at the nine points indicated



b) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(0) = 1$

$\frac{dy}{dx} = \frac{(x^3-3)(2y-1)}{1}$

$dy = (x^3-3)(2y-1)dx$

$\int \frac{dy}{2y-1} = \int x^3-3\,dx$

$u = 2y-1$ | $dy = \frac{du}{2}$

$\frac{du}{dy} = 2$

$\int \frac{1}{u} \cdot \frac{du}{2} = \int x^3-3\,dx$

$\frac{1}{2}\int \frac{1}{u}du = \int x^3-3\,dx$

$2\left(\frac{1}{2}\ln|2y-1| = \frac{x^4}{4} - 3x + C\right)$

$\ln|2y-1| = \frac{2x^4}{4} - 6x + c$

$e^{\ln|2y-1|} = e^{\frac{1}{2}x^4 - 6x + C}$

$|2y-1| = e^{\frac{1}{2}x^4 - 6x} \cdot \boxed{e^C} \leftarrow C$

$2y-1 = Ce^{\frac{1}{2}x^4-6x}$

$y = \frac{1}{2}Ce^{\frac{1}{2}x^4-6x} + \frac{1}{2}$

$y = \underline{C}e^{\frac{1}{2}x^4-6x} + \frac{1}{2}$ ↙ plug in (0,1)

c) Find the equation of the line tangent to $y = f(x)$ at the point where $x = 0$ and use it to approximate $f(0.2)$.

point: $(0,1)$

slope: $\left.\frac{dy}{dx}\right|_{(0,1)} = (0^3-3)(2-1) = -3$

$y - y_1 = m(x - x_1)$

$y - 1 = -3(x-0)$

$y = -3(x-0)+1$

$y(0.2) = -3(0.2)+1$

$\boxed{y(0.2) = 0.4}$

$1 = Ce^0 + \frac{1}{2}$

$1 = C + \frac{1}{2}$

$\frac{1}{2} = C$

$\boxed{y = \left(\frac{1}{2}\right)e^{\frac{1}{2}x^4-6x} + \frac{1}{2}}$

② $\int_0^3 \frac{3e^x}{\sqrt{1+2e^x}}dx =$

$\int \frac{3e^x}{(1+2e^x)^{1/2}}dx$

$u = 1+2e^x$

$\frac{du}{dx} = 2e^x$

$dx = \frac{du}{2e^x}$

$\int \frac{3\cancel{e^x}}{u^{1/2}} \cdot \frac{du}{2\cancel{e^x}}$

$\frac{3}{2}\int u^{-1/2}du$

$\frac{3}{2} \cdot \left(\frac{u^{1/2}}{1/2}\right)$

$\frac{3}{2} \cdot \frac{2}{1}u^{1/2}$

$3u^{1/2} \rightarrow \left.3(1+2e^x)^{1/2}\right]_0^3$

$3(1+2e^3)^{1/2} - 3(1+2e^0)^{1/2}$

$\boxed{3(1+2e^3)^{1/2} - 3(3)^{1/2}}$

(24)

3) Word Problem: The rise in population of a town is directly proprtional to the number present at any given time t.

a) Write the differential equation and the general solution

$p' = kp \Rightarrow P = Ce^{kt}$

b) Population doubled in the 50 years between 1920 and 1970. In 1998, the population was 75,500. What was the population in 1920?

let 1920 be $t=0$

(time, Population)
t, P

$(0, C)$

$(50, 2C)$

$(78, 75{,}500)$

$P = Ce^{kt}$

$2C = Ce^{k(50)}$

$2 = e^{50k}$

$\ln 2 = \ln e^{50k}$

$\frac{\ln 2}{50} = \frac{50k \cancel{\ln e}}{50}$

$\frac{\ln 2}{50} = k$

$P = Ce^{\left(\frac{\ln 2}{50}\right)t}$

$75{,}500 = Ce^{\left(\frac{\ln 2}{50}\right)(78)}$

$C = 25{,}606$

$\boxed{\text{Population was } \approx 25{,}606 \text{ in } 1920}$

4) ________ Match the differential equation with the slope field graphed to the right.

A) $\frac{dy}{dx} = \frac{x^2}{-y}$

→ $\boxed{\text{B) } \frac{dy}{dx} = \frac{x}{y^2}}$

C) $\frac{dy}{dx} = \frac{x^2}{y^2}$

D) $\frac{dy}{dx} = \frac{x}{y^3}$

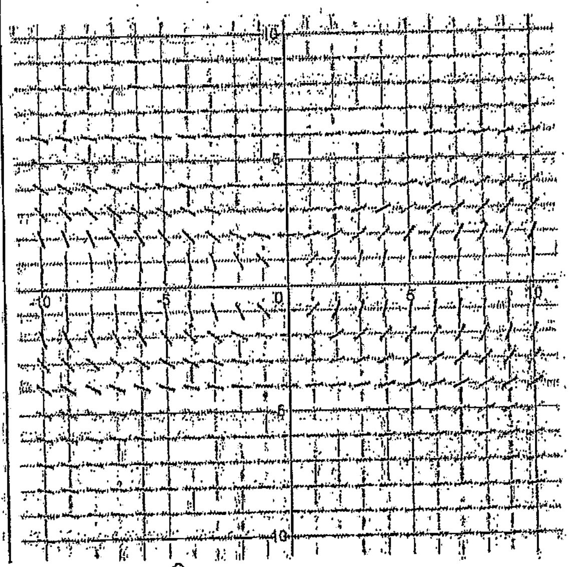
→ E) $\frac{dy}{dx} = \frac{y^2}{x}$

i) slope=0? (?)

ii) slope undefined? (?)

iii) pos. slope? Q1, Q4 (when $x>0$)

iv) neg. slope? Q2, Q3 (when $x<0$)



$\divideontimes \int \sin u\,du = -\cos u + C$

(5) $\int \sin\left(\frac{\pi x}{3}\right) - csc(5x)\,dx =$

$\int \sin\left(\frac{\pi x}{3}\right)dx - \int \csc(5x)\,dx$

$u = \frac{\pi x}{3}$

$\frac{du}{dx} = \frac{\pi}{3}$

$\pi dx = 3du$

$dx = \frac{3du}{\pi}$

$\int \sin u \cdot \frac{3}{\pi}du$

$\frac{3}{\pi}\int \sin u\,du$

$u = 5x$

$\frac{du}{dx} = 5$

$dx = \frac{du}{5}$

$\int \csc u \cdot \frac{du}{5}$

$\frac{1}{5}\int \csc u\,du$

$\int \csc u\,du = -\ln|\csc u + \cot u| + C$

$\frac{3}{\pi}\int \sin u\,du - \frac{1}{5}\int \csc u\,du$

$-\frac{3}{\pi}\cos\left(\frac{\pi x}{3}\right) - \left(-\frac{1}{5}\ln|\csc 5x + \cot 5x| + C\right.$

$\boxed{-\frac{3}{\pi}\cos\left(\frac{\pi x}{3}\right) + \frac{1}{5}\ln|\csc 5x + \cot 5x| + C}$

$$*\int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a}\operatorname{arcsec}\left(\frac{|u|}{a}\right)+C$$

6) $\int \frac{3x}{(4x^2)\sqrt{16x^4-7}}\,dx =$

$$\int \frac{3x}{4x^2\sqrt{(\ )^2-(\ )^2}}\,dx$$

$a=\sqrt{7}$

$u=4x^2$

$\frac{du}{dx}=8x$

$dx=\frac{du}{8x}$

$$\int \frac{3x}{4x^2\sqrt{(4x^2)^2-(\sqrt{7})^2}}\,dx$$

$$\int \frac{3\cancel{x}}{u\sqrt{u^2-a^2}}\cdot\frac{du}{8\cancel{x}} \rightarrow \frac{3}{8}\int \frac{du}{u\sqrt{u^2-a^2}}$$

$$\frac{3}{8}\cdot\frac{1}{\sqrt{7}}\operatorname{arcsec}\left(\frac{|4x^2|}{\sqrt{7}}\right)+C$$

$$\boxed{\frac{3}{8\sqrt{7}}\operatorname{arcsec}\left(\frac{|4x^2|}{\sqrt{7}}\right)+C}$$

$$*\int \frac{du}{a^2+u^2} = \frac{1}{a}\arctan\left(\frac{u}{a}\right)+C$$

7) $\int \frac{7}{x^2+16x+67}\,dx =$

*complete the square $\left(\frac{b}{2}\right)^2 \rightarrow \left(\frac{16}{2}\right)^2=8^2=64$

$$\underbrace{x^2+16x+64}_{(x+8)(x+8)}\underbrace{+67-64}_{3}$$

$(x+8)^2$

$a=\sqrt{3}$

$u=x+8$

$\frac{du}{dx}=1$

$dx=du$

$$\int \frac{7}{(x+8)^2+3}\,dx$$

$$7\int \frac{dx}{(x+8)^2+(\sqrt{3})^2}$$

$$7\int \frac{du}{u^2+a^2}$$

$$7\cdot\frac{1}{\sqrt{3}}\arctan\left(\frac{x+8}{\sqrt{3}}\right)+C$$

$$\boxed{\frac{7}{\sqrt{3}}\arctan\left(\frac{x+8}{\sqrt{3}}\right)+C}$$

8)

* ~~long division~~/synthetic division

$$\int_1^3 \frac{2x^3-5}{x+1}\,dx$$

$$\begin{array}{r|rrrr} -1 & 2x^3 & 0 & 0 & -5 \\ & \downarrow & -2 & 2 & -2 \\ \hline & 2x^2 & -2 & 2 & \frac{-7}{x+1} \end{array}$$

$$\int 2x^2-2x+2-\frac{7}{x+1}\,dx$$

$$\left.\frac{2x^3}{3}-\frac{\cancel{2}x^2}{\cancel{2}}+2x-7\ln|x+1|\right]_1^3$$

$$\boxed{\frac{2}{3}(3)^3-(3)^2+2(3)-7\ln|4|-\left(\frac{2}{3}-1+2-7\ln|2|\right)}$$

or

$$\boxed{\frac{46}{3}-7\ln 4-7\ln 2}$$

9)

*u-sub

$$\int \frac{5}{2x\ln x^3}\,dx \qquad \int \frac{5}{2x\cdot 3\ln x} \rightarrow \int \frac{5}{6x\ln x}\,dx$$

$$\frac{5}{6}\int \frac{1}{x\ln x}\,dx$$

$u=\ln x$

$\frac{du}{dx}=\frac{1}{x}$

$dx=x\,du$

$$\frac{5}{6}\int \frac{1}{\cancel{x}u}\cancel{x}\,du \rightarrow \frac{5}{6}\int \frac{1}{u}\,du$$

$$\frac{5}{6}\ln|u|+C$$

$$\boxed{\frac{5}{6}\ln|\ln x|+C}$$

Derivative & Integral Rules to memorize for Test

Derivative Rules:

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$$\frac{d}{dx}x^n = nx^{n-1}$$

Trig Derivatives:

$\frac{d}{dx}\sin u = \cos u * u'$	$\frac{d}{dx}\cos u = -\sin u * u'$
$\frac{d}{dx}\tan u = \sec^2 u * u'$	$\frac{d}{dx}\cot u = -csc^2 u * u'$
$\frac{d}{dx}\sec u = \sec u \tan u * u'$	$\frac{d}{dx}\csc u = -\csc u \cot u * u'$

$$\frac{d}{dx}e^u = e^u * u'$$

$$\frac{d}{dx}\ln u = \frac{u'}{u}$$

$$\frac{d}{dx}a^u = \ln a * \ a^u * u'$$

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$$\int csc^2 u\, du = -cot\, u + C \qquad \int csc\, u\, cot\, u\, du = -csc\, u + C$$

$$\int \frac{1}{u} du = \ln|u| + C$$

$$\int e^u du = e^u + C$$

$$\int a^u du = \left(\frac{1}{\ln a}\right) a^u + C$$

More Trig Integral Rules:

$$\int \tan u\, du = -\ln|\cos u| + C$$

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Arc-Trig Integral Rules

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Arc-Trig derivative Rules

19. $\frac{d}{dx}[\arcsin u] = \frac{u'}{\sqrt{1 - u^2}}$

20. $\frac{d}{dx}[\arccos u] = \frac{-u'}{\sqrt{1 - u^2}}$

21. $\frac{d}{dx}[\arctan u] = \frac{u'}{1 + u^2}$

22. $\frac{d}{dx}[\operatorname{arccot} u] = \frac{-u'}{1 + u^2}$

23. $\frac{d}{dx}[\operatorname{arcsec} u] = \frac{u'}{|u|\sqrt{u^2 - 1}}$

24. $\frac{d}{dx}[\operatorname{arccsc} u] = \frac{-u'}{|u|\sqrt{u^2 - 1}}$