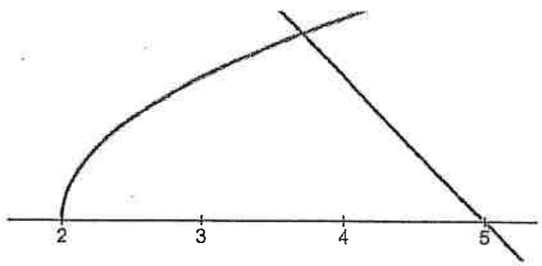
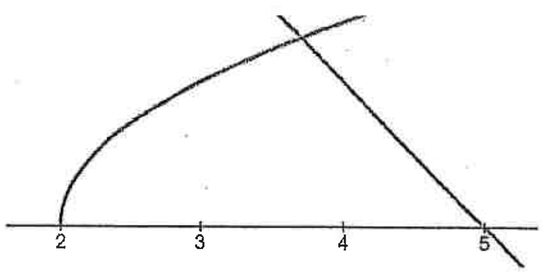
A.P. Calculus - Unit 8 Area & Volume Unit Review WS #1

1) Given the region below enclosed by $f(x) = \sqrt{x-2}$, $g(x) = 5 - x$, and the x-axis.

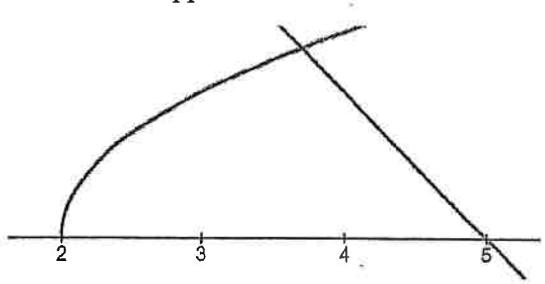
a) Find the area of the below region. (Write the integral notation(s) as well as the numeric approximation rounded to 3 decimal places)



b) Find the Volume of solid generated when the enclosed region is revolved about the line x = 1 (Write the integral notation as well as the numeric approximation rounded to 3 decimal places)

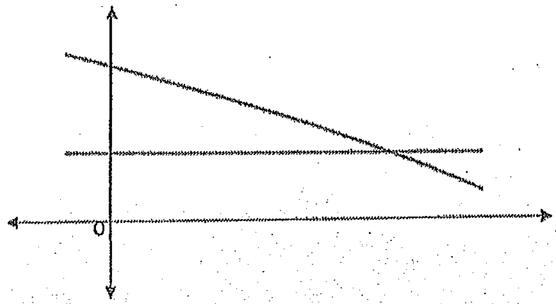


c) The enclosed region is the base of a solid. The cross section of the solid taken <u>perpendicular to the y-axis</u> is an equilateral triangle. Find the volume of the given solid. (Write the integral notation as well as the numeric approximation rounded to 3 decimal places)

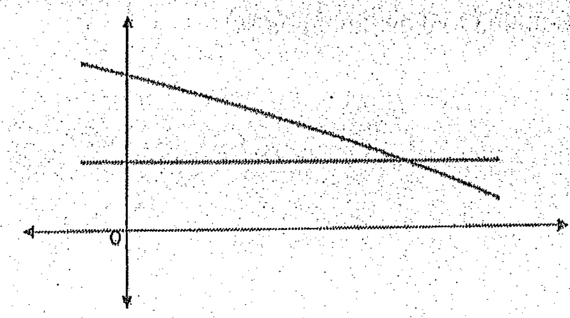


2) Given the region below enclosed by $f(x) = \ln(4 - x)$, the line $y = 1$, and the y-axis.

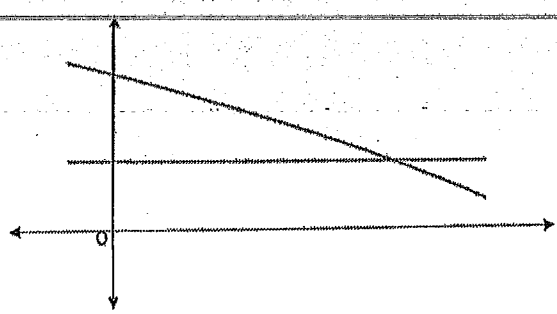
a) Find the Volume of solid generated when the enclosed region is revolved about the line $y = 1$ (Write the integral notation as well as the numeric approximation rounded to 3 decimal places)



b) Find the Volume of solid generated when the enclosed region is revolved about the line $y = 2$ (Write the integral notation as well as the numeric approximation rounded to 3 decimal places)



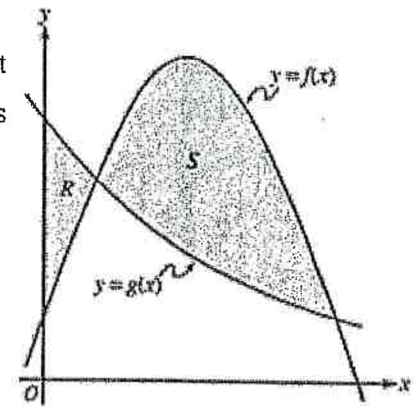
c) The enclosed region is the base of a solid. The cross section of the solid taken perpendicular to the x-axis is a rectangle whose height is twice the base. Find the volume of the given solid. (Write the integral notation as well as the numeric approximation rounded to 3 decimal places)



A.P. Calculus *Unit 8* Area & Volume Unit Review WS #2

1)

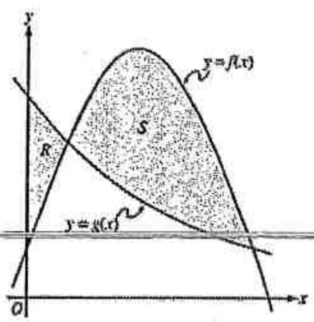
Let f and g be the functions given by $f(x) = \frac{1}{4} + \sin(\pi x)$ and $g(x) = 4^{-x}$. Let R be the shaded region in the first quadrant enclosed by the y-axis and the graphs of f and g, and let S be the shaded region in the first quadrant enclosed by the graphs of f and g, as shown in the figure above.



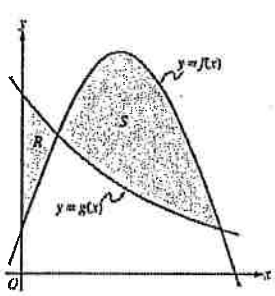
a) Find the area of S

b) Find the area of R

c) Find the volume of the solid generated when S is revolved about the horizontal line y = - 1.



d) The enclosed region R is the base of a solid. The cross section of the solid taken parallel to the y-axis is a isosceles right triangle with leg on base. Find the volume of the given solid. (Write the integral notation as well as the numeric approximation rounded to 3 decimal places)



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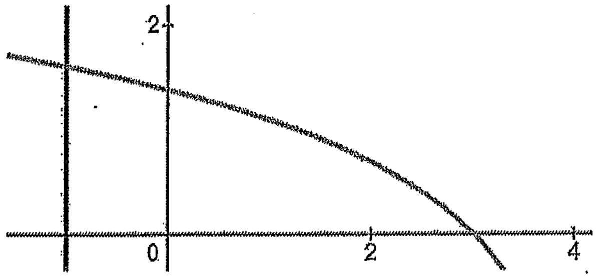
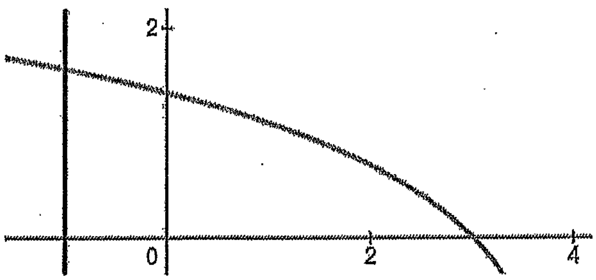
2) Given the region below enclosed by $f(x) = \ln(4-x)$, the line x = -1, and the x-axis.

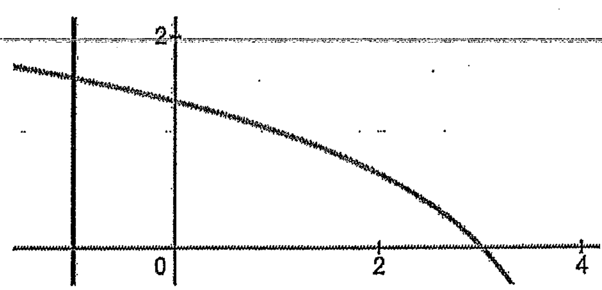
a) Find the Volume of solid generated when the enclosed region is revolved about the line x = -1 (Write the integral notation as well as the numeric approximation rounded to 3 decimal places)



b) Find the Volume of solid generated when the enclosed region is revolved about the line x = 4 (Write the integral notation as well as the numeric approximation rounded to 3 decimal places)



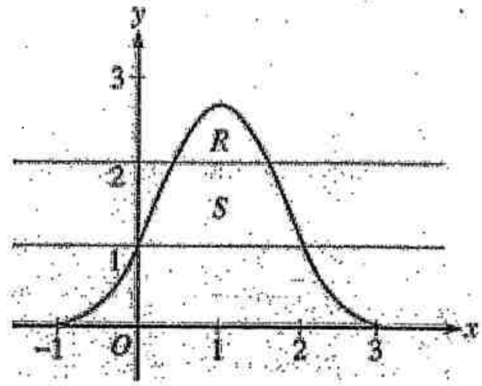
c) The enclosed region is the base of a solid. The cross section of the solid taken parallel to the x-axis is a rectangle whose height is 4. Find the volume of the given solid. (Write the integral notation as well as the numeric approximation rounded to 3 decimal places)



Unit 8 Area/Volume / FRQ Problems Worksheet #3

1)

Let R be the region bounded by the graph of $y = e^{2x-x^2}$ and the horizontal line $y = 2$, and let S be the region bounded by the graph of $y = e^{2x-x^2}$ and the horizontal lines $y = 1$ and $y = 2$, as shown above.



(a) Find the area of R.

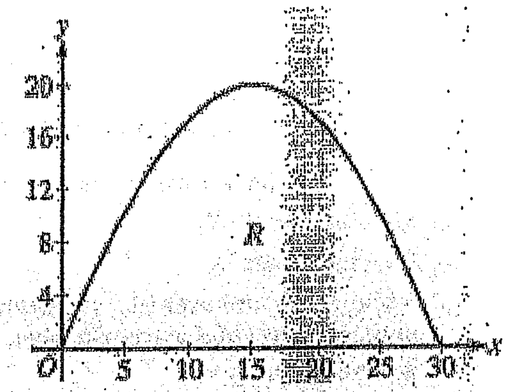
(b) Find the area of S.

(c) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when R is rotated about the horizontal line $y = 1$.

(6)

2)

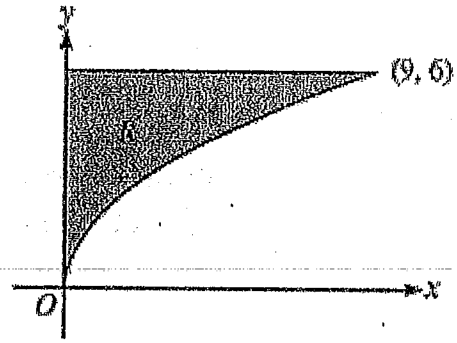
A baker is creating a birthday cake. The base of the cake is the region R in the first quadrant under the graph of $y = f(x)$ for $0 \le x \le 30$, where $f(x) = 20\sin\left(\frac{\pi x}{30}\right)$. Both x and y are measured in centimeters. The region R is shown in the figure above. The derivative of f is $f'(x) = \frac{2\pi}{3}\cos\left(\frac{\pi x}{30}\right)$.



(a) The region R is cut out of a 30-centimeter-by-20-centimeter rectangular sheet of cardboard, and the remaining cardboard is discarded. Find the area of the discarded cardboard.

(b) The cake is a solid with base R. Cross sections of the cake perpendicular to the x-axis are semicircles. If the baker uses 0.05 gram of unsweetened chocolate for each cubic centimeter of cake, how many grams of unsweetened chocolate will be in the cake?

3) (Non-Calculator)



Let R be the region in the first quadrant bounded by the graph of $y = 2\sqrt{x}$, the horizontal line $y = 6$, and the y-axis, as shown in the figure above.

(a) Find the area of R.

(b) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when R is rotated about the horizontal line $y = 7$.

(c) Region R is the base of a solid. For each y, where $0 \le y \le 6$, the cross section of the solid taken perpendicular to the y-axis is a rectangle whose height is 3 times the length of its base in region R. Write, but do not evaluate, an integral expression that gives the volume of the solid.

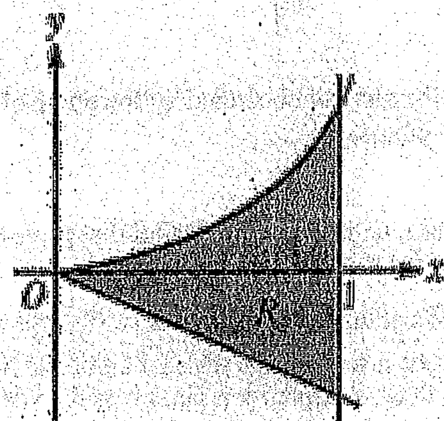
(8)

4)

Let R be the shaded region bounded by the graph of $y = xe^{x^2}$, the line $y = -2x$, and the vertical line $x = 1$, as shown in the figure above.

(a) Find the area of R.

(b) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when R is rotated about the horizontal line $y = -2$.



Calculus Chapter 8 Review Worksheet #4

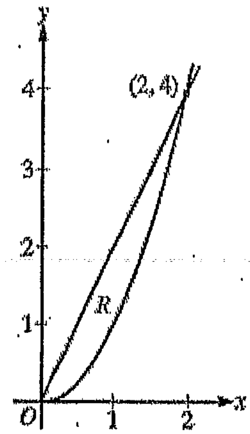
1. (Non-Calculator)

Let R be the region in the first quadrant enclosed by the graphs of $y = 2x$ and $y = x^2$, as shown in the figure above.

(a) Find the area of R.

(b) The region R is the base of a solid. For this solid, at each x the cross section perpendicular to the x-axis has area $A(x) = \sin\left(\frac{\pi}{2}x\right)$. Find the volume of the solid.

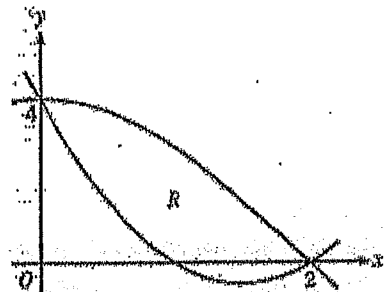
(c) Another solid has the same base R. For this solid, the cross sections perpendicular to the y-axis are squares. Write, but do not evaluate, an integral expression for the volume of the solid.



(d) Write an Integral Expression that gives the Volume of the Solid generated when R is rotated about $x = -1$.

2) (Non-Calculator)

Let $f(x) = 2x^2 - 6x + 4$ and $g(x) = 4\cos\left(\frac{1}{4}\pi x\right)$. Let R be the region bounded by the graphs of f and g, as shown in the figure above.



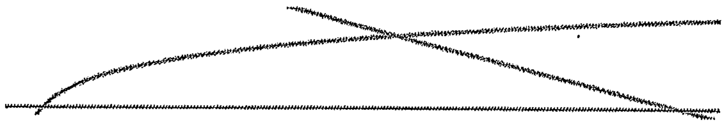
(a) Find the area of R.

(b) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when R is rotated about the horizontal line $y = 4$.

(c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a square. Write, but do not evaluate, an integral expression that gives the volume of the solid.

A.P. Calculus ___ Chapter 8 Area & Volume Unit Review WS #3

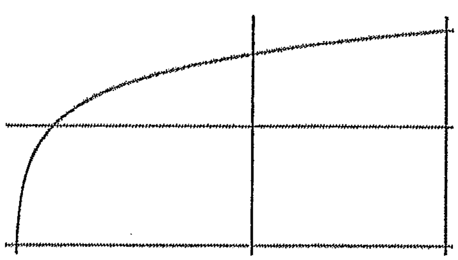
1) Given the region below enclosed by $f(x) = \ln(x-3)$, the line $y = 7 - \frac{1}{4}x$, and the x – axis.



2) Given the region below enclosed by $f(x) = \ln(x+6)$, the line $y = -3$, and $x = 5$.

a) Find the Volume of solid generated when the enclosed region is revolved about the line $y = -4$

~~(Write the integral notation as well as the numeric approximation rounded to 3 decimal places)~~



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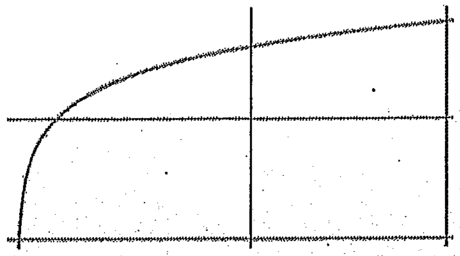
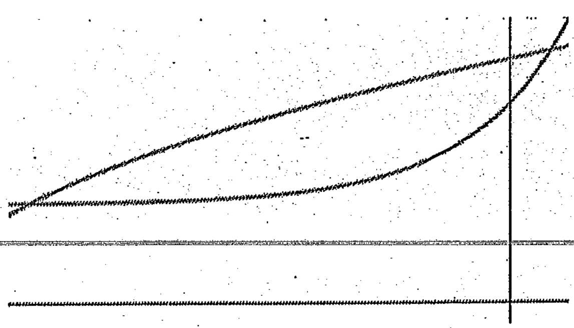
2) Given the region below enclosed by $f(x) = \ln(x+6)$, the line $y = -3$, and $x = 5$.

b) Find the Volume of solid generated when the enclosed region is revolved about the line x = 5 (Write the integral notation as well as the numeric approximation rounded to 3 decimal places)



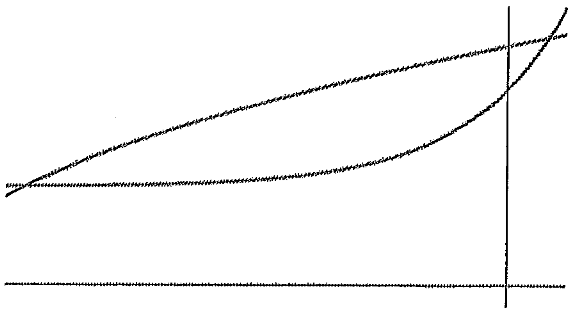
3) Given the region below enclosed by $f(x) = \sqrt{x+6}$, the $g(x) = e^x + 1$

a) Find the Volume of solid generated when the enclosed region is revolved about the line x = –6 (Write the integral notation as well as the numeric approximation rounded to 3 decimal places)



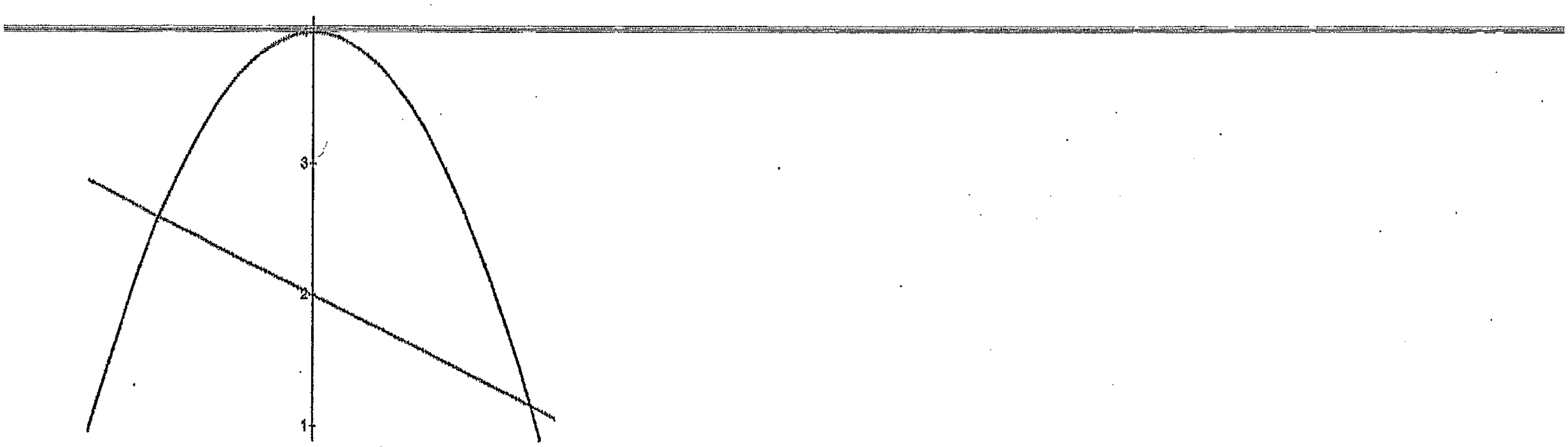
3) Given the region below enclosed by $f(x) = \sqrt{x+6}$, the $g(x) = e^x + 1$

b) The enclosed region is the base of a solid. The cross section of the solid taken parallel to the y-axis is a isosceles right triangle with hypotenuse on base. Find the volume of the given solid. (Write the integral notation as well as the numeric approximation rounded to 3 decimal places)



4) Given the region below enclosed by $f(x) = -x^2 + 4$ and $g(x) = -\frac{1}{2}x + 2$

Find the Volume of solid generated when the enclosed region is revolved about the line y = 4 (Write the integral notation as well as the numeric approximation rounded to 3 decimal places)



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Formulas Need to be Memorized

Unit 8 Area & Volume Formula Sheet

$$Area = \int_{x_1}^{x_2} (Top\ \ graph - Bottom\ graph)dx$$

(in the forms of " y = __ ")

$$Area = \int_{y_1}^{y_2} (Right\ graph - Left\ graph)dy$$

(in the form of " x = ___")

Disc Method: (Top – Bottom) – Vertical Radius – Horizontal AOR

$$V = \pi \int_{x_1}^{x_2} [R(x)]^2\, dx$$

(expression(s) used above has form: " y = ___")

Disc Method: (Right – Left) – Horizontal Radius Vertical AOR

$$V = \pi \int_{y_1}^{y_2} [R(y)]^2\, dy$$

(expression(s) used above has form: " x = ___")

Washer Method: (Top – Bottom) , Vertical Radius (Horizontal AOR)

$$V = \pi \int_{x_1}^{x_2} [R(x)]^2 - [r(x)]^2\, dx$$

(expression(s) used above has form: " y = ___")

Washer Method: (Right – Left) , Horizontal Radius (Vertical AOR)

$$V = \pi \int_{y_1}^{y_2} [R(y)]^2 - [r(y)]^2 dy$$

(expression(s) used above has form: " x = ___")

Top-Bottom Vertical base

$$V = \int_{x_1}^{x_2} [Area\ \text{of cross section}]dx$$

*Note: All values in integral are in terms of x (in the form of " y = ____")

Right-Left Horizontal base

$$V = \int_{y_1}^{y_2} [Area\ \text{of cross section}]dy$$

*Note: All values in integral are in terms of y (in the forms of " x = _______")

Area formulas for Cross sections:

1. Square: $A = (base)^2$	2. Isosceles Right Triangle (leg on base): $A = \frac{1}{2}(base)^2$	3. Isosceles Right Triangle (hypotenuse on base): $A = \frac{1}{4}(base)^2$
4. Rectangle: A = (base)(height)	5. Equilateral Triangle: $A = \frac{\sqrt{3}}{4}(base)^2$	6. Semicircle: $A = \frac{\pi}{8}(base)^2$