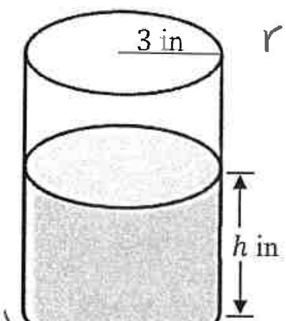
Key

Unit 7 Differential Equations and Slope Fields Quiz Review WS 1

1) Mr. Bean's favorite addiction (rhymes with Poctor Depper) is put into a cylindrical container with radius 3 inches, as shown in the figure above. Let h be the depth of the soda in the container, measured in inches, where h is a function of time t, measured in minutes. The volume V of soda in the container is changing at the rate of $-\frac{\pi}{2}\sqrt{h}$ cubic inches per minute throughout the morning. Given that $h = 9$ at the start of 1[st] period ($t = 0$), solve the differential equation $\frac{dh}{dt}$ for h as a function of t. (The volume V of a cylinder with radius r and height h is $V = \pi r^2 h$.)



r is constant

$\frac{dV}{dt} = -\frac{\pi}{2}\sqrt{h}$

$V = \pi(3)^2 h$

$V = 9\pi h$

$\frac{dV}{dt} = 9\pi\left(\frac{dh}{dt}\right)$

$-\frac{\pi}{2}\sqrt{h} = 9\pi\left(\frac{dh}{dt}\right)$

$9\pi\frac{dh}{dt} = \frac{-\pi\sqrt{h}}{2}$

$18\cancel{\pi}\,dh = -\cancel{\pi}\sqrt{h}\,dt$

$\frac{dh}{\sqrt{h}} = -\frac{1}{18}dt$

$\int h^{-1/2}dh = -\frac{1}{18}\int 1\,dt$

$\frac{h^{1/2}}{1/2} = -\frac{1}{18}t + C$

$2h^{1/2} = -\frac{1}{18}t + C$

$2(9)^{1/2} = -\frac{1}{18}(0) + C$

plug in $h(0) = 9$

$6 = C$

$\left(2h^{1/2} = -\frac{1}{18}t + 6\right)\frac{1}{2}$

$h^{1/2} = -\frac{1}{36}t + 3$

$\boxed{h = \left(-\frac{1}{36}t + 3\right)^2}$

2) The rate at which a baby koala bear gains weight is proportional to the difference between its adult weight and its current weight. At time $t = 0$, when the bear is first weighed, its weight is 2 pounds. If $B(t)$ is the weight of the bear, in pounds, at time t days after it is first weighed, then $\frac{dB}{dt} = \frac{1}{4}(20 - B)$. Find $y = B(t)$, the particular solution to the differential equation.

$dB = \frac{1}{4}(20-B)\,dt$

$\int \frac{dB}{20-B} = \int \frac{1}{4}dt$

$u = 20 - B$, $\frac{du}{dB} = -1$, $dB = -1\,du$

$\int \frac{-1\,du}{u}$

$-\ln|20-B| = \frac{1}{4}t + C$

$\ln|20-B| = -\frac{1}{4}t + C$

$e \qquad e$

$|20-B| = e^{-1/4t}\cdot e^{c}$

$|20-B| = Ce^{-1/4t}$

$20 - B = \pm Ce^{-1/4t}$

$20 - B = Ce^{-1/4t}$

$20 - Ce^{-1/4t} = B$

plug in $B(0) = 2$

$20 - Ce^{-1/4(0)} = 2$

$20 - C = 2$

$18 = C$

$\boxed{B(t) = 20 - 18e^{-1/4t}}$

3) Consider the differential equation $\frac{dy}{dx} = e^y(4x - 1)$. Let $y = f(x)$ be the particular solution to the differential equation that passes through (2,0).

(a) Write an equation for the line tangent to the graph of f at the point (2,0). Use the tangent line to approximate $f(2.2)$.

$\left.\frac{dy}{dx}\right|_{(2,0)} = e^0(4(2)-1) = 1(7) = 7$

point: $(2,0)$

slope: $m = 7$

$y - 0 = 7(x-2)$

$y = 7(x-2)$

$y(2.2) = 7(2.2-2) = \boxed{1.4}$

(b) Find $y = f(x)$, the particular solution to the differential equation that passes through (2,0).

$\frac{dy}{dx} = \frac{e^y(4x-1)}{1}$

$dy = e^y(4x-1)\,dx$

$\frac{dy}{e^y} = 4x - 1\,dx$

$\int e^{-y}dy = \int 4x - 1\,dx$

$-e^{-y} = \frac{4x^2}{2} - 1x + C$

$-e^{-y} = 2x^2 - 1x + C$

plug in $(2,0)$

$-e^{0} = 2(2)^2 - 1(2) + C$

$C = -7$

$-e^{-y} = 2x^2 - 1x - 7$

$e^{-y} = -2x^2 + x + 7$

$\ln e^{-y} = \ln(-2x^2 + x + 7)$

$\boxed{y = -\ln(-2x^2 + x + 7)}$

4) Consider the differential equation $\frac{dy}{dx} = 6 - 2y$. Let $y = f(x)$ be the particular solution to the differential equation with the initial condition $f(0) = 4$.

a. Write an equation for the line tangent to the graph of $y = f(x)$ at $x = 0$. Use the tangent line to approximate $f(0.6)$.

$\left.\frac{dy}{dx}\right|_{(0,4)} = 6-2(4) = -2$

point: $(0,4)$

slope: $m=-2$

$y-4=-2(x-0)$

$y=-2x+4$

$y(0.6)=-2(0.6)+4$

$y(0.6)=-1.2+4$

$y(0.6)=\boxed{2.8}$

b. Find the value of $\frac{d^2y}{dx^2}$ at the point $(0, 4)$. Is the graph of $y = f(x)$ concave up or concave down at the point $(0, 4)$? Give a reason for your answer.

$\frac{d^2y}{dx^2} = 0-2\left(\frac{dy}{dx}\right)$

$\frac{d^2y}{dx^2} = -2(6-2y)$

$\frac{d^2y}{dx^2} = -12+4y$

$\left.\frac{d^2y}{dx^2}\right|_{(0,4)} = -12+4(4) = 4$

$f(x)$ is concave up at $(0,4)$ since $\frac{d^2y}{dx^2} > 0$ at $(0,4)$

$\boxed{y=e^{-2x}+3}$

c. Find $y = f(x)$, the particular solution to the differential equation with the initial condition $f(0) = 4$.

$\frac{dy}{dx} = \frac{6-2y}{1}$

$1\,dy = (6-2y)\,dx$

$\int \frac{1}{6-2y}\,dy = \int 1\,dx$

$u=6-2y$

$\frac{du}{dy}=-2$

$dy = \frac{du}{-2}$

$\int \frac{1}{u}\cdot\frac{du}{-2} = \int 1\,dx$

$-\frac{1}{2}\ln|6-2y| = x+c$

$\ln|6-2y| = -2x+c$

$e^{\ln|6-2y|} = e^{-2x+c}$

$|6-2y| = e^{-2x}\cdot e^{c}$

$6-2y = Ce^{-2x}$

$6-Ce^{-2x} = 2y$

plug in $(0,4)$

$6-Ce^{-2(0)} = 2(4)$

$6-C(1)=8$

$-2=C$

$\frac{6--2e^{-2x}}{2} = \frac{2y}{2}$

5) During a certain epidemic, the number of people that are infected at any time increases at a rate proportional to the number of people that are infected at that time. If 500 people are infected when the epidemic is first discovered, and 800 people are infected 5 days later, how many people are infected 10 days after the epidemic is first discovered?

$P' = kP$

$P = Ce^{kt}$

(time, People)

$(0, 500)$

$(5, 800)$

$(10, —)$

$500 = Ce^{k(0)}$

$500 = C$

$P = 500e^{kt}$

$800 = 500e^{k(5)}$

$\frac{8}{5} = e^{5k}$

$\ln\left(\frac{8}{5}\right) = \ln e^{5k}$

$\ln(8/5) = 5k\cancel{\ln e}$

$\frac{1}{5}\ln(8/5) = k$

$P = 500e^{\frac{1}{5}\ln\left(\frac{8}{5}\right)t}$

$P = 500e^{\frac{1}{5}\ln(8/5)(10)}$

$\boxed{P = 1280 \text{ people}}$

6) A population of fruit flies is increasing at an exponential rate. If on the 3rd day there were 400 fruit flies, and the 7th day there were 600 fruit flies, approximately how many flies were in the original population (day 0)?

$P = Ce^{kt}$

(t, P)

$(3, 400)$

$(7, 600)$

$400 = Ce^{3k}$

$\frac{400}{e^{3k}} = C$

$600 = Ce^{7k}$

$\frac{600}{e^{7k}} = C$

set these equal!

$\frac{400}{e^{3k}} = \frac{600}{e^{7k}}$

$400e^{7k} = 600e^{3k}$

$\frac{e^{7k}}{e^{3k}} = \frac{600}{400}$

$e^{4k} = 1.5$

$\ln e^{4k} = \ln 1.5$

$4k = \ln 1.5$

$k = \frac{\ln 1.5}{4}$

$\frac{600}{e^{7\left(\frac{\ln 1.5}{4}\right)}} = C$

$\boxed{C \simeq 295 \text{ flies}}$

7) ______ . A radioactive substance has a rate of decay that can be modeled by the equation $\frac{dy}{dt} = ky$, where k is a constant and t is measured in years. If the half-life of the radioactive substance is 300 years, then what is the value of k?

$y = Ce^{kt}$

(t, y)
$(0, C)$
$(300, \frac{1}{2}C)$

$\frac{1}{2}\cancel{C} = \cancel{C}e^{300k}$
$\frac{1}{2} = e^{300k}$

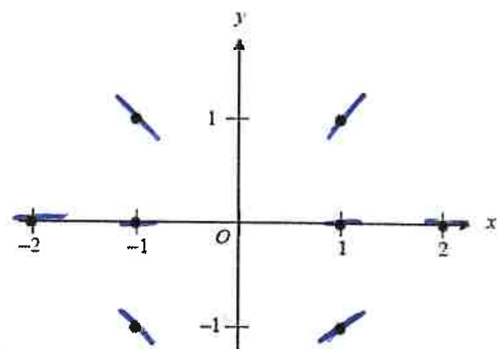
$\ln(\frac{1}{2}) = \ln e^{300k}$
$\ln(\frac{1}{2}) = 300k$
$\frac{\ln(1/2)}{300} = k$

$\boxed{k = -0.00231}$

8)

Consider the differential equation $\frac{dy}{dx} = \frac{y^2}{x}$, where $x \neq 0$.

a. On the axes provided, sketch a slope field for the given differential equation at the eight points indicated.



b. Find the particular solution $y = f(x)$ to the differential equation with the initial condition $f(1) = -1$.

$\frac{dy}{dx} = \frac{y^2}{x}$
$x\,dy = y^2\,dx$

$\frac{dy}{y^2} = \frac{dx}{x}$
$\int y^{-2}\,dy = \int \frac{1}{x}\,dx$

$\frac{y^{-1}}{-1} = \ln|x| + C$
$-\frac{1}{y} = \ln|x| + C$ plug in $(1,-1)$

$\frac{-1}{(-1)} = \ln|1| + C$
$1 = C$
$-\frac{1}{y} = \ln|x| + 1$

$\frac{1}{y} = -\ln|x| - 1$

$\boxed{y = \frac{1}{-\ln|x| - 1} \text{ or } \frac{-1}{\ln|x| + 1}}$

c. Write an equation for the tangent line to the curve $y = f(x)$ through the point $(1, -1)$. Then use your tangent line equation to estimate the value of $f(1.2)$.

$\left.\frac{dy}{dx}\right|_{(1,-1)} = \frac{1^2}{1} = 1$

point: $(1,-1)$
slope: $m = 1$

$y - (-1) = 1(x-1)$
$y + 1 = x - 1$
$y = x - 2$

$y(1.2) = 1.2 - 2$
$\boxed{y(1.2) = -0.8}$

9. For what value of k, if any, will $y = k\cos(2x) + 3\sin(4x)$ be a solution to the differential equation $y'' + 16y = -6\cos(2x)$?

$y' = k(-\sin(2x))\cdot 2 + 3\cos(4x)\cdot 4$
$y' = -2k\sin(2x) + 12\cos(4x)$
$y'' = -2k\cos(2x)\cdot 2 - 12\sin(4x)\cdot 4$
$y'' = -4k\cos(2x) - 48\sin(4x)$

$-4k\cos(2x) - 48\sin(4x) + 16[k\cos(2x) + 3\sin(4x)] = -6\cos(2x)$
$-4k\cos(2x) - \cancel{48\sin(4x)} + 16k\cos(2x) + \cancel{48\sin(4x)} = -6\cos(2x)$
$12k\cos(2x) = -6\cos(2x)$
$k = \frac{-6\cancel{\cos(2x)}}{12\cancel{\cos(2x)}}$

$\boxed{k = \frac{-1}{2}}$

Find the value of k of each equation that would be a solution to the given differential equation.

10) $y = 3ke^{2x} + \cos(4x)$

Diff Eq: $\boxed{\dfrac{y''}{2} + 8y = 15e^{2x}}$

$y' = 3ke^{2x}(2) - \sin(4x)(4)$

$y' = 6ke^{2x} - 4\sin(4x)$

$y'' = 6ke^{2x}(2) - 4\cos(4x)(4)$

$y'' = 12ke^{2x} - 16\cos(4x)$

$\dfrac{12ke^{2x} - 16\cos(4x)}{2} + 8\left[3ke^{2x} + \cos(4x)\right] = 15e^{2x}$

$6ke^{2x} - 8\cancel{\cos(4x)} + 24ke^{2x} + 8\cancel{\cos(4x)} = 15e^{2x}$

$30ke^{2x} = 15e^{2x}$

$k = \dfrac{15e^{2x}}{30e^{2x}} \rightarrow \boxed{k = \frac{1}{2}}$

11) $y = k\sin(-x) + 2\cos(3x)$

Diff Eq: $\boxed{2y'' + 18y = 32\sin(-x)}$

$y' = k\cos(-x)\cdot(-1) + 2(-\sin(3x))\cdot 3$

$y' = -k\cos(-x) - 6\sin(3x)$

$y'' = -k(-\sin(-x))\cdot(-1) - 6\cos(3x)(3)$

$y'' = -k\sin(-x) - 18\cos(3x)$

$2\left[-k\sin(-x) - 18\cos(3x)\right] + 18\left[k\sin(-x) + 2\cos(3x)\right] = 32\sin(-x)$

$\underline{-2k\sin(-x)} - 36\cancel{\cos(3x)} + \underline{18k\sin(-x)} + 36\cancel{\cos(3x)} = 32\sin(-x)$

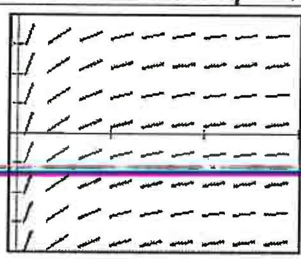
$16k\sin(-x) = 32\sin(-x)$

$k = \dfrac{32\cancel{\sin(-x)}}{16\cancel{\sin(-x)}} \rightarrow \boxed{k = 2}$

12)

The slope field from a certain differential equation is shown for each problem. The multiple choice answers are either differential equations OR a specific solution to that differential equation.

1.



$\boxed{\text{(A) } y = \ln x}$

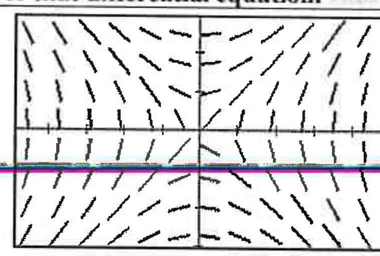
(B) $y = e^x$

(C) $y = e^{-x}$

(D) $y = \cos x$

(E) $y = x^2$

2.



(A) $\frac{dy}{dx} = x + y$

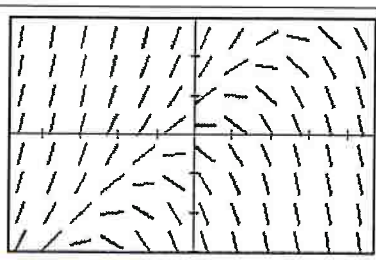
$\boxed{\text{(B) } \frac{dy}{dx} = \frac{x}{y}}$

(C) $\frac{dy}{dx} = \frac{y}{x}$

(D) $\frac{dy}{dx} = (x - 1)y$

(E) $\frac{dy}{dx} = x(y - 1)$

3.



$\boxed{\text{(A) } \frac{dy}{dx} = y - x}$

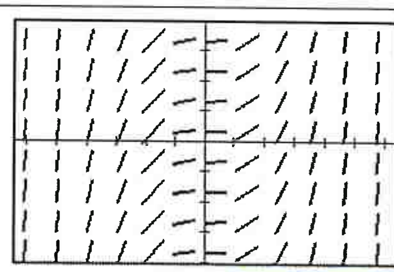
(B) $\frac{dy}{dx} = -\frac{x}{y}$

(C) $\frac{dy}{dx} = -\frac{y}{x}$

(D) $\frac{dy}{dx} = y(x - 1)$

(E) $\frac{dy}{dx} = x(y - 1)$

4.



(A) $y = \sin x$

(B) $y = \cos x$

(C) $y = x^2$

$\boxed{\text{(D) } y = \frac{1}{6}x^3}$

(E) $y = \frac{1}{4}x^4$

(24) 4) Non-Calculator $\boxed{2017}$ (30a)

4. At time $t = 0$, a boiled potato is taken from a pot on a stove and left to cool in a kitchen. The internal temperature of the potato is 91 degrees Celsius (°C) at time $t = 0$, and the internal temperature of the potato is greater than 27°C for all times $t > 0$. The internal temperature of the potato at time t minutes can be modeled by the function H that satisfies the differential equation $\frac{dH}{dt} = -\frac{1}{4}(H - 27)$, where $H(t)$ is measured in degrees Celsius and $H(0) = 91$.

(a) Write an equation for the line tangent to the graph of H at $t = 0$. Use this equation to approximate the internal temperature of the potato at time $t = 3$.

(b) Use $\frac{d^2H}{dt^2}$ to determine whether your answer in part (a) is an underestimate or an overestimate of the internal temperature of the potato at time $t = 3$.

(c) For $t < 10$, an alternate model for the internal temperature of the potato at time t minutes is the function G that satisfies the differential equation $\frac{dG}{dt} = -(G - 27)^{2/3}$, where $G(t)$ is measured in degrees Celsius and $G(0) = 91$. Find an expression for $G(t)$. Based on this model, what is the internal temperature of the

a) $(x, y) \rightarrow (t, H)$

$(0, 91)$

$\left.\frac{dH}{dt}\right|_{(0,91)} = \frac{-1}{4}(91-27) = -16$

$y - y_1 = m(x - x_1)$

$H - H_1 = m(t - t_1)$

$H - 91 = -16(t - 0)$

$\boxed{H = -16(t-0)+91}$

$H(3) = -16(3-0)+91 = \boxed{43\,°C}$

b) $\frac{dH}{dt} = \frac{-1}{4}H + \frac{27}{4}$

$\frac{d^2H}{dt^2} = \frac{-1}{4}\frac{dH}{dt} + 0$

$\frac{d^2H}{dt^2} = \frac{-1}{4}\left[\frac{-1}{4}(H-27)\right]$

$\frac{d^2H}{dt^2} = \frac{1}{16}\left[43-27\right] > 0$

(at t=3)

Since H is concave up, the approximation is an underestimate

c) Next page

$\boxed{4}$ c) $\dfrac{dG}{dt} = \dfrac{-(G-27)^{2/3}}{1}$

$dG = -(G-27)^{2/3}\,dt$

$\dfrac{dG}{(G-27)^{2/3}} = -1\,dt$

$\int (G-27)^{-2/3}\,dG = \int -1\,dt$

$u = G-27$

$\dfrac{du}{dG} = 1$

$dG = du$

$\int u^{-2/3}\,du \rightarrow \dfrac{u^{1/3}}{1/3} = -1t + c$

$3(G-27)^{1/3} = -1t + C$

$t \quad G$

$(0, 91)$

$3(91-27)^{1/3} = -1(0) + C$

$3(64)^{1/3} = C$

$3(4) = C$

$\boxed{12 = C}$

$3(G-27)^{1/3} = -1t + 12$

$(G-27)^{1/3} = \dfrac{-t+12}{3}$

$G - 27 = \left(\dfrac{12-t}{3}\right)^3$

$\boxed{G = 27 + \left(\dfrac{12-t}{3}\right)^3}$

The internal temperature of the potato at $t = 3$ mins is $27 + \left(\dfrac{12-3}{3}\right)^3 = \boxed{\text{54 degrees Celsius.}}$

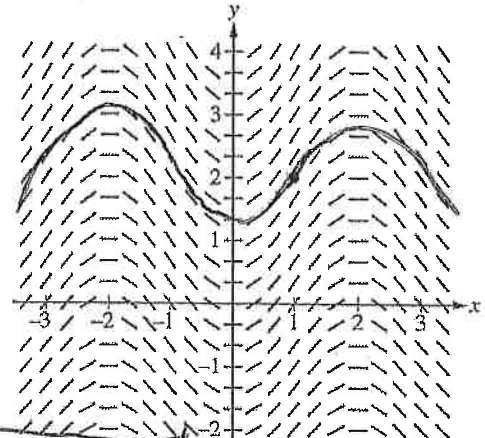
5. Consider the differential equation $\frac{dy}{dx} = \frac{1}{2}\sin\left(\frac{\pi}{2}x\right)\sqrt{y+7}$. Let $y = f(x)$ be the particular solution to the differential equation with the initial condition $f(1) = 2$. The function f is defined for all real numbers.

1 (a) A portion of the slope field for the differential equation is given below. Sketch the solution curve through the point $(1, 2)$.

2 (b) Write an equation for the line tangent to the solution curve in part (a) at the point $(1, 2)$. Use the equation to approximate $f(0.8)$.

1 (c) It is known that $f''(x) > 0$ for $-1 \le x \le 1$. Is the approximation found in part (b) an overestimate or an underestimate for $f(0.8)$? Give a reason for your answer.

5 (d) Use separation of variables to find $y = f(x)$, the particular solution to the differential equation $\frac{dy}{dx} = \frac{1}{2}\sin\left(\frac{\pi}{2}x\right)\sqrt{y+7}$ with the initial condition $f(1) = 2$.



b) point: $(1,2)$

Slope: $\left.\frac{dy}{dx}\right|_{(1,2)} = \frac{1}{2}\sin\left(\frac{\pi}{2}\right)\sqrt{2+7} = \frac{1}{2}(3) = \frac{3}{2}$

$y - y_1 = m(x - x_1)$

$y - 2 = \frac{3}{2}(x-1)$

$y = \frac{3}{2}(x-1) + 2$

$y(0.8) = \frac{3}{2}(0.8-1) + 2$

$f(0.8) \approx \boxed{\frac{3}{2}(0.8-1)+2 \text{ or } 1.7}$

c) Since $f''(x) > 0$ (graph is concave up), the tangent line is therefore below the curve, and the approximation is an underestimate.

d) $\frac{dy}{dx} = \frac{1}{2}\sin\left(\frac{\pi}{2}x\right)\sqrt{y+7}$

$\frac{dy}{\sqrt{y+7}} = \frac{1}{2}\sin\left(\frac{\pi}{2}x\right)dx$

$\int (y+7)^{-1/2}dy = \frac{1}{2}\int \sin\left(\frac{\pi}{2}x\right)dx$

$u = y+7$, $\frac{du}{dy} = 1$ | $\int u^{-1/2}du$

$u = \frac{\pi}{2}x$, $\frac{du}{dx} = \frac{\pi}{2}$, $dx = \frac{2}{\pi}du$

$\frac{1}{2}\int \sin u \cdot \frac{2}{\pi}du$

$\frac{1}{\pi}\int \sin u\, du$

$\frac{u^{1/2}}{1/2} = \frac{1}{\pi}(-\cos u) + C$

$2(y+7)^{1/2} = -\frac{1}{\pi}\cos\left(\frac{\pi}{2}x\right) + C$

plug in $(1,2)$ to find C

$2(2+7)^{1/2} = -\frac{1}{\pi}\cos\left(\frac{\pi}{2}(1)\right) + C$

$2(\sqrt{9}) = 0 + C$

$6 = C$

$2(y+7)^{1/2} = -\frac{1}{\pi}\cos\left(\frac{\pi}{2}x\right) + 6$

$(y+7)^{1/2} = \frac{-1}{2\pi}\cos\left(\frac{\pi}{2}x\right) + 3$

$y+7 = \left[\frac{-1}{2\pi}\cos\left(\frac{\pi}{2}x\right)+3\right]^2$

$\boxed{y = \left[\frac{-1}{2\pi}\cos\left(\frac{\pi}{2}x\right)+3\right]^2 - 7}$

24 4) Non-Calculator $\boxed{2017}$ 30a

4. At time $t = 0$, a boiled potato is taken from a pot on a stove and left to cool in a kitchen. The internal temperature of the potato is 91 degrees Celsius (°C) at time $t = 0$, and the internal temperature of the potato is greater than 27°C for all times $t > 0$. The internal temperature of the potato at time t minutes can be modeled by the function H that satisfies the differential equation $\frac{dH}{dt} = -\frac{1}{4}(H - 27)$, where $H(t)$ is measured in degrees Celsius and $H(0) = 91$.

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$H - H_1 = m(t - t_1)$

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$H - 91 = -16(t-0)$

$\boxed{H = -16(t-0)+91}$

$H(3) = -16(3-0)+91 = \boxed{43\,°C}$

b) $\frac{dH}{dt} = \frac{-1}{4}H + \frac{27}{4}$

$\frac{d^2H}{dt^2} = \frac{-1}{4}\frac{dH}{dt} + 0$

$\frac{d^2H}{dt^2} = \frac{-1}{4}\left[\frac{-1}{4}(H-27)\right]$

$\frac{d^2H}{dt^2} = \frac{1}{16}\left[43-27\right] > 0$

Since H is concave up, (at t=3)
the approximation is an underestimate.

c) Next page

2017 30b

$\boxed{4}$ c) $\frac{dG}{dt} = \frac{-(G-27)^{2/3}}{1}$

$dG = -(G-27)^{2/3}\,dt$

$\frac{dG}{(G-27)^{2/3}} = -1\,dt$

$\int (G-27)^{-2/3}\,dG = \int -1\,dt$

$u = G-27$

$\frac{du}{dG} = 1$

$dG = du$

$\int u^{-2/3}\,du \to \frac{u^{1/3}}{1/3} = -1t + c$

$3(G-27)^{1/3} = -1t + C$

t G
(0, 91)

$3(91-27)^{1/3} = -1(0) + C$

$3(64)^{1/3} = C$

$3(4) = C$

$12 = C$

$3(G-27)^{1/3} = -1t + 12$

$(G-27)^{1/3} = \frac{-t+12}{3}$

$G - 27 = \left(\frac{12-t}{3}\right)^3$

$\boxed{G = 27 + \left(\frac{12-t}{3}\right)^3}$

The internal temperature of the potato at $t = 3$ mins is $27 + \left(\frac{12-3}{3}\right)^3 = \boxed{54 \text{ degrees Celsius.}}$