A.P. Calculus AB
NO CALCULATORS!

Unit 6
~~4.3 – 4.5 Quiz~~ **Review**

1. Given $f(x) = x^2 - 2x + 3$, find a) average value in the interval $[0, 3]$ b) find the value of c guaranteed by the theorem

2. Given $f(x) = \sec^2 x$, find the average value in the interval $\left[\frac{-\pi}{4}, \frac{\pi}{4}\right]$

3. If $f(x) = \int_{-2}^{-2x^2} \frac{t}{4-t^3}\,dt$, find $\frac{d}{dx} f(x)$.

4. If $f(x) = \int_{-x}^{3\sqrt{x}} 1-2t\,dt$, find $\frac{d}{dx} f(x)$.

5. Let $\int_{-3}^{6} g(x)\,dx = 10$ and $\int_{3}^{0} g(x)\,dx = -4$

a) If g(x) is even, find $\int_{-6}^{3} g(x)\,dx$

b) If g(x) is odd, find $\int_{0}^{6} g(x)\,dx$

6. If $\int_{3} f(x)\,dx = -4$

a) $\int_{7}^{3} 2f(x)\,dx$

b) $\int_{7}^{3} [3f(x)-2]\,dx$

7. Evaluate $\int \frac{2}{x^2}\sec\left(\frac{3}{x}\right)\tan\left(\frac{3}{x}\right)dx$

8. Evaluate $\int 5x\sqrt{2-x}\,dx$.

9. Evaluate $\int_{4}^{9} \frac{x+1}{\sqrt{x}}\,dx$

10. Evaluate $\int_{0}^{\frac{\pi}{3}} \tan^2 x \sec^2 x \, dx$

A.P. Calculus AB Unit 6 ~~4.3 – 4.5~~ Quiz Review Solutions
NO CALCULATORS!

Avg. value theorem: $f(c) = \frac{1}{b-a}\int_a^b f(x)\,dx$

1. Given $f(x) = x^2 - 2x + 3$, find a) average value in the interval [0, 3] b) find the value of c guaranteed by the theorem

$f(c) = \frac{1}{3-0}\int_0^3 x^2 - 2x + 3\,dx$

$= \frac{1}{3} \cdot \frac{x^3}{3} - \frac{2x^2}{2} + 3x \Big]_0^3$

$\frac{1}{3} \cdot \frac{27}{3} - 3^2 + 9$

$= \frac{1}{3}(9 - 9 + 9)$

$= 3$

$f(c) = 3$

Avg. value = 3

b) $x^2 - 2x + 3 = 3$

$x^2 - 2x = 0$

$x(x-2) = 0$ $x = 0,\ x = 2$

$\boxed{c = 2}$, $\boxed{c = 0}$

2. Given $f(x) = \sec^2 x$, find average value in the interval $[-\frac{\pi}{4}, \frac{\pi}{4}]$

$f(c) = \frac{1}{\frac{\pi}{4} - -\frac{\pi}{4}}\int_{-\pi/4}^{\pi/4} \sec^2 x\,dx = \frac{1}{\pi/2} \cdot \tan x\Big]_{-\pi/4}^{\pi/4} = \frac{2}{\pi}\left[\tan\frac{\pi}{4} - \tan\left(-\frac{\pi}{4}\right)\right] = \frac{2}{\pi}[1 - (-1)]$

$f(c) = \frac{2}{\pi} \cdot 2 = \boxed{\frac{4}{\pi}}$

SFTC: $\frac{d}{dx}\int_a^{p(x)} f(t)\,dt = f(p(x)) \cdot p'(x)$

$\frac{d}{dx}\left[\int_{g(x)}^{p(x)} f(t)\,dt\right] = f(p(x)) \cdot p'(x) - f(g(x)) \cdot g'(x)$

3. If $f(x) = \int_{-2}^{-2x^2} \frac{t}{4 - t^3}\,dt$, find $\frac{d}{dx}f(x)$. use $\boxed{\text{SFTC}}$

$= \frac{-2x^2}{4 - (-2x^2)^3} \cdot -4x = \frac{8x^3}{4 + 8x^6}$

$= \boxed{\frac{2x^3}{1 + 2x^6}}$

4. If $f(x) = \int_{-x}^{3\sqrt{x}} 1 - 2t\,dt$, find $\frac{d}{dx}f(x)$.

$\frac{d}{dx}\left[\int_{-x}^{3\sqrt{x}} 1 - 2t\,dt\right] = [1 - 2(3\sqrt{x})] \cdot 3 \cdot \frac{1}{2}x^{-1/2} - [1 - 2(-x)](-1)$

$(1 - 6\sqrt{x})\frac{3}{2\sqrt{x}} + 1 + 2x$

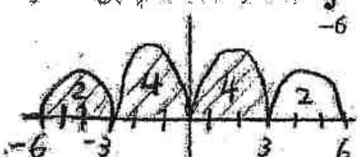
$\frac{3}{2\sqrt{x}} - \frac{18\sqrt{x}}{2\sqrt{x}} + 1 + 2x$

$\frac{3}{2\sqrt{x}} - 9 + 1 + 2x$

$= \boxed{\frac{3}{2\sqrt{x}} - 8 + 2x}$

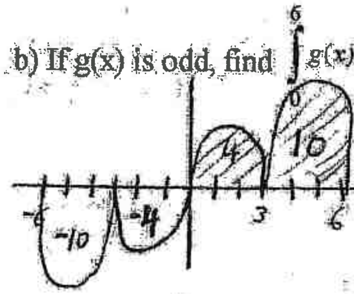
5. Let $\int_{-3}^{6} g(x)\,dx = 10$ and $\int_{3}^{0} g(x)\,dx = -4$ $= \int_0^3 g(x)\,dx = 4$

a) If g(x) is even, find $\int_{-6}^{3} g(x)\,dx$



$\int_{-6}^{3} g(x)\,dx = \boxed{10}$

b) If g(x) is odd, find $\int_{0}^{6} g(x)\,dx$



$\int_0^6 g(x)\,dx = \boxed{14}$

6. If $\int_3^7 f(x)\,dx = -4$

a) $\int_7^3 2f(x)\,dx = 2\left[-\int_3^7 f(x)\,dx\right]$

$2\cdot(-(-4)) = \boxed{8}$

b) $\int_7^3 [3f(x)-2]\,dx = 3\int_7^3 f(x)\,dx - \int_7^3 2\,dx$

$3\cdot(4) = 12$

$2x\Big]_7^3 = 6-14 = -8$

$= 12-(-8) = \boxed{20}$

7. Evaluate $\int \frac{2}{x^2}\sec\left(\frac{3}{x}\right)\tan\left(\frac{3}{x}\right)dx$

$u = \frac{3}{x} = 3x^{-1}$

$\frac{du}{dx} = -3x^{-2}$

$\frac{du}{dx} = \frac{-3}{x^2}$

$dx = -\frac{x^2}{3}du$

$\int \frac{2}{\cancel{x^2}}\sec(u)\tan(u)\cdot -\frac{\cancel{x^2}}{3}du$

$= -\frac{2}{3}\int \sec u \tan u\,du$

$= -\frac{2}{3}\sec u + C$

$= \boxed{-\frac{2}{3}\sec\left(\frac{3}{x}\right)+C}$

8. Evaluate $\int 5x\sqrt{2-x}\,dx = \int 5x(2-x)^{1/2}dx$

$u = 2-x$

$\frac{du}{dx} = -1$

$dx = -du$

$x = 2-u$

$\int 5x\cdot u^{1/2}(-du)$

$\int 5(2-u)u^{1/2}\cdot(-du)$

$= \int -10u^{1/2} + 5u^{3/2}\,du$

$= \frac{-10u^{3/2}}{3/2} + \frac{5u^{5/2}}{5/2} + C$

$= \frac{2}{3}(-10u^{3/2}) + \frac{2}{5}(5u^{5/2}) + C$

$= -\frac{20}{3}u^{3/2} + 2u^{5/2} + C$

$= \boxed{-\frac{20}{3}(2-x)^{3/2} + 2(2-x)^{5/2} + C}$

9. Evaluate $\int_4^9 \frac{x+1}{\sqrt{x}}dx = \int (x+1)x^{-1/2}dx$

$\int_4^9 x^{1/2} + x^{-1/2}\,dx$

$= \frac{x^{3/2}}{3/2} + \frac{x^{1/2}}{1/2}$

$\frac{2}{3}x^{3/2} + 2x^{1/2}\Big]_4^9$

$= \frac{2}{3}(9)^{3/2} + 2(9)^{1/2} - \left(\frac{2}{3}(4)^{3/2} + 2(4)^{1/2}\right)$

$\frac{2}{3}(27) + 2(3) - \frac{2}{3}(8) - 2(2)$

$18 + 6 - \frac{16}{3} - 4 = \boxed{\frac{44}{3}}$

10. Evaluate $\int_0^{\frac{\pi}{3}} \tan^2 x \sec^2 x\,dx$

if $x=0$, $u = \tan 0 = 0$

if $x = \pi/3$, $u = \tan(\pi/3) = \sqrt{3}$

$u = \tan x$

$\frac{du}{dx} = \sec^2 x$

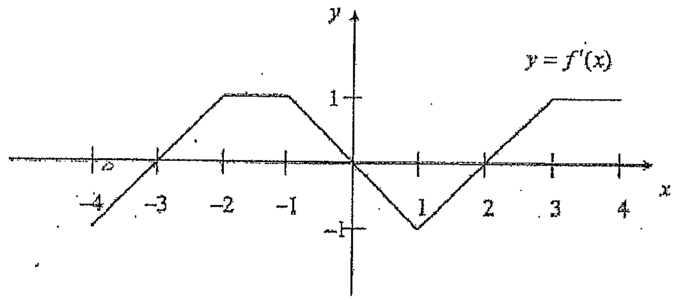
$dx = \frac{du}{\sec^2 x}$

$\int_0^{\pi/3} u^2\cdot\cancel{\sec^2 x}\cdot\frac{du}{\cancel{\sec^2 x}}$

$\frac{u^3}{3}\Big]_0^{\sqrt{3}} = \frac{1}{3}(\sqrt{3})^3 - \frac{1}{3}(0)^3 = \frac{1}{3}(3\sqrt{3}) = \boxed{\sqrt{3}}$

AP Calculus Chapter 4 Morning Review Session (part 1: Non-Calculator)

1. Find the average value of f '(x) on [-4, 4]



2. a) Find the average value of $f(x) = 4 - x^2$ on [0, 2].

b) Find the c-value guaranteed by the average value theorem.

3. $$\int \frac{x-2}{\sqrt[4]{x^2-4x}}\,dx$$

4. $$\int_{-5}^{2} |x+3|\,dx$$

5. $\int_{\sqrt{7}}^{0} x\sqrt{16-x^2}\,dx$

6. $\int \sqrt[3]{\cos x}\,\sin x\,dx$

7. $\int x\sqrt{1-x}\,dx$

8. Find $\frac{d}{dx}\left[\int_{-4x}^{\sqrt{x}} 1-t^2\,dt\right]$

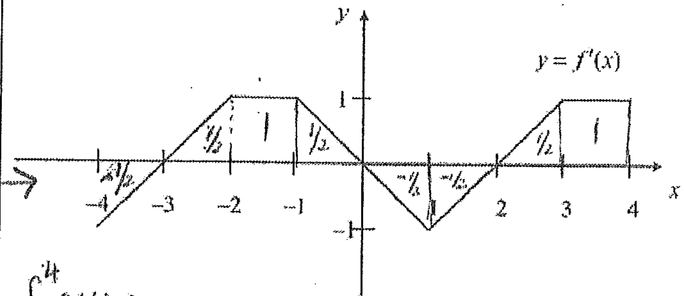
9. Given f''(x) = 1 – 2x and f'(-1) = 6 and f(0) = 14 find the below

   a. Find the specific equation for f'(x)

   b. Find the specific equation for f(x)

AP Calculus Chapter 4 Morning Review Session (part 1: Non-Calculator)

1. Find the average value of f '(x) on [-4, 4]



$$\text{Avg. value} = \frac{1}{b-a}\int_a^b f'(x)\,dx$$

$$= \frac{1}{4-(-4)}\int_{-4}^{4} f'(x)\,dx \qquad \int_{-4}^{4} f'(x)\,dx = 2$$

$$= \frac{1}{8}(2) = \frac{2}{8} = \boxed{\frac{1}{4}}$$

$$\int_{-4}^{4} f'(x)\,dx = -\frac{1}{2}+\frac{1}{2}+1+\frac{1}{2}-\frac{1}{2}-\frac{1}{2}+\frac{1}{2}+1 = 2$$

2. a) Find the average value of $f(x) = 4 - x^2$ on [0, 2].

$$\text{Avg. value} = \frac{1}{2-0}\int_0^2 4-x^2\,dx$$

$$\int_0^2 4-x^2\,dx = 4x - \frac{x^3}{3}\Big]_0^2 = 8 - \frac{8}{3} - \left(4(0) - \frac{0^3}{3}\right)$$

$$\text{Avg. value} = \frac{1}{2}\left(8-\frac{8}{3}\right) = \frac{1}{2}\left(\frac{16}{3}\right) = \boxed{\frac{8}{3}}$$

b) Find the c-value guaranteed by the average value theorem.

set f(x) = Avg. value

$$4 - x^2 = \frac{8}{3}$$

$$-x^2 = \frac{8}{3} - 4$$

$$-x^2 = -\frac{4}{3}$$

$$x^2 = \frac{4}{3}$$

$$x = \pm\sqrt{\frac{4}{3}}$$

$$\boxed{c = \frac{2}{\sqrt{3}}}$$

3. $\displaystyle\int \frac{x-2}{\sqrt[4]{x^2-4x}}\,dx = \int \frac{x-2}{(x^2-4x)^{1/4}}\,dx$

$u = x^2 - 4x$

$\frac{du}{dx} = 2x - 4$

$dx = \frac{du}{2x-4}$

$$\int \frac{x-2}{u^{1/4}}\cdot\frac{du}{2x-4}$$

$$\int \frac{\cancel{x-2}}{u^{1/4}}\cdot\frac{du}{2\cancel{(x-2)}}$$

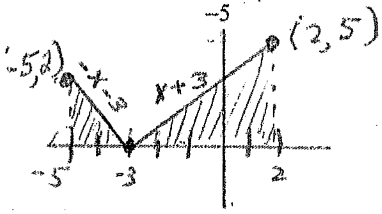
$$\frac{1}{2}\int u^{-1/4}\,du$$

$$= \frac{1}{2}\left(\frac{u^{3/4}}{3/4}\right)$$

$$= \frac{1}{2}\cdot\frac{4}{3}u^{3/4} + C$$

$$= \boxed{\frac{2}{3}(x^2-4x)^{3/4} + C}$$

4. $\displaystyle\int_{-5}^{2} |x+3|\,dx$



$$\frac{1}{2}(2)(2) + \frac{1}{2}(5)(5)$$

$$2 + \frac{25}{2} = \boxed{14.5 \text{ or } \frac{29}{2}}$$

$\boxed{\text{OR}}$

$$\int_{-5}^{-3} -x-3\,dx + \int_{-3}^{2} x+3\,dx$$

$$-\frac{x^2}{2} - 3x\Big]_{-5}^{-3} + \frac{x^2}{2} + 3x\Big]_{-3}^{2}$$

$$-\frac{9}{2} + 9 - \left(-\frac{25}{2} + 15\right) \qquad \frac{4}{2} + 6 - \left(\frac{9}{2} - 9\right)$$

$$2 + 12.5 = \boxed{14.5 \text{ or } \frac{29}{2}}$$

5. $\int_{\sqrt{7}}^{0} x\sqrt{16-x^2}\,dx$

if $x=\sqrt{7}$, $u=16-\sqrt{7}^2=9$
if $x=0$, $u=16-0=16$

$u=16-x^2$
$\frac{du}{dx}=-2x$
$dx=\frac{du}{-2x}$

$\int x\cdot u^{1/2}\cdot\frac{du}{-2x}$

$-\frac{1}{2}\int u^{1/2}du$

$-\frac{1}{2}\left(\frac{u^{3/2}}{3/2}\right)$

$-\frac{1}{2}\cdot\frac{2}{3}u^{3/2}$

$-\frac{1}{3}u^{3/2}\Big]_9^{16}=-\frac{1}{3}(16)^{3/2}-\left(-\frac{1}{3}(9)^{3/2}\right)$

$=-\frac{1}{3}(64)+\frac{1}{3}(27)$

$=\boxed{\frac{-37}{3}}$

6. $\int \sqrt[3]{\cos x}\sin x\,dx$

$u=\cos x$
$\frac{du}{dx}=-\sin x$
$dx=\frac{du}{-\sin x}$

$\int u^{1/3}\cdot\cancel{\sin x}\cdot\frac{du}{-\cancel{\sin x}}$

$-1\int u^{1/3}du$

$-\frac{u^{4/3}}{4/3}+C$

$\boxed{-\frac{3}{4}(\cos x)^{4/3}+C}$

7. $\int x\sqrt{1-x}\,dx$

$u=1-x \longrightarrow x=1-u$
$\frac{du}{dx}=-1$
$dx=-du$

$\int x\cdot u^{1/2}(-du)$

$\int(1-u)u^{1/2}(-du)$

$\int -u^{1/2}+u^{3/2}du$

$-\frac{u^{3/2}}{3/2}+\frac{u^{5/2}}{5/2}+C$

$-\frac{2}{3}u^{3/2}+\frac{2}{5}u^{5/2}+C$

$\boxed{-\frac{2}{3}(1-x)^{3/2}+\frac{2}{5}(1-x)^{5/2}+C}$

8. Find $\frac{d}{dx}\left[\int_{-4x}^{\sqrt{x}} 1-t^2\,dt\right]$

$\boxed{\text{use SFTC}}$

$=\left(1-(\sqrt{x})^2\right)\cdot\frac{1}{2}x^{-1/2}-\left(1-(-4x)^2\right)(-4)$

$=\boxed{\frac{1-x}{2\sqrt{x}}+4-64x^2}$

9. Given f''(x) = 1 – 2x and f'(-1) = 6 and f(0) = 14 find the below

a. Find the specific equation for f'(x)

$f'(x)=\int f''(x)dx=\int 1-2x\,dx=x-\frac{2x^2}{2}+C$

$f'(x)=x-x^2+C$

$6=(-1)-(-1)^2+C$

$6=-1-1+C$

$8=C$ $\quad\boxed{f'(x)=x-x^2+8}$

b. Find the specific equation for f(x)

$f(x)=\int f'(x)dx=\int x-x^2+8\,dx=\frac{x^2}{2}-\frac{x^3}{3}+8x+K$

$f(x)=\frac{x^2}{2}-\frac{x^3}{3}+8x+K$

$14=0-0+0+K$

$14=K$

$\boxed{f(x)=\frac{x^2}{2}-\frac{x^3}{3}+8x+14}$