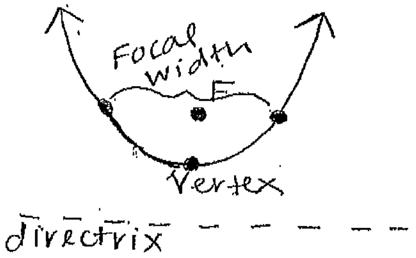
Accel Algebra 2

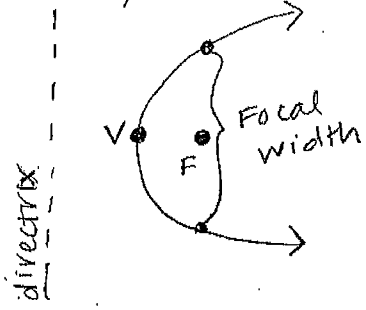
10.01 Notes - Graphing Parabolas

Parabola: a conic section where the distance from 1 fixed point (focus) and a line (directrix) is equal.

Vertical: x^2



Horizontal: y^2



Vertical Axis of Symmetry	Horizontal Axis of Symmetry
$(x-h)^2 = \boxed{4p}(y-k)$	$(y-k)^2 = 4p(x-h)$
Vertex: $\underline{(h,k)}$ Axis of Symmetry: $x = h$ Focus: $(h,\ k+p)$ Directrix: $y = k - p$ Focal width: $\lvert 4p \rvert$	Vertex: (h,k) Axis of Symmetry: $y = k$ Focus: $(h+p, k)$ Directrix: $x = h - p$ Focal width: $\lvert 4p \rvert$

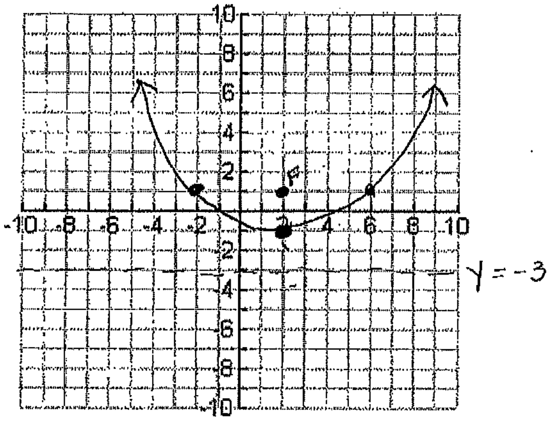
Examples: Graph the parabola. State the vertex, AOS, focus, directrix, and focal width.

1. $(x-2)^2 = \underset{\sim}{8}(y+1)$

x^2
$+p$

$4p = 8$
$P = 2$

Vertex: $(2,-1)$
Axis of Symmetry: $X = 2$
Focus: $(2,1)$
Directrix: $Y = -3$
Focal Width: 8

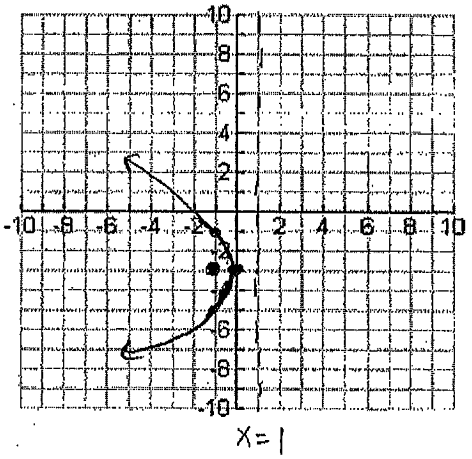


2. $(y+3)^2 = \boxed{-4}x$

y^2
$-p$

$-4 = 4p$
$P = -1$

Vertex: $(0,-3)$
Axis of Symmetry: $Y = -3$
Focus: $(-1,-3)$
Directrix: $X = 1$
Focal Width: 4



Practice: Graph the parabola. State the vertex, AOS, focus, directrix, and focal width.

up 1. $(x+4)^2 = 6(y-2)$

$4p = 6$

$p = 3/2$ or 1.5

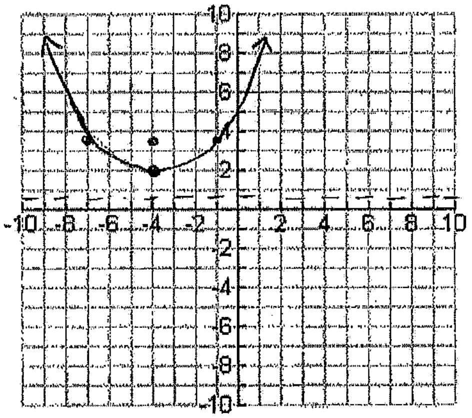
Vertex: (-4, 2)

Axis of Symmetry: $x = -4$

Focus: (-4, 3.5)

Directrix: $y = .5$

Focal Width: 6



2. $(y-3)^2 = 12(x-1)$

right

$4p = 12$

$p = 3$

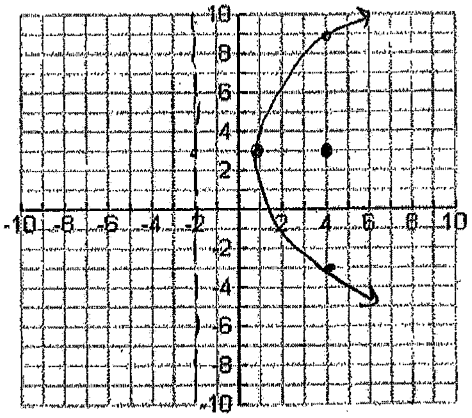
Vertex: (1, 3)

Axis of Symmetry: $y = 3$

Focus: (4, 3)

Directrix: $x = -2$

Focal Width: 12



3. $(x-5)^2 = -4(y+5)$

down

$4p = -4$

$p = -1$

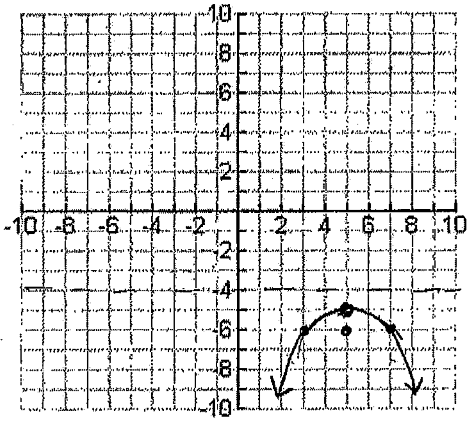
Vertex: (5, -5)

Axis of Symmetry: $x = 5$

Focus: (5, -6)

Directrix: $y = -4$

Focal Width: 4



4. $(y+2)^2 = -8(x+2)$

left

$4p = -8$

$p = -2$

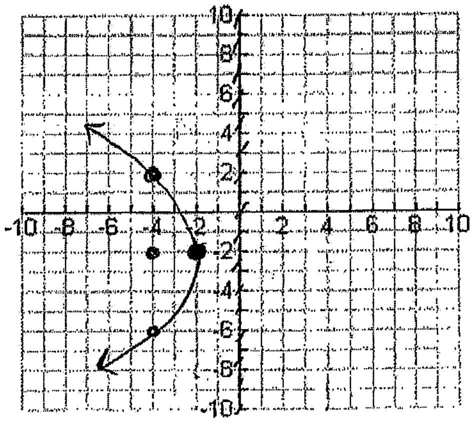
Vertex: (-2, -2)

Axis of Symmetry: $y = -2$

Focus: (-4, -2)

Directrix: $x = 0$

Focal Width: 8



Accel Algebra 2

10.01 Notes - Graphing Parabolas

Parabola: a conic section where the distance from 1 fixed point (focus) and a line (directrix) is equal.

Figure:

Vertex:

Axis of Symmetry:

Focus:

Directrix:

Focal Width:

Vertical Axis of Symmetry	Horizontal Axis of Symmetry
$(x-h)^2 = 4p(y-k)$	$(y-k)^2 = 4p(x-h)$
Vertex: (h,k) Axis of Symmetry: $x = h$ Focus: $(h,\ k+p)$ Directrix: $y = k - p$ Focal width: $\lvert 4p \rvert$	Vertex: (h,k) Axis of Symmetry: $y = k$ Focus: $(h+p, k)$ Directrix: $x = h - p$ Focal width: $\lvert 4p \rvert$

Examples: Graph the parabola. State the vertex, AOS, focus, directrix, and focal width.

+ up

1. $(x-2)^2 = 8(y+1)$ $4p = 8$ $p = 2$

-left

2. $(y+3)^2 = -4x$ $4p = -4$ $p = -1$

Vertex: (2,-1)

Axis of Symmetry: X = 2

Focus: (2,1)

Directrix: y = -3

Focal Width: 8

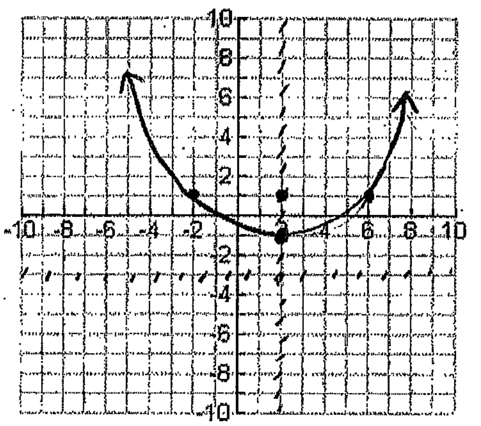
Vertex: (0,-3)

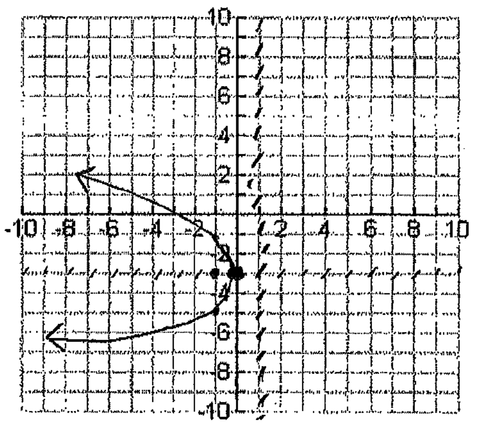
Axis of Symmetry: y = -3

Focus: (-1,-3)

Directrix: x = 1

Focal Width: 4

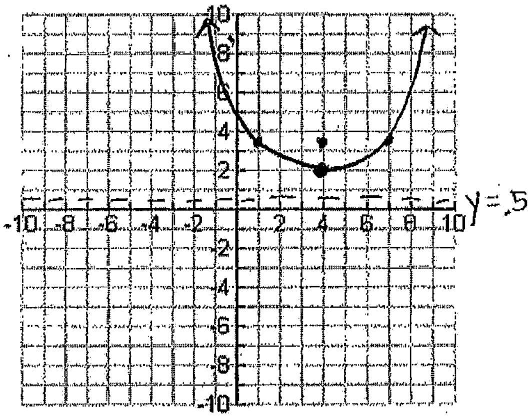




Practice: Graph the parabola. State the vertex, AOS, focus, directrix, and focal width.

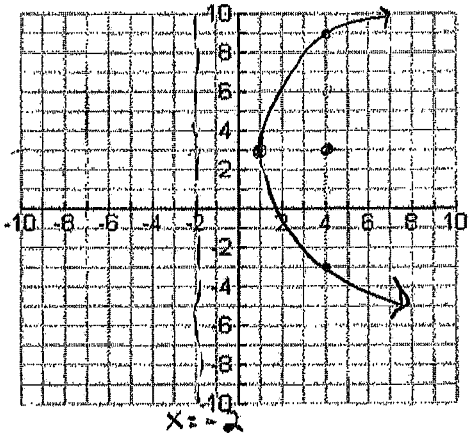
up 1. $(x+4)^2 = 6(y-2)$ $4p = 6$
$p = 3/2$ or 1.5

Vertex: (4, 2)
Axis of Symmetry: $x = 4$
Focus: (4, 3.5)
Directrix: $y = .5$
Focal Width: 6



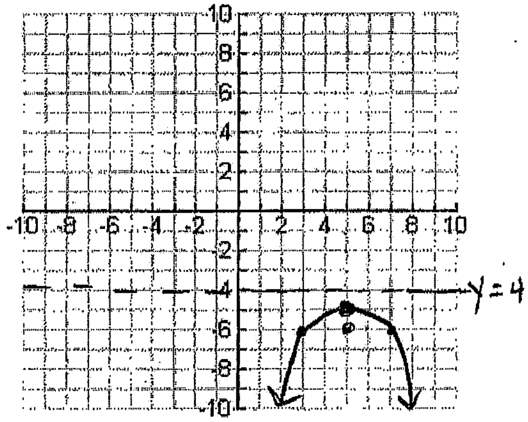
right 2. $(y-3)^2 = 12(x-1)$ $4p = 12$
$p = 3$

Vertex: (1, 3)
Axis of Symmetry: $y = 3$
Focus: (4, 3)
Directrix: $x = -2$
Focal Width: 12



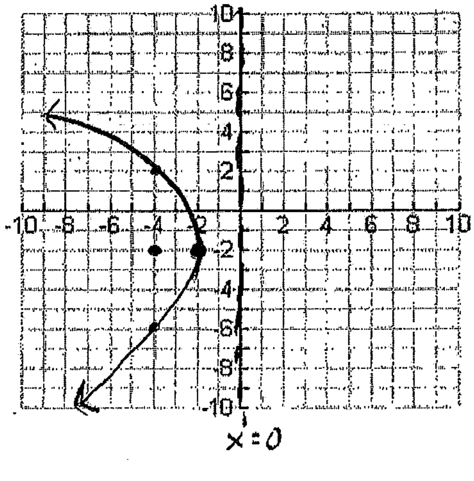
down 3. $(x-5)^2 = -4(y+5)$ $4p = -4$
$p = -1$

Vertex: (5, -5)
Axis of Symmetry: $X = 5$
Focus: (5, -6)
Directrix: $y = 4$
Focal Width: 4



left 4. $(y+2)^2 = -8(x+2)$ $4p = -8$
$p = -2$

Vertex: (-2, -2)
Axis of Symmetry: $y = -2$
Focus: (-4, -2)
Directrix: $X = 0$
Focal Width: 8



Accel Algebra 2

Name: ____________________

10.05 Completing the Square - Parabolas

Write the standard form of the equation of each parabola.

1. $y^2 + 12x = 2y - 13$

$y^2 - 2y \underline{+1} = -12x - 13 \underline{+1}$

$(y-1)^2 = -12x - 12$

Standard Equation: $(y-1)^2 = -12(x+1)$

2. $x^2 + 10x + 25 = -8y + 24$

$(x+5)^2 = -8y + 24$

Standard Equation: $(x+5)^2 = -8(y-3)$

3. $$\frac{3x^2 - 30y - 18x + 87 = 0}{3}$$

$x^2 - 6x - 10y + 29 = 0$

$x^2 - 6x \underline{+9} = 10y - 29 \underline{+9}$

$(x-3)^2 = 10y - 20$

Standard Equation: $(x-3)^2 = 10(y-2)$

4. $\dfrac{12x - 15 = 3y^2 + 6y}{3}$

$4x-5=y^2+2y$

$4x-5\underline{+1}=y^2+2y\underline{+1}$

$4x-4=(y+1)^2$

Standard Equation: $(y+1)^2=4(x-1)$

5. $x^2 + 8x + 14y = -44$

$x^2+8x\underline{+16}=-14y-44\underline{+16}$

$(x+4)^2=-14y-28$

Standard Equation: $(x+4)^2=-14(y+2)$

6. $\dfrac{2y^2 - 4y + 12x + 50 = 0}{2}$

$y^2-2y+6x+25=0$

$y^2-2y\underline{+1}=-6x-25\underline{+1}$

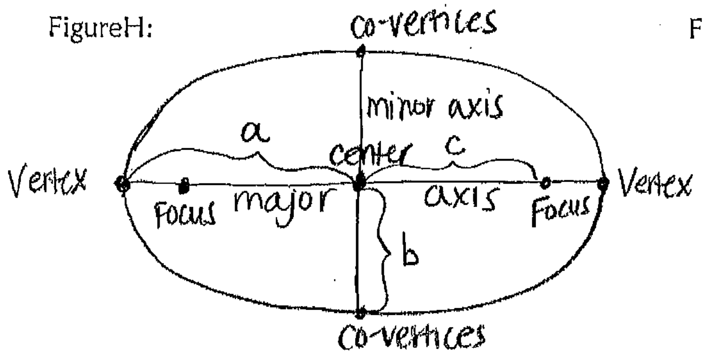
$(y-1)^2=-6x-24$

Standard Equation: $(y-1)^2=-6(x+4)$

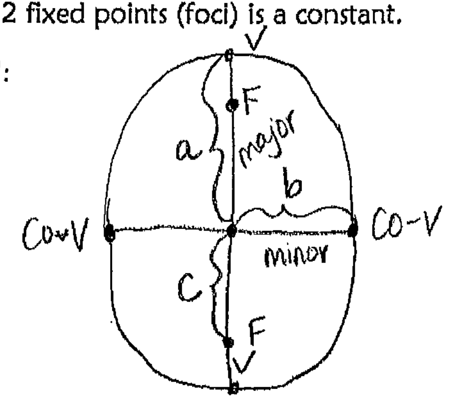
10.02 Ellipses 10.02 Practice [illegible] State the center vertices foci and eccentricity

Ellipse: A conic section where the sum of the distance from 2 fixed points (foci) is a constant.

FigureH:



FigureV:



Horizontal Major Axis	Vertical Major Axis
$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1, where\ a>b$	$\frac{(x-h)^2}{b^2}+\frac{(y-k)^2}{a^2}=1, where\ a>b$
Center: (h, k) Vertices: (h ± a, k) Co-vertices: (h, k ± b) Foci: (h ± c, k), $a^2-b^2=c^2$ **Eccentricity** = $\frac{c}{a}$	Center: (h, k) Vertices: (h, k ± a) Co-vertices: (h ± b, k) Foci: (h, k ± c), $a^2-b^2=c^2$ **Eccentricity** = $\frac{c}{a}$

Examples: Graph the ellipse. State the center, vertices, co-vertices, foci, and eccentricity.

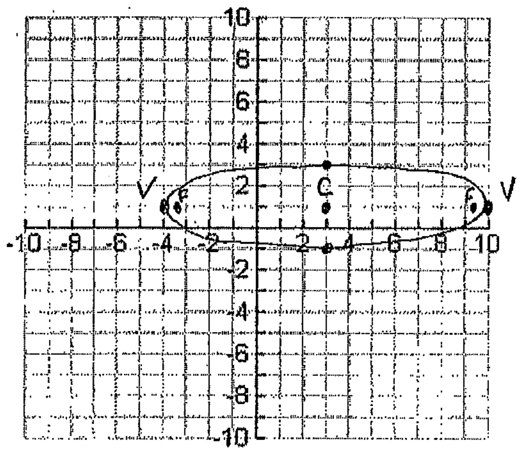
1. $\frac{(x-3)^2}{\underset{a^2}{49}}+\frac{(y-1)^2}{\underset{b^2}{4}}=1$ Center: (3, 1)

Vertices: (-4, 1) (10, 1)

Co-Vertices: (3, -1) (3, 3)

Foci: $(3 \pm 3\sqrt{5}, 1)$ E: $3\sqrt{5}/7$

$c^2 = 49-4$ $c = 3\sqrt{5}$



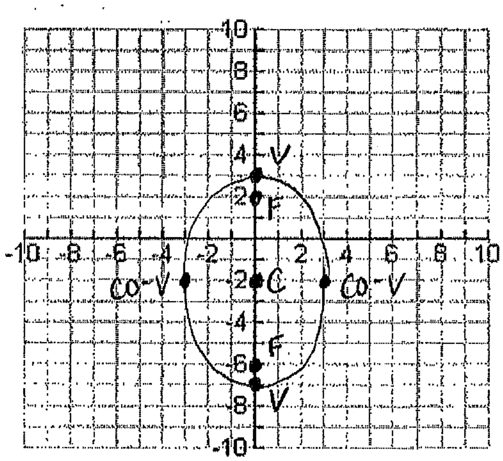
2. $\frac{x^2}{\underset{b^2}{9}}+\frac{(y+2)^2}{\underset{a^2}{25}}=1$ Center: (0, -2)

Vertices: (0, 3) (0, -7)

Co-Vertices: (-3, -2) (3, -2)

Foci: (0, -6) (0, 2) E: 4/5

$c^2 = 25-9$ $c = 4$



10.01 Practice: Graph the ellipse. State the center, vertices, co-vertices, foci, and eccentricity.

1. $\frac{(x-3)^2}{\underset{a^2}{81}}+\frac{(y+5)^2}{\underset{b^2}{25}}=1$

$c^2=81-25$

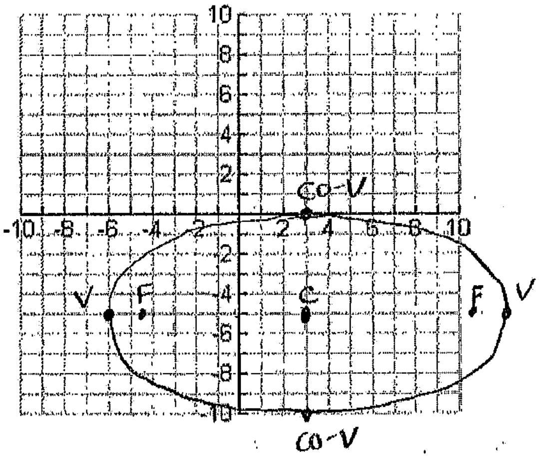
$c=2\sqrt{14}$

Center: $(3,-5)$ Eccentricity: $\frac{2\sqrt{14}}{9}$

Vertices: $(-6,-5)\ (12,-5)$

Co-Vertices: $(3,-10)\ (3,0)$

Foci: $(3\pm2\sqrt{14},-5)$



2. $\frac{(x+2)^2}{64}+\frac{(y-6)^2}{1}=1$

$c^2=64-1$

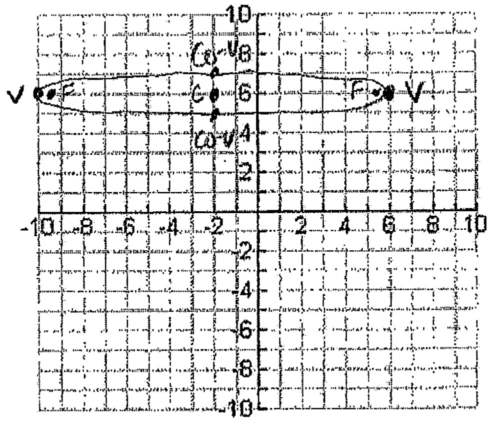
$c=3\sqrt{7}$

Center: $(-2,6)$ Eccentricity: $3\sqrt{7}/8$

Vertices: $(-10,6)(6,6)$

Co-Vertices: $(-2,5)\ (-2,7)$

Foci: $(-2\pm3\sqrt{7},6)$



3. $\frac{x^2}{\underset{b^2}{9}}+\frac{y^2}{\underset{a^2}{64}}=1$

$c^2=64-9$

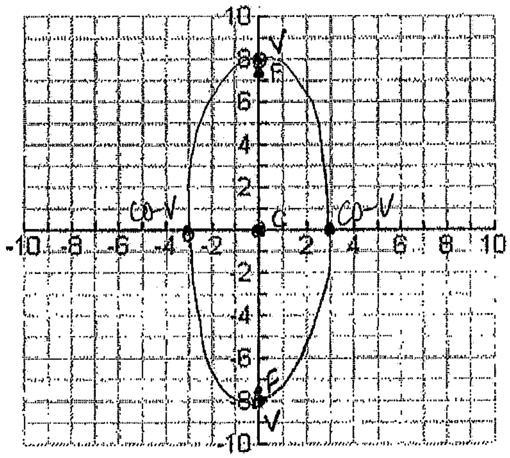
$c=\sqrt{55}$

Center: $(0,0)$ Eccentricity: $\frac{\sqrt{55}}{8}$

Vertices: $(0,\pm8)$

Co-Vertices: $(\pm3,0)$

Foci: $(0,\pm\sqrt{55})$



4. $\frac{(x+2)^2}{\underset{b^2}{16}}+\frac{(y+3)^2}{\underset{a^2}{36}}=1$

$c^2=36-16$

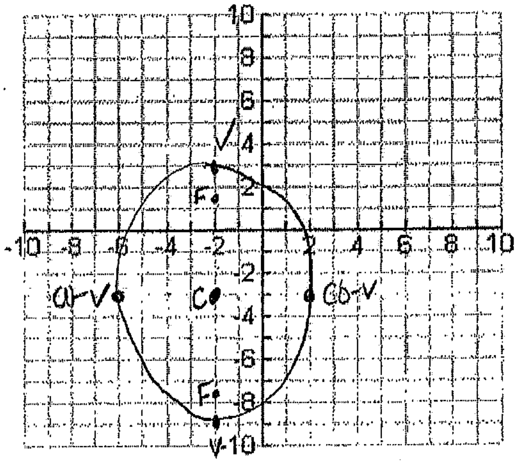
$c=2\sqrt{5}$

Center: $(-2,-3)$ Eccentricity: $\frac{2\sqrt{5}}{6}=\frac{\sqrt{5}}{3}$

Vertices: $(-2,-9)\ (-2,3)$

Co-Vertices: $(-6,-3)\ (2,-3)$

Foci: $(-2,-3\pm2\sqrt{5})$

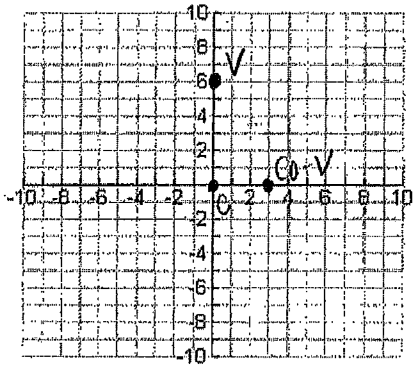


10.04 Writing Equations

Write the equation in standard form that meets each set of conditions.

Ellipses

1. The length of the semi-major axis is twice the length of the horizontal semi-~~major~~ minor axis, the center is at the origin, and b=3.



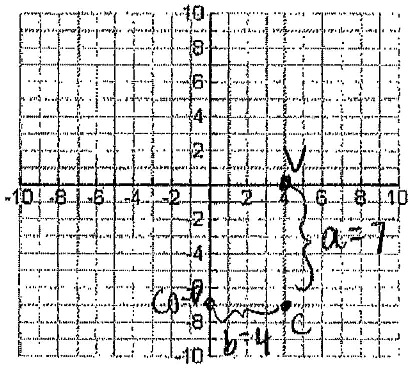
$a =$ 6

$b =$ 3

$c =$ $3\sqrt{3}$

Standard Form: $\frac{x^2}{9}+\frac{y^2}{36}=1$

2. The ellipse is tangent to the x-axis and the y-axis, and the center is (4, -7)



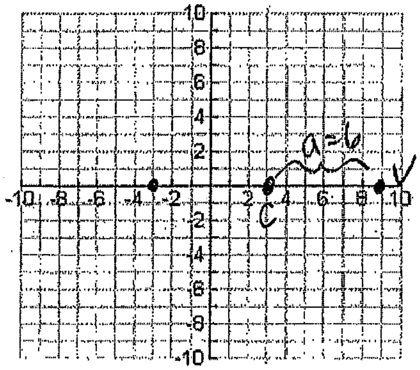
$a =$ 7

$b =$ 4

$c =$

Standard Form: $\frac{(x-4)^2}{16}+\frac{(y+7)^2}{49}=1$

3. The ~~vertical~~ horizontal major axis is ~~20~~ 12 units. The center is at (3,0) and the eccentricity equals ~~3/5~~ 2/3.



$a =$ 6

$b =$

$c =$ 4

$16=36-b^2$

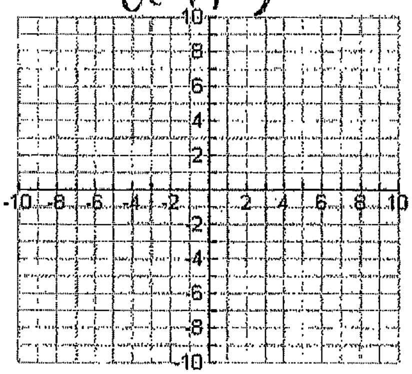
$20=b^2$

Standard Form: $\frac{(x-3)^2}{36}+\frac{y^2}{20}=1$

4. Foci: $(-6, 9 \pm 2\sqrt{30})$. The eccentricity equals $\frac{2\sqrt{30}}{13}$. $\frac{c}{a}$

C (vertical)

C(-6,9)



$a =$ 13

$b =$

$c =$ $2\sqrt{30}$

$120=169-b^2$

$49=b^2$

Standard Form: $\frac{(x+6)^2}{49}+\frac{(y-9)^2}{169}=1$

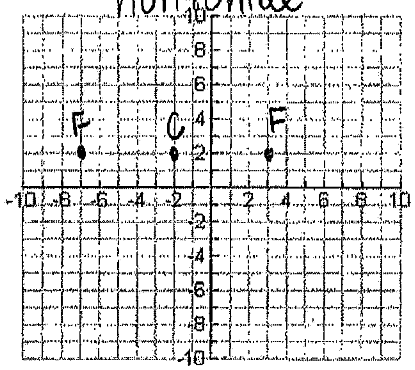
Hyperbolas

5. The length of the $\overset{\text{major}}{\text{transverse}}$ axis is 6 units and the foci are at (3, 2) and (-7, 2).

a=3

$C(-2,2)$

horizontal



$a =$ 3

$b =$ ______

$c =$ 5

$25=9+b^2$

$16=b^2$

Standard Form: $\dfrac{(x+2)^2}{9} - \dfrac{(y-2)^2}{16} = 1$

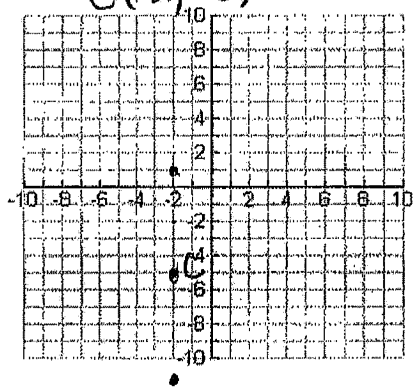
6. The endpoints of the conjugate axis: (-2, 1) (-2, -11)

Eccentricity = $\frac{5}{4}$ $\frac{c}{a}$

$4^2+6^2 \neq 5^2$

$8^2+6^2=10^2$ ✓

$C(-2,-5)$



$a =$ 8

$b =$ 6

$c =$ 10

Standard Form: $\dfrac{(x+2)^2}{64} - \dfrac{(y+5)^2}{36} = 1$

7. Vertices (-3, 6) (-3, -4)

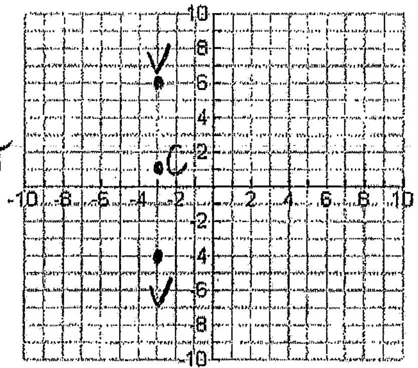
Perimeter of Central Rectangle = 72

$C(-3,1)$ $a=5$

$P=4a+4b$

$72=20+4b$ $b=13$

vert



$a =$ 5

$b =$ 13

$c =$ ______

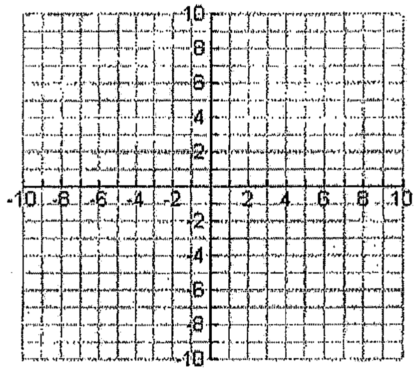
Standard Form: $\dfrac{(y-1)^2}{25} - \dfrac{(x+3)^2}{169} = 1$

8. Foci: $(7 \pm \overset{c}{7\sqrt{2}}, -3)$.

Asymptotes: $y = x - 10$ and $y = -x + 4$

$C(7,-3)$ horiz

$a=b$ $m=\frac{b}{a}=1$



$a =$ 7

$b =$ 7

$c =$ $7\sqrt{2}$

Standard Form: $\dfrac{(x-7)^2}{49} - \dfrac{(y+3)^2}{49} = 1$

$a^2+b^2=(7\sqrt{2})^2$

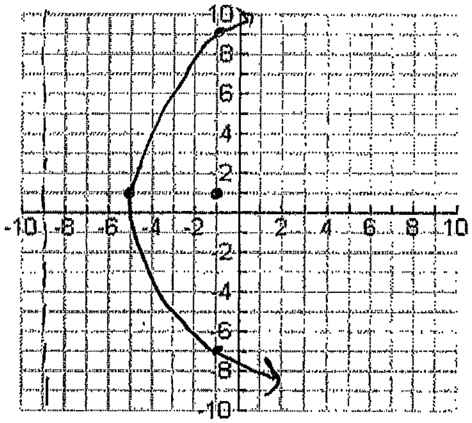
since $a=b \rightarrow a^2+a^2=98$

$2a^2=98$

$a^2=49$

Parabolas $(y-k)^2 = 4p(x-h)$

9. Vertex: (-5, 1). Focus: (-1, 1)



h: -5 k: 1 p: 4

Standard Form: $(y-1)^2 = 16(x+5)$

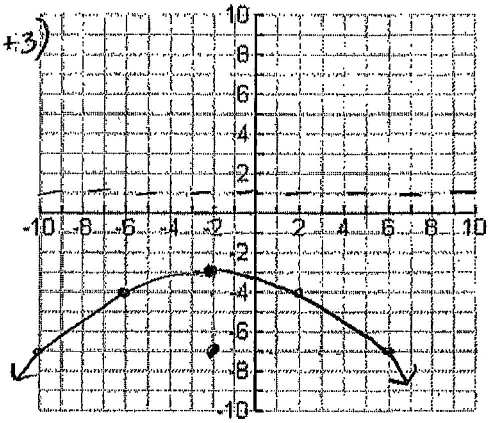
$(x-h)^2 = 4p(y-k)$

10. Passes through the point ~~[illegible]~~ $(-6,-4)$, opens ~~left~~, vertex is (-2, -3).
down

$(-6+2)^2 = 4p(-4+3)$

$16 = -4p$

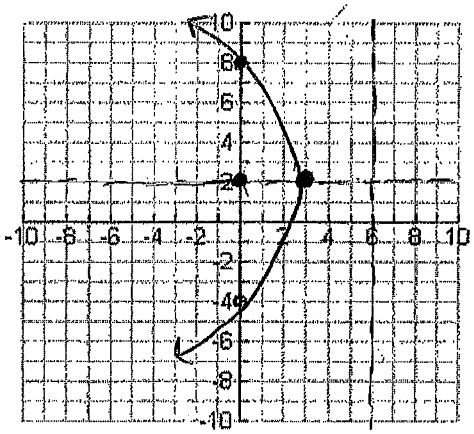
$p = -4$



h: -2 k: -3 p: -4

Standard Form: $(x+2)^2 = -16(y+3)$

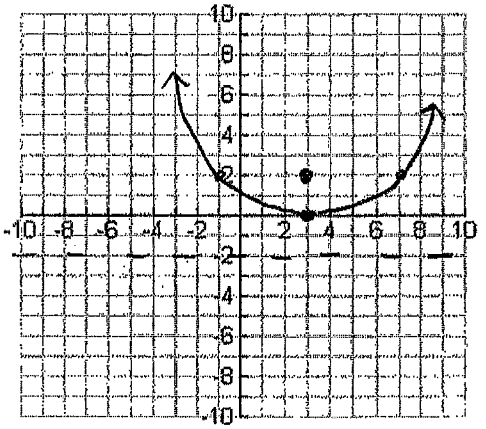
11. axis of symmetry: y = 2
focus: (0, 2), p = -3.



h: 3 k: 2 p: -3

Standard Form: $(y-2)^2 = -12(x-3)$

12. The focus is at (3, 2), the distance from the focus to the vertex is 2 units, and the function has a minimum.



h: 3 k: 0 p: 2

Standard Form: $(x-3)^2 = 8y$

Accelerated Pre-Calculus

Name: Key

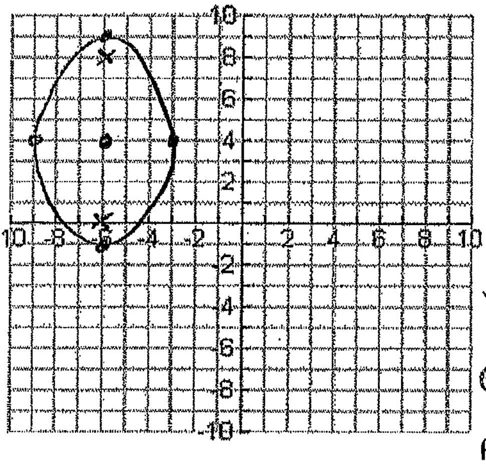
8.03 Ellipses – Day 2

Date: ____________

Review: For each ellipse, identify the coordinates of the center, the vertices, and co-vertices. Then graph.

1. $\frac{(x+6)^2}{9}+\frac{(y-4)^2}{25}=1$

2. $\frac{(x-1)^2}{81}+\frac{y^2}{64}=1$



center
(−6,4)

a = 5 ↕

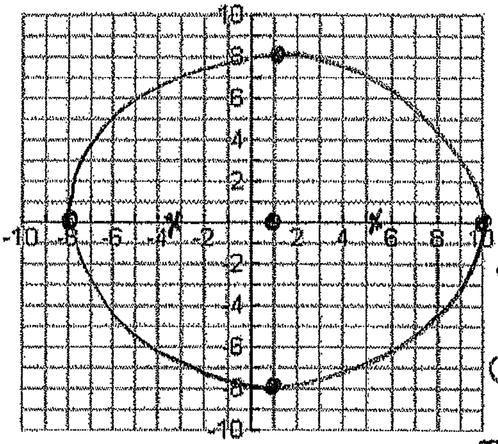
b = 3 ↔

c = 4 ↕

Vertices
(−6,9) (−6,−1)

CoVertices
(−9,4) (−3,4)

Foci
(−6,8) (−6,0)



center
(1,0)

a = 9 ↔

b = 8 ↕

c = $\sqrt{17}$ ↔

Vertices
(10,0) (−8,0)

CoVertices
(1,8) (1,−8)

Foci
$(1\pm\sqrt{17},0)$

How would you identify the characteristics of the ellipse if the equation is not in standard form?

Write the standard form of the equation of each ellipse and then graph the equation. List the coordinates of the center, foci, and the major and minor axis vertices. State the eccentricity of the ellipse.

3. $2x^2 + 18y^2 + 8x + 108y + 99 = 1$

C: $2x^2+8x+18y^2+108y=-98$

A: $2(x^2+4x+\underline{4})+18(y^2+6y+\underline{9})=-98$

B: $(\frac{4}{2})^2\nearrow$ $(\frac{6}{2})^2\nearrow$ $+2(4)$ $+18(9)$

factor: $2(x+2)^2+18(y+3)^2=72$

divide: $\frac{2(x+2)^2}{72}+\frac{18(y+3)^2}{72}=\frac{72}{72}$

a = 6 ↔

b = 2 ↕

c = $\sqrt{32}=4\sqrt{2}$ ↔

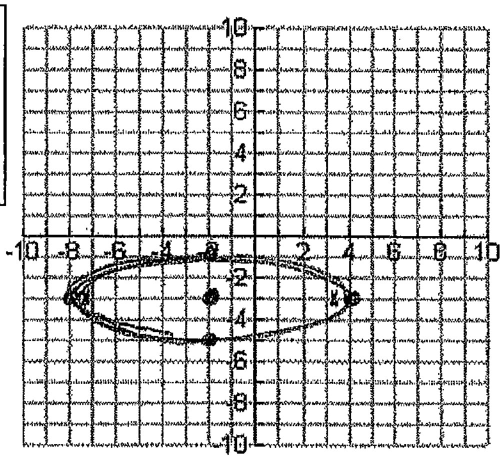
Standard Form: $\frac{(x+2)^2}{36}+\frac{(y+3)^2}{4}=1$

Center: (−2,−3) Vertices: (−8,−3) (4,−3)

Co–Vertices: (−2,−1) (−2,−5)

Foci: $(-2\pm4\sqrt{2},-3)$ Eccentricity = $\frac{4\sqrt{2}}{6}=\frac{2\sqrt{2}}{3}$

≈ 0.943



4. $9x^2 + 4y^2 - 72x + 40y + 100 = 0$

C: $9x^2-72x+4y^2+40y=-100$

A: $9(x^2-8x+\underline{16})+4(y^2+10y+\underline{25})=-100$

B: $(\frac{-8}{2})^2\nearrow$ $(\frac{10}{2})^2\nearrow$ $+9(\underline{16})$ $+4(\underline{25})$

factor: $9(x-4)^2+4(y+5)^2=144$

divide: $\frac{9(x-4)^2}{144}+\frac{4(y+5)^2}{144}=\frac{144}{144}$

a = 6 ↕

b = 4 ↔

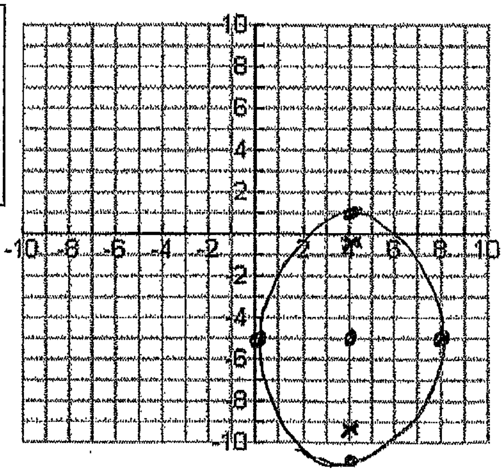
c = $\sqrt{20}=2\sqrt{5}$ ↕

Standard Form: $\frac{(x-4)^2}{16}+\frac{(y+5)^2}{36}=1$

Center: (4,−5) Vertices: (4,1) (4,−11)

Co–Vertices: (0,−5) (8,−5)

Foci: $(4,-5\pm2\sqrt{5})$ Eccentricity = $\frac{2\sqrt{5}}{6}=\frac{\sqrt{5}}{3}$



5. $49x^2 + 16y^2 + 160y - 384 = 0$

C : $49x^2+16y^2+160y = 384$

A : $49x^2 + 16(y^2+10y+\underline{25}) = 384 + 16(25)$

B : $(\frac{10}{2})^2 \nearrow$

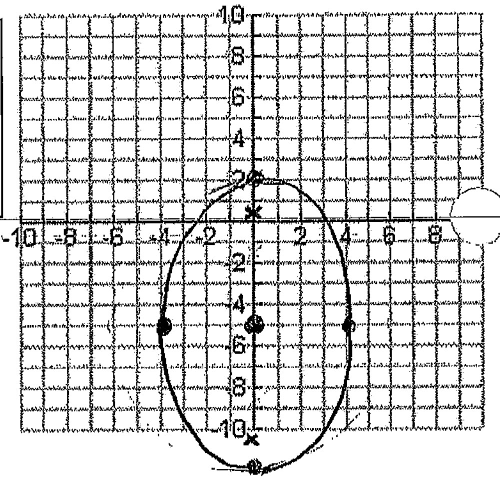
factor: $\frac{49x^2}{784}+\frac{16(y+5)^2}{784}=\frac{784}{784}$

divide: 784 784 784

$a = \underline{7 \updownarrow}$

$b = \underline{4 \leftrightarrow}$

$c = \underline{\sqrt{33} \updownarrow}$



Standard Form: $\frac{x^2}{16}+\frac{(y+5)^2}{49}=1$

Center: (0,-5) Vertices: (0,-12) (0,2)

Co-Vertices: (4,-5) (-4,-5)

Foci: $(0,-5\pm\sqrt{33})$ Eccentricity = $\frac{\sqrt{33}}{7}$

6. $3x^2 + y^2 + 18x - 2y + 4 = 0$

C: $3x^2+18x+y^2-2y=-4$

A: $3(x^2+6x+\underline{9})+(y^2-2y+\underline{1})=-4 + 3(\underline{9}) + 1(\underline{1})$

B: $(\frac{6}{2})^2 \nearrow$ $(\frac{-2}{2})^2 \nearrow$

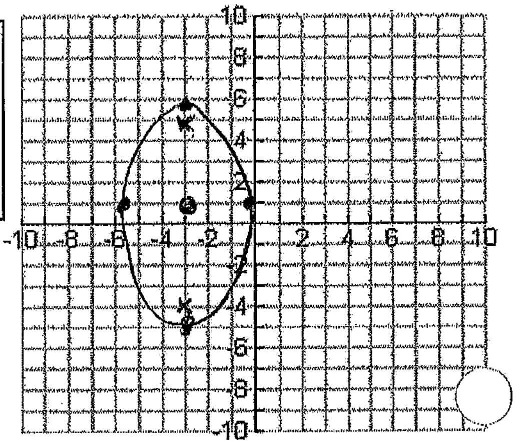
factor: $\frac{3(x+3)^2}{24}+\frac{(y-1)^2}{24}=\frac{24}{24}$

divide: 24 24 24

$a = \underline{\sqrt{24}=2\sqrt{6}\updownarrow}$

$b = \underline{\sqrt{8}=2\sqrt{2}\leftrightarrow}$

$c = \underline{4 \updownarrow}$



Standard Form: $\frac{(x+3)^2}{8}+\frac{(y-1)^2}{24}=1$

Center: (-3,1) Vertices: $(-3, 1\pm2\sqrt{6})$

Co-Vertices: $(-3\pm2\sqrt{2}, 1)$

Foci: (-3,5) (-3,-3) Eccentricity = $\frac{4}{2\sqrt{6}}=\frac{2}{\sqrt{6}}=\frac{2\sqrt{6}}{6}=\frac{\sqrt{6}}{3}$

7. $18y^2 + 12x^2 - 144y - 48x = -120$

C: $12x^2-48x+18y^2-144y=-120$

A: $12(x^2-4x+\underline{4})+18(y^2-8y+\underline{16})=-120 + 12(4) + 18(16)$

B:

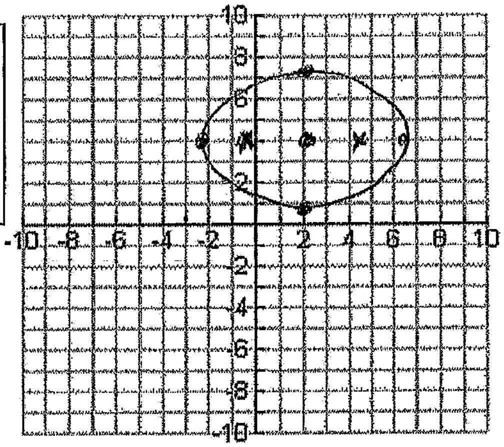
factor: $\frac{12(x-2)^2}{216}+\frac{18(y-4)^2}{216}=\frac{216}{216}$

divide: 216 216 216

$a = \underline{\sqrt{18}=3\sqrt{2}\leftrightarrow}$

$b = \underline{\sqrt{12}=2\sqrt{3}\updownarrow}$

$c = \underline{\sqrt{6}\leftrightarrow}$



Standard Form: $\frac{(x-2)^2}{18}+\frac{(y-4)^2}{12}=1$

Center: (2,4) Vertices: $(2\pm3\sqrt{2}, 4)$

Co-Vertices: $(2, 4\pm2\sqrt{3})$

Foci: $(2\pm\sqrt{6}, 4)$ Eccentricity = $\frac{\sqrt{6}}{3\sqrt{2}}=\frac{\sqrt{3}}{3}$

Accelerated Pre-Calculus

Name: Key

8.04 Ellipses – Day 3

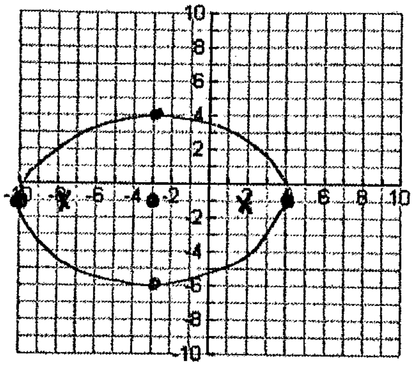
Date:

Write the equation of the ellipse in standard form that meets each set of conditions. Calculate a, b, and c. Graph, then list the coordinates of the center, foci, vertices, and co-vertices.

1. The center is at (–3, –1), the length of the horizontal semi-major axis is 7 units, and the length of the semi-minor axis is 5 units.

$49-25=c^2$

$24=c^2$



a = 7↔

b = 5↕

c = $2\sqrt{6}$↔

Standard Form: $\frac{(x+3)^2}{49}+\frac{(y+1)^2}{25}=1$

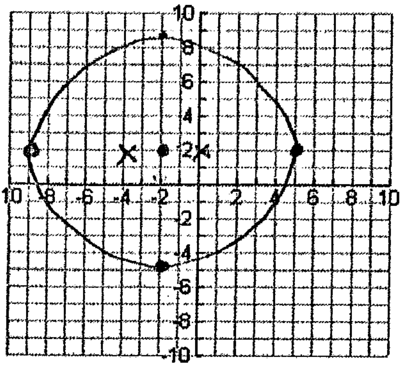
Vertices: (-10,-1)(4,-1) Co–Vertices: (-3,4) (-3,-6)

Foci: $(-3\pm2\sqrt{6},-1)$ Eccentricity: $2\sqrt{6}/7$

2. The foci are at ~~(–2, 0)~~ (0,2) and ~~(2, 0)~~ (-4,2) and a = 7.

$49-b^2=4$

$b^2=45$



a = 7↔

b = $3\sqrt{5}$↕

c = 2↔

Standard Form: $\frac{(x+2)^2}{49}+\frac{(y-2)^2}{45}=1$

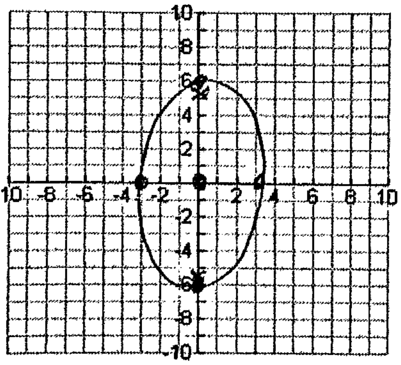
Vertices: (-9,2)(5,2) Co–Vertices: $(-2, 2\pm3\sqrt{5})$

Center: (-2, 2) Eccentricity = 2/7

3. The length of the semi-major axis is twice 2 the length of the horizontal semi-minor axis, the center is at the origin, and b = 3.

$36-9=c^2$

$27=c^2$



a = 6 ↕

b = 3↔

c = $3\sqrt{3}$ ↕

Standard Form: $\frac{x^2}{9}+\frac{y^2}{36}=1$

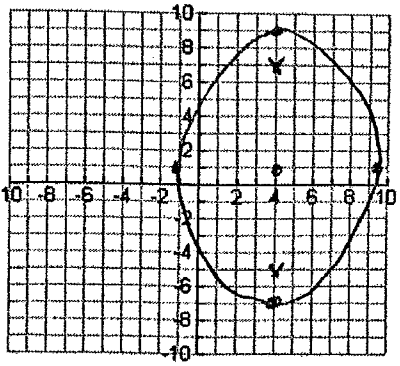
Vertices: $(0, \pm6)$ Co–Vertices: $(\pm3, 0)$

Foci: $(0, \pm3\sqrt{3})$ Eccentricity: $3\sqrt{3}/6 = \sqrt{3}/2$

4. The major axis endpoints are ~~(5, 15)~~, ~~(5, –5)~~ (4,9) (4,-7) and the foci are at ~~(5, 13)~~ and ~~(5, –3)~~. (4,7) (4,-5)

$64-b^2=36$

$b^2=28$



a = 8

b = $2\sqrt{7}$

c = 6

Standard Form: $\frac{(x-4)^2}{28}+\frac{(y-1)^2}{64}=1$

Vertices: (4,9)(4,-7) Co–Vertices: $(4\pm2\sqrt{7}, 1)$

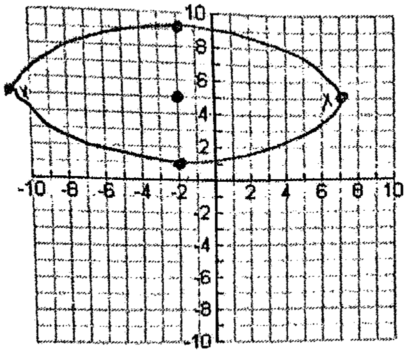
Center: (4,1) Eccentricity = 6/8 = 3/4

5. The endpoints of the major axis are (-11, 5) and (7, 5). The endpoints of the minor axis are (-2, 9) and (-2, 1).

$81-16=c^2$

$65=c^2$

$\sqrt{65}=c$



a = 9 ↔

b = 4 ↕

c = $\sqrt{65}$ ↔

Standard Form: $\frac{(x+2)^2}{81}+\frac{(y-5)^2}{16}=1$

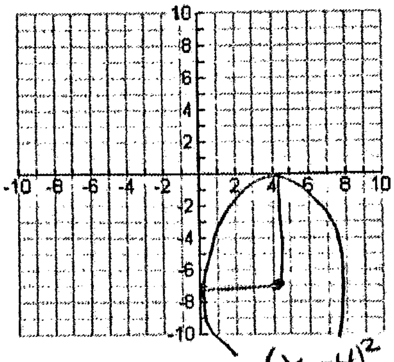
Center: (−2, 5) Foci: (−2 ± $\sqrt{65}$, 5)

Eccentricity: $\sqrt{65}/9$

6. The ellipse is tangent to the x-axis and the y-axis and the center is (4, -7).

$49-16=c^2$

$33=c^2$



a = 7 ↕

b = 4 ↔

c = $\sqrt{33}$

Standard Form: $\frac{(x-4)^2}{16}+\frac{(y+7)^2}{49}=1$

Vertices: (4, 0) (4, −14) Co-Vertices: (0, −7) (8, −7)

Foci: (4, −7 ± $\sqrt{33}$) Eccentricity: $\sqrt{33}/7$

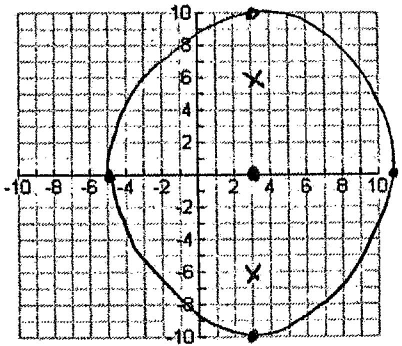
midpoint = center (1, 2)

7. The vertical major axis is 20 units, $a=10$ the center is at (3, 0), and the eccentricity equals $\frac{3}{5}$.

$\frac{c}{a}=\frac{3}{5}=\frac{c}{10}$

$100-b^2=36$

$b^2=64$



a = 10 ↕

b = 8 ↔

c = 6 ↕

Standard Form: $\frac{(x-3)^2}{64}+\frac{y^2}{100}=1$

Vertices: (3, ±10) Co-Vertices: (11, 0) (−5, 0)

Foci: (3, ±6)

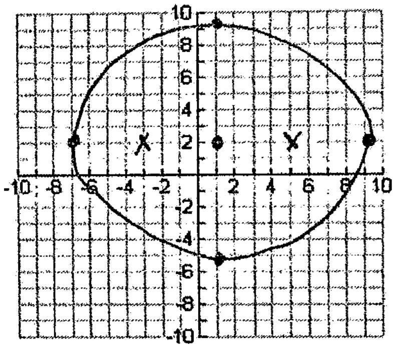
8. The foci are at (−3, 2) and (5, 2) and the eccentricity equals 0.5

$\frac{c}{a}=\frac{1}{2}=\frac{4}{a}$

$64-b^2=16$

$b^2=48$

$b=$ [illegible]



a = 8 ↔

b = $4\sqrt{3}$ ↕

c = 4 ↔

Standard Form: $\frac{(x-1)^2}{64}+\frac{(y-2)^2}{48}=1$

Vertices: (9, 2) (−7, 2) Co-Vertices: (1, 2 ± $4\sqrt{3}$)

Center: (1, 2)

9. The eccentricity equals $\frac{\sqrt{7}}{4}$, and the co-vertices are (2, -9) and (-4, -9). ← center = midpoint (−1, −9)

distance = 3

b

Standard Form: $\frac{(x+1)^2}{9}+\frac{(y+9)^2}{16}=1$

$\frac{c}{a}=\frac{\sqrt{7}}{4}$

$a^2-b^2=c^2$

$a^2-9=c^2$

check c and a?

$4^2-9\overset{?}{=}\sqrt{7}^2$

$16-9=7$ ✓

$a=4$ $c=\sqrt{7}$

a = 4 ↕

b = 3 ↔

c = $\sqrt{7}$

10. Foci: (−6, 9 + 2√30) and (−6, 9 − 2√30) the eccentricity equals $\frac{2\sqrt{30}}{13}$.

Standard Form: $\frac{(x+6)^2}{49}+\frac{(y-9)^2}{169}=1$

center @ (−6, 9)

$169-b^2=120$

$b^2=49$

a = 13 ↕

b = 7

c = $2\sqrt{30}$

8.02 Practice: Graph the ellipse. State the center, vertices, co-vertices, foci, and eccentricity.

1. $\frac{(x-3)^2}{81} + \frac{(y+5)^2}{25} = 1$

a^2 b^2

$a = 9 \leftrightarrow$

$b = 5 \updownarrow$

$c = 2\sqrt{14} \leftrightarrow$

Center: $(3, -5)$ Eccentricity: $2\sqrt{14}/9 \approx .831$

Vertices: $(12, -5)$ $(-6, -5)$

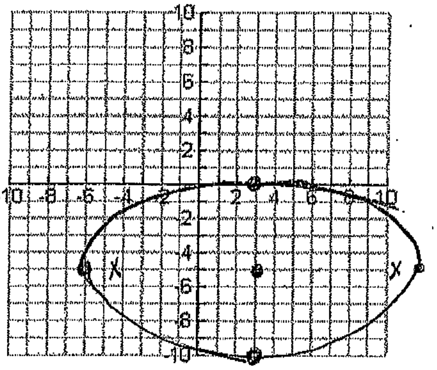
Co-Vertices: $(3, 0)$ $(3, -10)$

Foci: $(3 \pm 2\sqrt{14}, -5)$

$81 - 25 = c^2$

$56 = c^2$

$\sqrt{56} = c$



2. $\frac{(x+2)^2}{64} + \frac{(y-6)^2}{1} = 1$

$a^2 \nearrow$ $1 \leftarrow b^2$

$a = 8 \leftrightarrow$

$b = 1 \updownarrow$

$c = 3\sqrt{7} \leftrightarrow$

Center: $(-2, 6)$ Eccentricity: $3\sqrt{7}/8 \approx .992$

Vertices: $(-10, 6)$ $(6, 6)$

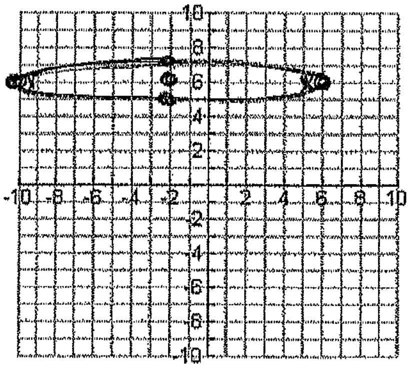
Co-Vertices: $(-2, 7)$ $(-2, 5)$

Foci: $(-2 \pm 3\sqrt{7}, 6)$

$64 - 1 = c^2$

$63 = c^2$

$\sqrt{63} = c$



3. $\frac{x^2}{9} + \frac{y^2}{64} = 1$

$b^2 \nearrow$ $\nwarrow a^2$

$a = 8 \updownarrow$

$b = 3 \leftrightarrow$

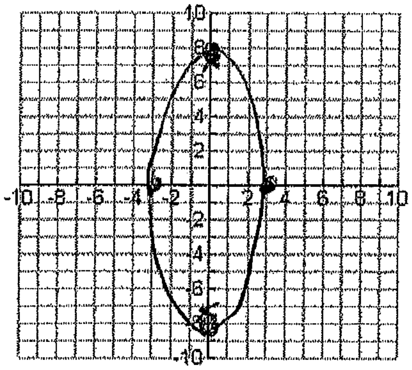
$c = \sqrt{55} \updownarrow$

Center: $(0, 0)$ Eccentricity: $\sqrt{55}/8 \approx .927$

Vertices: $(0, 8)$ $(0, -8)$

Co-Vertices: $(3, 0)$ $(-3, 0)$

Foci: $(0, \pm\sqrt{55})$



4. $\frac{(x+2)^2}{16} + \frac{(y+3)^2}{36} = 1$

$b^2 \nearrow$ $\leftarrow a^2$

$a = 6 \updownarrow$

$b = 4 \leftrightarrow$

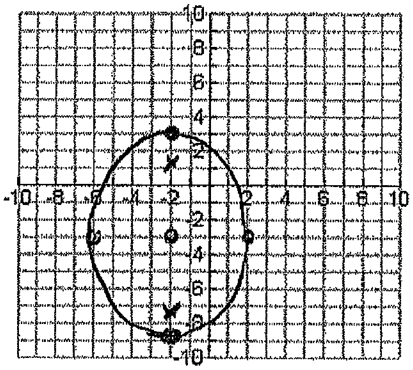
$c = 2\sqrt{5} \updownarrow$

Center: $(-2, -3)$ Eccentricity: $\frac{2\sqrt{5}}{6} = \frac{\sqrt{5}}{3} \approx .745$

Vertices: $(-2, 3)$ $(-2, -9)$

Co-Vertices: $(-6, -3)$ $(2, -3)$

Foci: $(-2, -3 \pm 2\sqrt{5}$



Use completing the square to factor the following equations.

5. $16x^2 + 9y^2 - 64x - 18y - 71 = 0$

C: $16x^2 - 64x + 9y^2 - 18y = 71$

A: $16(x^2 - 4x + \underline{4}) + 9(y^2 - 2y + \underline{1}) = 71 + 16(4) + 9(1)$

B: $\left(\frac{-4}{2}\right)^2 \nearrow$ $\left(\frac{-2}{2}\right)^2 \nearrow$

factor: $16(x-2)^2 + 9(y-1)^2 = 144$

divide: $\frac{16(x-2)^2}{144} + \frac{9(y-1)^2}{144} = \frac{144}{144}$

Ellipse: $\frac{(x-2)^2}{9} + \frac{(y-1)^2}{16} = 1$

6. $4x^2 + 25y^2 - 16x + 250y + 541 = 0$

C: $4x^2 - 16x + 25y^2 + 250y = -541$

A: $4(x^2 - 4x + \underline{4}) + 25(y^2 + 10y + \underline{25}) = -541 + 4(4) + 25(25)$

B: $\left(\frac{-4}{2}\right)^2 \nearrow$ $\left(\frac{10}{2}\right)^2 \nearrow$

factor. $4(x-2)^2 + 25(y+5)^2 = 100$

divide: $\frac{4(x-2)^2}{100} + \frac{25(y+5)^2}{100} = \frac{100}{100}$

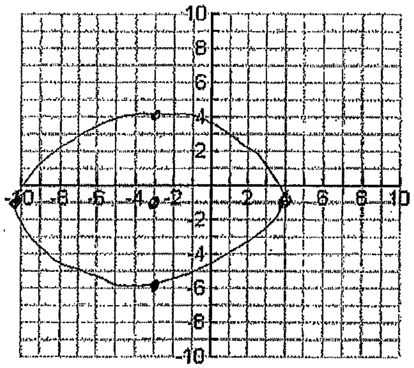
Ellipse: $\frac{(x-2)^2}{25} + \frac{(y+5)^2}{4} = 1$

10.04 Extra Practice

Write the equation in standard form that meets each set of conditions.

Ellipses

1. The center is at $(\overset{h}{-3}, \overset{k}{-1})$, the length of the horizontal semi-major axis is 7 units, and the length of the semi-minor axis is 5 units.



$a =$ 7

$b =$ 5

$c =$ ______

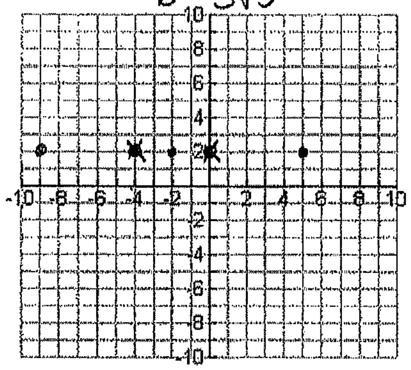
Standard Form: $\frac{(x+3)^2}{49} + \frac{(y+1)^2}{25} = 1$

2. The foci are at (0, 2) and (−4, 2) and a=7. Center: $(-2, 2)$

$49 - b^2 = 4$

$\sqrt{45} = \sqrt{b^2}$

$b = 3\sqrt{5}$



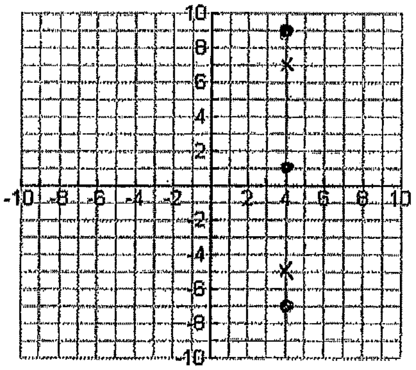
$a =$ 7

$b = 3\sqrt{5}$

$c =$ 2

Standard Form: $\frac{(x+2)^2}{49} + \frac{(y-2)^2}{45} = 1$

3. The major axis endpoints are (4, 9), (4, −7), and the foci are at (4, 7) and (4, −5). Center: $(4, 1)$



$a =$ 8

$b = 2\sqrt{7}$

$c =$ 6

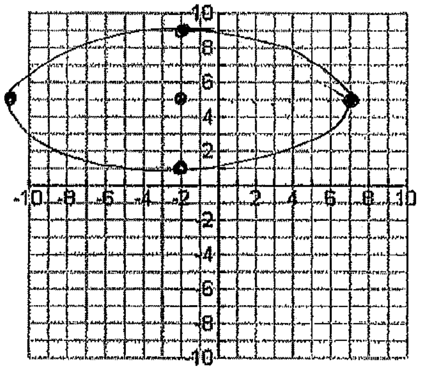
$64 - b^2 = 36$

$28 = b^2$

$b = 2\sqrt{7}$

Standard Form: $\frac{(x-4)^2}{28} + \frac{(y-1)^2}{64} = 1$

4. The endpoints of the major axis are (−11, 5) and (7, 5). The endpoints of the minor axis are (−2, 9) and (−2, 1)



$a =$ 9

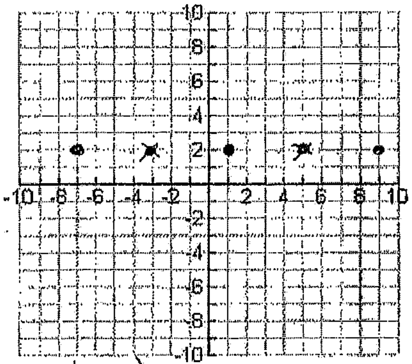
$b =$ 4

$c =$ ______

C: $(-2, 5)$

Standard Form: $\frac{(x+2)^2}{81} + \frac{(y-5)^2}{16} = 1$

5. The foci are at (−3, 2) and (5, 2) and the eccentricity equals 0.5. $= \frac{1}{2} = \frac{c}{a} = \boxed{\frac{4}{8}}$



$a =$ 8

$b =$ $4\sqrt{3}$

$c =$ 4

$64 - b^2 = 16$

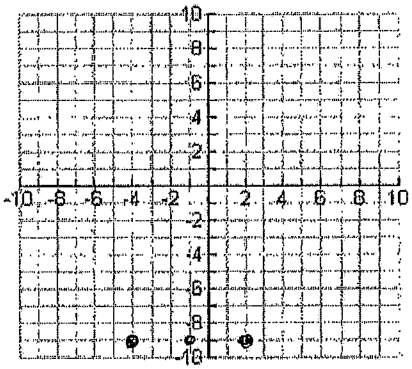
$48 = b^2$

$b = 4\sqrt{3}$

C: (1, 2)

Standard Form: $\frac{(x-1)^2}{64} + \frac{(y-2)^2}{48} = 1$

6. The eccentricity equals $\frac{\sqrt{7}}{4}$, and the co-vertices are (2, −9) and (−4, −9).



$a =$ 4

$b =$ 3

$c =$ $\sqrt{7}$

$a^2 - b^2 = c^2$

$16 - 9 \overset{?}{=} (\sqrt{7})^2$

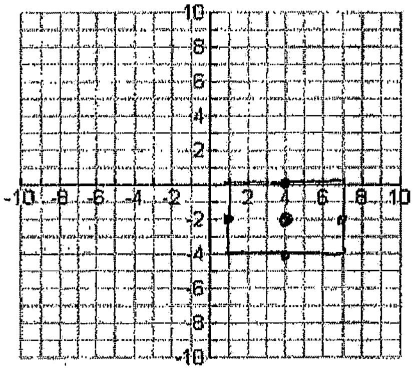
$7 = 7$ ✓

C: (−1, −9)

Standard Form: $\frac{(x+1)^2}{16} + \frac{(y+9)^2}{9} = 1$

Hyperbolas

7. The center is at $(\overset{h}{4}, \overset{k}{-2})$, a=2, b=3, and the transverse axis is vertical.



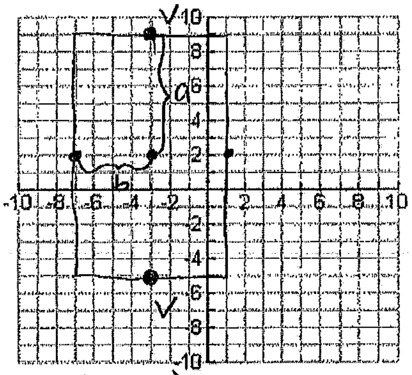
$a =$ 2

$b =$ 3

$c =$ ______

Standard Form: $\frac{(y+2)^2}{4} - \frac{(x-4)^2}{9} = 1$

8. The length of the conjugate axis is 8 units and the vertices are (−3, 9) and (−3, −5).



$a =$ 7

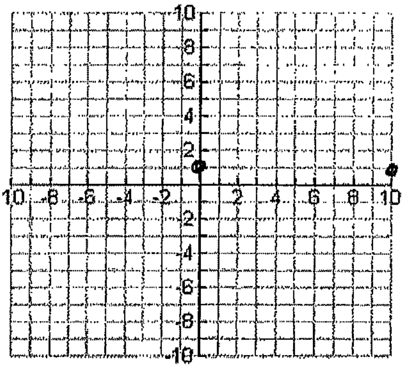
$b =$ 4

$c =$ ______

C: (−3, 2)

Standard Form: $\frac{(y-2)^2}{49} - \frac{(x+3)^2}{16} = 1$

9. The center is (0, 1), one focus is at (10, 1) and the eccentricity is $\frac{5}{3} = \frac{c}{a} = \frac{10}{6}$



$a =$ 6

$b =$ 8

$c =$ 10

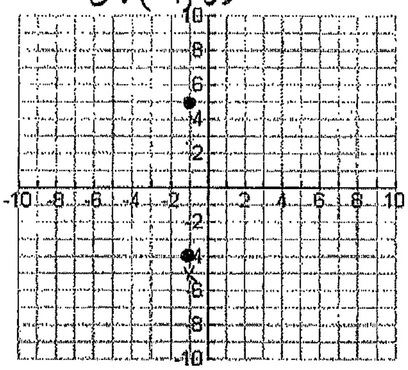
$36 + b^2 = 100$

$b^2 = 64$

$b = 8$

Standard Form: $\frac{x^2}{36} - \frac{(y-1)^2}{64} = 1$

10. Endpoints of the transverse axis:
(−1, 14), (−1, −4). $a = 9$
Foci: (−1, 15) (−1, −5)
C: (−1, 5)



$a =$ 9

$b = \sqrt{19}$

$c =$ 10

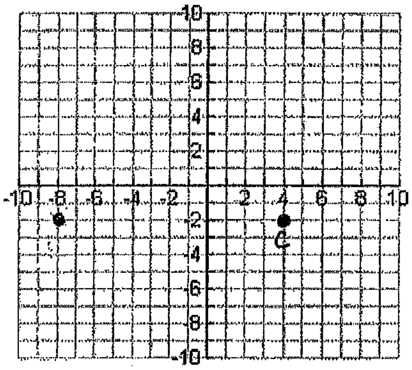
$81 + b^2 = 100$

$b^2 = 19$

$b = \sqrt{19}$

Standard Form: $\frac{(y-5)^2}{81} - \frac{(x+1)^2}{19} = 1$

11. Endpoints of the conjugate axis:
(16, −2), (−8, −2)
Foci: $(4, -2 \pm 4\sqrt{13})$
h k c



$a =$ 8

$b =$ 12

$c = 4\sqrt{13}$

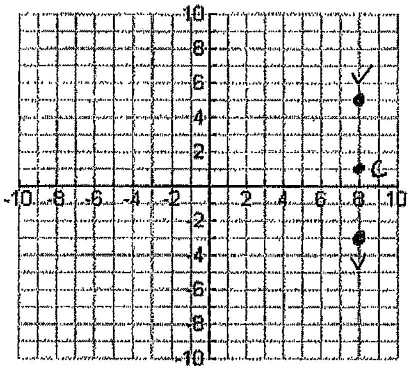
$a^2 + 144 = 208$

$a^2 = 64$

$a = 8$

Standard Form: $\frac{(y+2)^2}{64} - \frac{(x-4)^2}{144} = 1$

12. Vertices: (8, 5), (8, −3)
Eccentricity = $\frac{\sqrt{10}}{2} = \frac{c}{a} = \frac{2\sqrt{10}}{4}$



$a =$ 4

$b = 2\sqrt{6}$

$c = 2\sqrt{10}$

$16 + b^2 = 40$

$b^2 = 24$

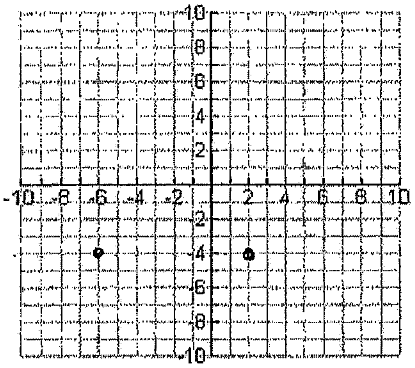
$b = 2\sqrt{6}$

Standard Form: $\frac{(y-1)^2}{16} - \frac{(x-8)^2}{24} = 1$

13. Vertices: (2, −4), (−14, −4) C: (-6, -4)

Asymptotes: $y = \frac{9}{8}x + \frac{11}{4}$ and

$y = -\frac{9}{8}x - \frac{43}{4}$ $m = \pm\frac{b}{a} = \pm\frac{9}{8}$



$a =$ 8

$b =$ 9

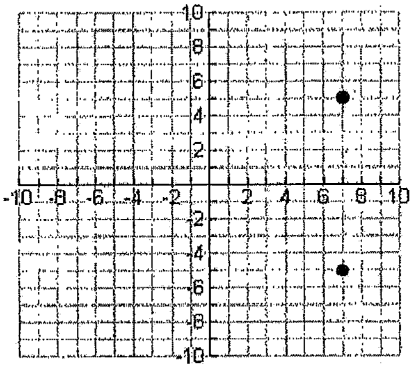
$c =$ ______

Standard Form: $\frac{(x+6)^2}{64} - \frac{(y+4)^2}{81} = 1$

14. Vertices: (7, 5), (7, −15) C: (7, -5)

Asymptotes: $y = \frac{5}{3}x - \frac{50}{3}$ and

$y = -\frac{5}{3}x + \frac{20}{3}$ $m = \pm\frac{a}{b} = \boxed{\frac{10}{b}} = \frac{5}{3}$



$a =$ 10

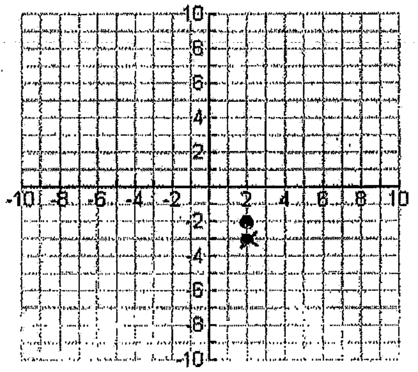
$b =$ 6

$c =$ ______

Standard Form: $\frac{(y+5)^2}{100} - \frac{(x-7)^2}{36} = 1$

Parabolas

15. Vertex: (2, −2). Focus: (2, −3)

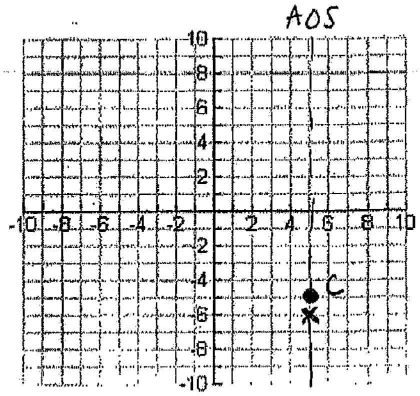


x^2

h: 2 k: -2 p: -1

Standard Form: $(x-2)^2 = -4(y+2)$

16. axis of symmetry: x=5
focus: (5, −6), p = -1

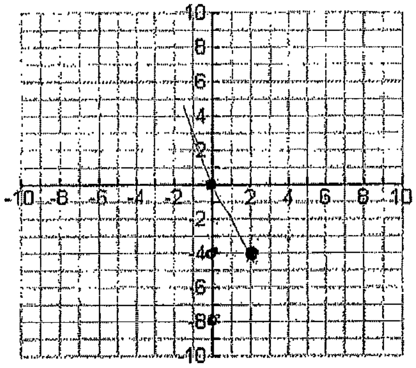


x^2

h: 5 k: -5 p: -1

Standard Form: $(x-5)^2 = -4(y+5)$

17. Passes through the point $(\overset{x}{0}, \overset{y}{-8})$ ~~$(\overset{x}{4}, \overset{y}{\sqrt{3}})$~~, opens ~~down~~ left; vertex is (2, -4).



h: 2 k: -4 p: 1

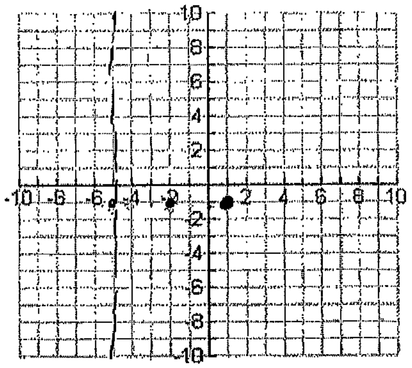
Standard Form: $(y+4)^2 = -8(x-2)$

$(-8+4)^2 = 4p(0-2)$

$16 = -8p$

$p = -2$

18. The focus is at (1, −1), ~~the distance from the focus to the vertex is 1 unit, and the function has a minimum.~~ directrix: $x = -5$



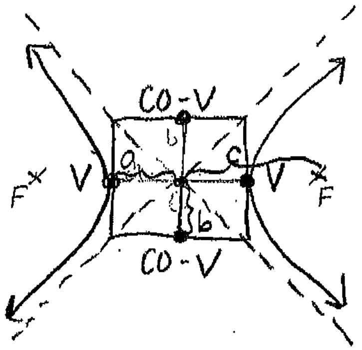
h: -2 k: -1 p: 3

Standard Form: $(y+1)^2 = 12(x+2)$

10.03 Hyperbolas

Hyperbola: a conic section where the difference of the distance from 2 fixed points (foci) is a constant.

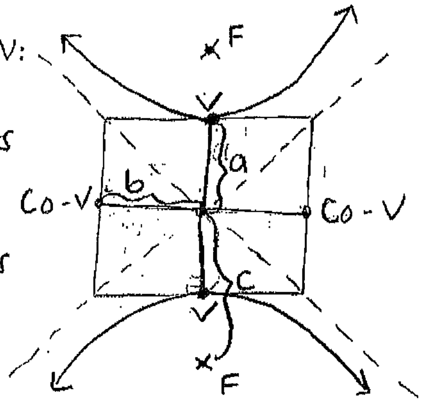
FigureH:



*Transverse axis connects Vertices

*Conjugate axis connects Co-Vertices

FigureV:



Horizontal Transverse Axis	Vertical Transverse Axis
$\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1$	$\frac{(y-k)^2}{a^2}-\frac{(x-h)^2}{b^2}=1$
Center: (h, k) Vertices: (h ± a, k) Foci: (h ± c, k), $a^2+b^2=c^2$ Asymptotes: y – k = ± $\frac{b}{a}$ (x – h) Eccentricity = $\frac{c}{a}$	Center: (h, k) Vertices: (h, k ± a) Foci: (h, k ± c), $a^2+b^2=c^2$ Asymptotes: y – k = ± $\frac{a}{b}$ (x – h) Eccentricity = $\frac{c}{a}$

Examples: Graph the Hyperbola. State the center, vertices, foci, asymptotes, and eccentricity.

1. $\frac{(x+1)^2}{16}-\frac{(y-1)^2}{9}=1$

$a = 4$ $b = 3$

$16 + 9 = c^2$

$25 = c^2$

$c = 5$

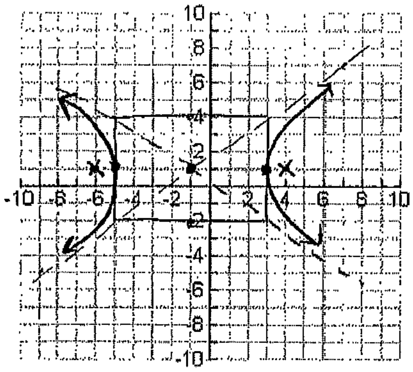
Center: (-1, 1)

Vertices: (-5, 1) (3, 1)

Foci: (-6, 1) (4, 1)

Asymptotes: $y - 1 = \pm \frac{3}{4}(x+1)$

Eccentricity: $5/4$



2. $\frac{(y+3)^2}{4}-\frac{(x+4)^2}{25}=1$

$a = 2$ $b = 5$

$4 + 25 = c^2$

$c = \sqrt{29}$

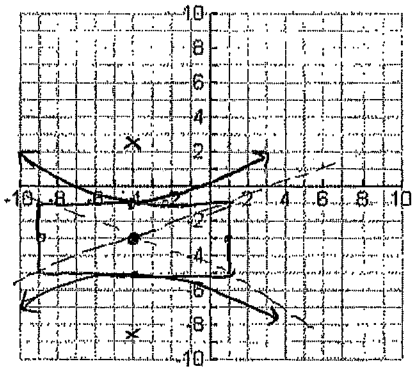
Center: (-4, -3)

Vertices: (-4, -1) (-4, -5)

Foci: $(-4, -3 \pm \sqrt{29})$

Asymptotes: $y + 3 = \pm 2/5 (x+4)$

Eccentricity: $\sqrt{29}/2$



10.02 Practice: Graph the Hyperbola. State the center, vertices, foci, and asymptotes.

1. $\frac{(x-3)^2}{4} - \frac{y^2}{81} = 1$ $a=2 \quad b=9$

$4+81=c^2$

$c=\sqrt{85}$

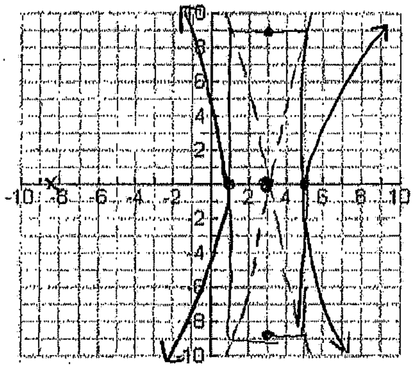
Center: $(3,0)$

Vertices: $(1,0)\ (5,0)$

Foci: $(3 \pm \sqrt{85}, 0)$

Asymptotes: $y = \pm \frac{9}{2}(x-3)$

Eccentricity: $\sqrt{85}/2$



2. $(y+6)^2 - \frac{(x+2)^2}{16} = 1$ $a=1 \quad b=4$

$1+16=c^2$

$c=\sqrt{17}$

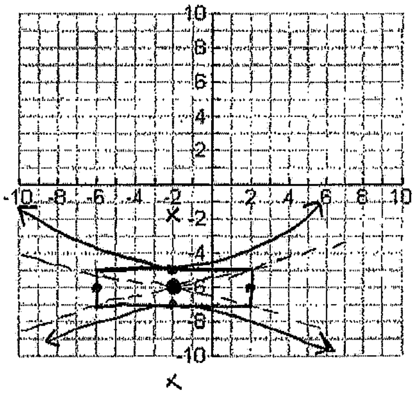
Center: $(-2,-6)$

Vertices: $(-2,-5)\ (-2,-7)$

Foci: $(-2,-6 \pm \sqrt{17})$

Asymptotes: $y+6 = \pm \frac{1}{4}(x+2)$

Eccentricity: $\sqrt{17}$



3. $\frac{(x+1)^2}{64} - \frac{(y-2)^2}{36} = 1$ $a=8 \quad b=6$

$64+36=c^2$

$100=c^2$

$c=10$

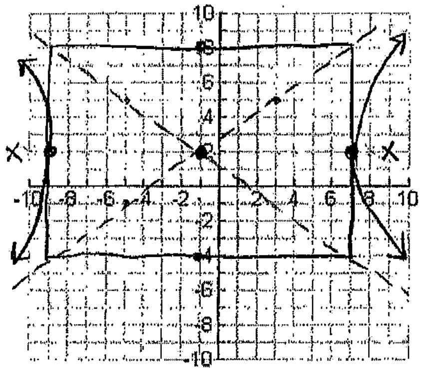
Center: $(-1,2)$

Vertices: $(-9,2)\ (7,2)$

Foci: $(-11,2)\ (9,2)$

Asymptotes: $y-2 = \pm 3/4(x+1)$

Eccentricity: $10/8 = 5/4$



4. $\frac{(y-1)^2}{49} - \frac{(x-5)^2}{9} = 1$ $a=7 \quad b=3$

$49+9=c^2$

$58=c^2$

$c=\sqrt{58}$

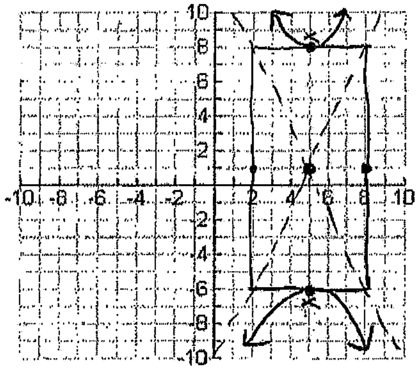
Center: $(5,1)$

Vertices: $(5,8)\ (5,-6)$

Foci: $(5, 1 \pm \sqrt{58})$

Asymptotes: $y-1 = \pm \frac{7}{3}(x-5)$

Eccentricity: $\sqrt{58}/7$



Accelerated Pre-Calculus

8.08 Hyperbolas – Day 1

Name: Key

Date: ____________

Hyperbola: a conic section where the difference of the distance from 2 fixed points (foci) is a constant.

FigureH:

FigureV:

Identify each: Center Vertices Foci

Asymptotes Transverse axis Conjugate axis Eccentricity

Horizontal Transverse Axis	Vertical Transverse Axis
$\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1$	$\frac{(y-k)^2}{a^2}-\frac{(x-h)^2}{b^2}=1$
Center: (h, k) Vertices: $(h \pm a, k)$ Foci: $(h \pm c, k)$, $a^2+b^2=c^2$ Asymptotes: $y-k=\pm\frac{b}{a}(x-h)$ Eccentricity $=\frac{c}{a}$	Center: (h, k) Vertices: $(h, k \pm a)$ Foci: $(h, k \pm c)$, $a^2+b^2=c^2$ Asymptotes: $y-k=\pm\frac{a}{b}(x-h)$ Eccentricity $=\frac{c}{a}$

Examples: Graph the Hyperbola. State the center, vertices, foci, asymptotes, and eccentricity.

1. $\frac{(x+1)^2}{16}-\frac{(y-1)^2}{9}=1$

) (

$a=4 \leftrightarrow$

$b=3 \updownarrow$

$c=5 \leftrightarrow$

$a^2+b^2=c^2$

$16+9=c^2$

$25=c^2$

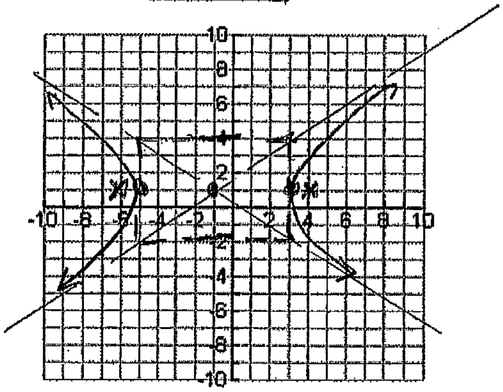
Center: (-1, 1)

Vertices: (3, 1) (-5, 1)

Foci: (4, 1) (-6, 1)

Asymptotes: $y-1=\pm 3/4(x+1)$

Eccentricity: $5/4=1.25$



2. $\frac{\overset{-k}{(y+3)^2}}{4}-\frac{\overset{-h}{(x+4)^2}}{25}=1$

$\smile$ $\frown$

$a=2 \updownarrow$

$b=5 \leftrightarrow$

$c=\sqrt{29} \updownarrow$

$a^2+b^2=c^2$

$4+25=c^2$

$29=c^2$

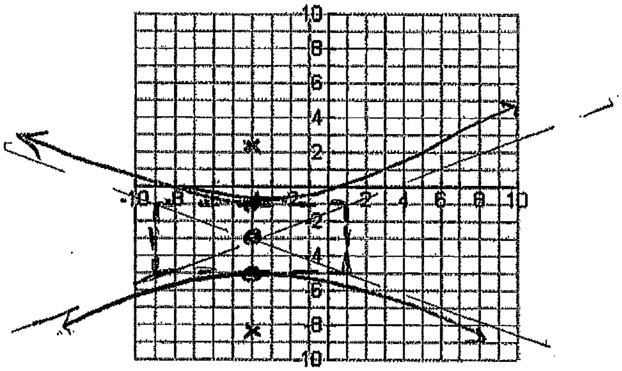
Center: ☆ (-4, -3)

Vertices: (-4, -1) (-4, -5)

Foci: $(-4, -3 \pm \sqrt{29})$

Asymptotes: $y+3=\pm 2/5(x+4)$

Eccentricity: $\sqrt{29}/2$



8.12 Practice: Graph the Hyperbola. State the center, vertices, foci, and asymptotes.

1. $\frac{(x-3)^2}{4} - \frac{y^2}{81} = 1$

) (

$a = 2$ ↔
$b = 9$ ↕
$c = \sqrt{85}$ ↔

$a^2 + b^2 = c^2$
$4 + 81 = c^2$
$85 = c^2$

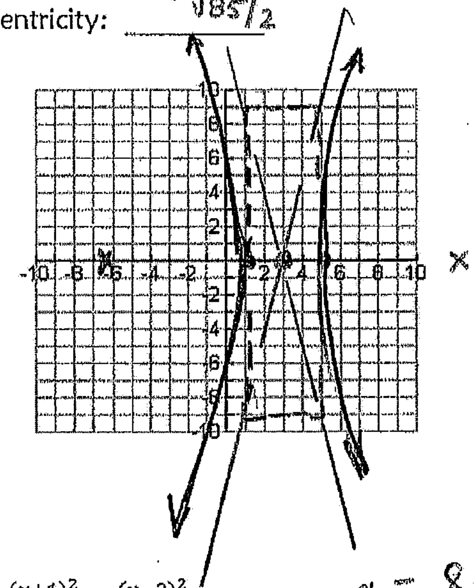
Center: (3,0)

Vertices: (1,0) (5,0)

Foci: $(3 \pm \sqrt{85}, 0)$

Asymptotes: $y = \pm \frac{9}{2}(x-3)$

Eccentricity: $\sqrt{85}/2$



2. $\overset{-k}{(y+6)^2} - \frac{\overset{-h}{(x+2)^2}}{16} = 1$

$\smile$
$\frown$

$a = 1$ ↕
$b = 4$ ↔
$c = \sqrt{17}$ ↕

$a^2 + b^2 = c^2$
$1 + 16 = c^2$
$17 = c^2$

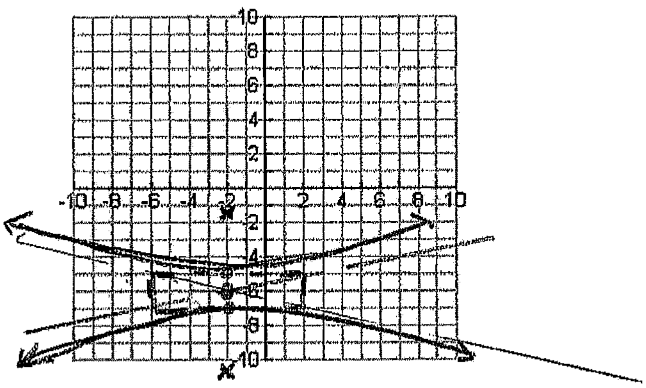
Center: ☆ (-2,-6)

Vertices: (-2,-7) (-2,-5)

Foci: $(-2, -6 \pm \sqrt{17})$

Asymptotes: $y + 6 = \pm \frac{1}{4}(x+2)$

Eccentricity: $\sqrt{17}$



3. $\frac{(x+1)^2}{64} - \frac{(y-2)^2}{36} = 1$

) (

$a = 8$ ↔
$b = 6$ ↕
$c = 10$ ↔

$a^2 + b^2 = c^2$
$64 + 36 = c^2$
$100 = c^2$

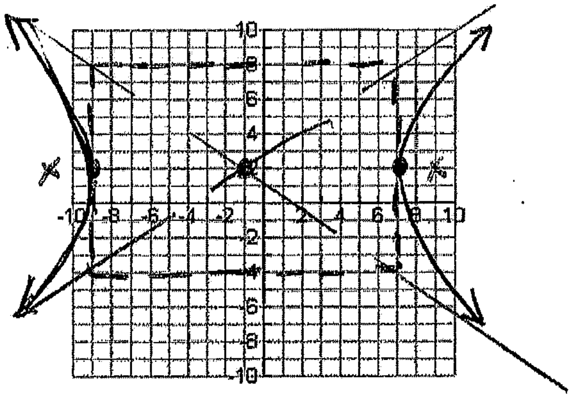
Center: (-1,2)

Vertices: (-9,2) (7,2)

Foci: (-11,2) (9,2)

Asymptotes: $y - 2 = \pm \frac{3}{4}(x+1)$

Eccentricity: $\frac{10}{8} = \frac{5}{4} = 1.25$



4. $\frac{\overset{-k}{(y-1)^2}}{49} - \frac{\overset{-h}{(x-5)^2}}{9} = 1$

$\smile$
$\frown$

$a = 7$ ↕
$b = 3$ ↔
$c = \sqrt{58}$ ↕

$a^2 + b^2 = c^2$
$49 + 9 = c^2$
$58 = c^2$
$\sqrt{58} = c$

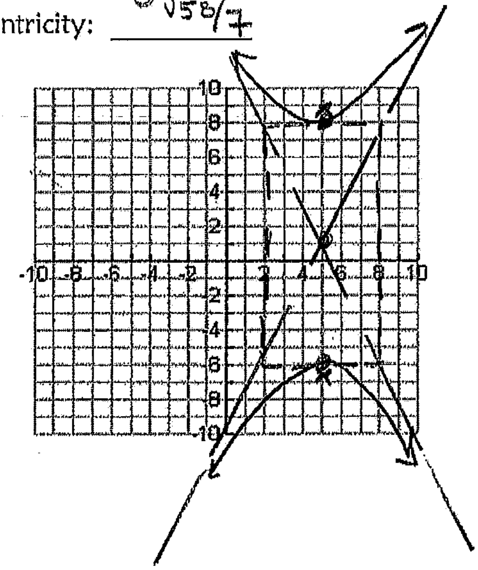
Center: ☆ (5,1)

Vertices: (5,8) (5,-6)

Foci: $(5, 1 \pm \sqrt{58})$

Asymptotes: $y - 1 = \pm \frac{7}{3}(x-5)$

Eccentricity: $\sqrt{58}/7$



Accelerated Pre-Calculus

Name: Key

8.13 Hyperbolas – Day 2

Date: ____________

Write the standard form of the equation of each hyperbola and list the coordinates of the center, vertices, & foci, the equation of the asymptotes, and the eccentricity. Then graph the hyperbola.

★ ← move x to after y! Factor -9 out carefully!

1. $-9x^2 + 4y^2 + 90x - 16y - 353 = 0$

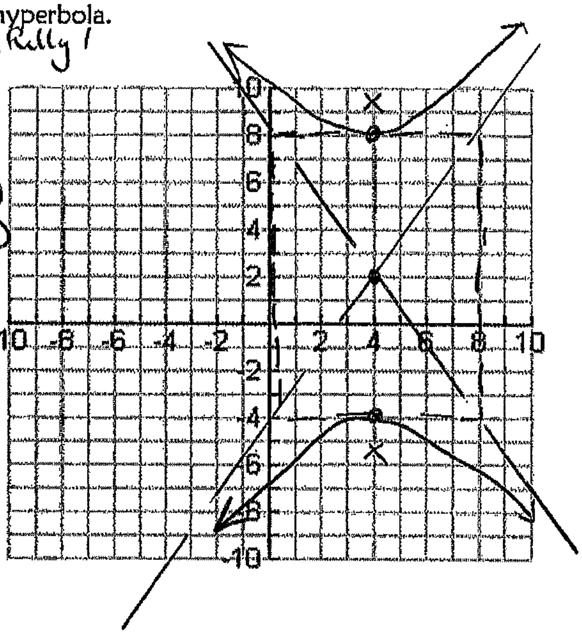
C $4y^2 - 16y - 9x^2 + 90x = 353$

A $4(y^2 - 4y + \underline{4}) - 9(x^2 - 10x + \underline{25}) = 353$ $+4(\underline{4})$ $-9(\underline{25})$

B $(\frac{-4}{2})^2 \nearrow$ $(\frac{-10}{2})^2 \nearrow$

factor $\frac{4(y-2)^2}{144} - \frac{9(x-5)^2}{144} = \frac{144}{144}$

divide



a=6↕
b=4↔
c=2√13↕

Standard Form: $\frac{(y-2)^2}{36} - \frac{(x-5)^2}{16} = 1$

(h,k) not (k,h) ★ Center: (5,2) Vertices: (5,8) (5,-4)

Eccentricity = $\frac{2\sqrt{13}}{6} = \frac{\sqrt{13}}{3}$ Foci: $(5, 2 \pm 2\sqrt{13})$

Asymptotes: $y - 2 = \pm \frac{3}{2}(x-5)$

★ y will be 2nd. Factor -4 out carefully!

2. $x^2 - 4y^2 + 6x - 8y - 11 = 0$

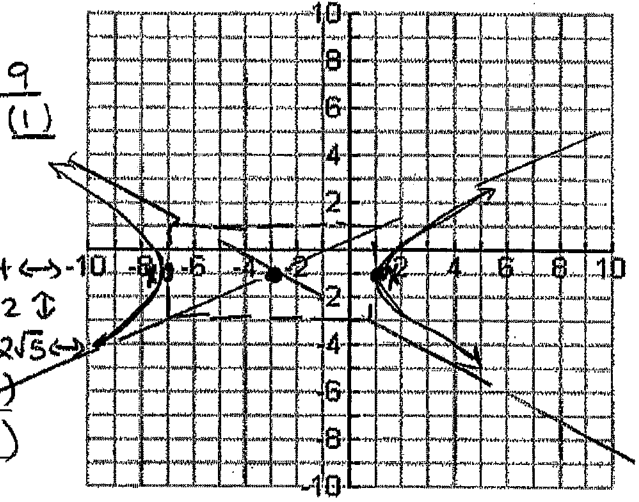
C: $x^2 + 6x - 4y^2 - 8y = 11$

A: $(x^2 + 6x + \underline{9}) - 4(y^2 + 2y + \underline{1}) = 11 + \underline{9}$ $-4(\underline{1})$

B: $(\frac{6}{2})^2 \nearrow$ $(\frac{2}{2})^2 \nearrow$

factor: $\frac{(x+3)^2}{16} - \frac{4(y+1)^2}{16} = \frac{16}{16}$

divide:



a=4↔
b=2↕
c=2√5↔

Standard Form: $\frac{(x+3)^2}{16} - \frac{(y+1)^2}{4} = 1$

Center: (-3,-1) Vertices: (1,-1) (-7,-1)

Eccentricity = $\frac{2\sqrt{5}}{4} = \frac{\sqrt{5}}{2}$ Foci: $(-3 \pm 2\sqrt{5}, -1)$

Asymptotes: $y + 1 = \pm \frac{1}{2}(x+3)$

★ x will be 2nd. Factor -4 out carefully!

3. $-4x^2 + 9y^2 - 90y - 24x + 153 = 0$

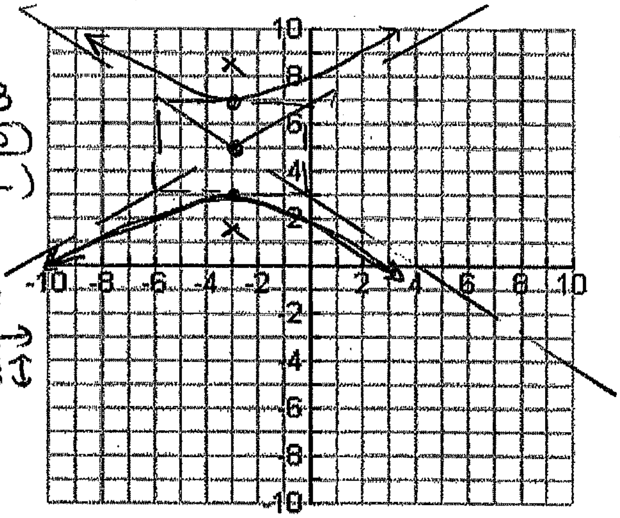
C: $9y^2 - 90y - 4x^2 - 24x = -153$

A: $9(y^2 - 10y + \underline{25}) - 4(x^2 + 6x + \underline{9}) = -153$ $+9(\underline{25})$ $-4(\underline{9})$

B: $(\frac{-10}{2})^2 \nearrow$ $(\frac{6}{2})^2 \nearrow$

factor: $\frac{9(y-5)^2}{36} - \frac{4(x+3)^2}{36} = \frac{36}{36}$

divide:



a=2↕
b=3↔
c=√13↕

Standard Form: $\frac{(y-5)^2}{4} - \frac{(x+3)^2}{9} = 1$

★ Center: (-3,5) Vertices: (-3,7) (-3,3)

Eccentricity = $\sqrt{13}/2$ Foci: $(-3, 5 \pm \sqrt{13})$

Asymptotes: $y - 5 = \pm 2/3(x+3)$

★ y is 2nd variable. factor −49 out carefully!

4. $36x^2 - 49y^2 - 72x - 294y - 2169 = 0$

C: $36x^2 - 72x - 49y^2 - 294y = 2169$

A: $36(x^2 - 2x + \underline{1}) - 49(y^2 + 6y + \underline{9}) = 2169$ $+36(\underline{1})$ $-49(\underline{9})$

B: $\left(\frac{2}{2}\right)^2 \nearrow$ $\left(\frac{6}{2}\right)^2 \nearrow$

factor: $\frac{36(x-1)^2}{1764} - \frac{49(y+3)^2}{1764} = \frac{1764}{1764}$

divide:

Standard Form: $\frac{(x-1)^2}{49} - \frac{(y+3)^2}{36} = 1$

a = 7 ↔

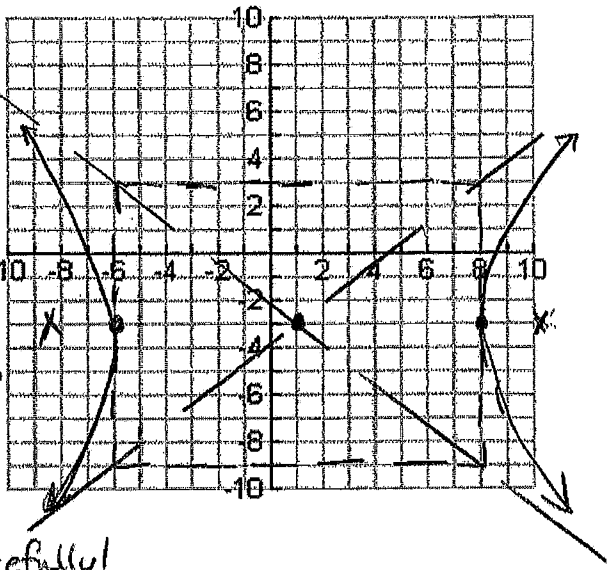
b = 6 ↕

c = $\sqrt{85}$ ↔

Center: (1, −3) Vertices: (8, −3) (−6, −3)

Eccentricity = $\sqrt{85}/7$ Foci: $(1 \pm \sqrt{85}, -3)$

Asymptotes: $y + 3 = \pm 6/7 (x - 1)$



★ y is 2nd variable. factor −9 out carefully!

5. $x^2 - 9y^2 - 72y - 153 = 0$

C: $x^2 - 9y^2 - 72y = 153$

A: $x^2 - 9(y^2 + 8y + \underline{16}) = 153 - 9(\underline{16})$

B: $\left(\frac{0}{2}\right)^2$ $\left(\frac{8}{2}\right)^2 \nearrow$

factor: $\frac{x^2}{9} - \frac{9(y+4)^2}{9} = \frac{9}{9}$

divide:

Standard Form: $\frac{x^2}{9} - (y+4)^2 = 1$

a = 3 ↔

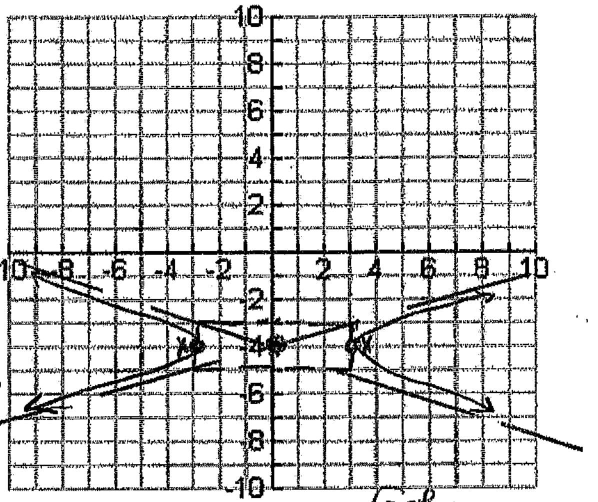
b = 1 ↕

c = $\sqrt{10}$ ↔

Center: (0, −4) Vertices: (±3, −4)

Eccentricity = $\sqrt{10}/3$ Foci: $(\pm\sqrt{10}, -4)$

Asymptotes: $y + 4 = \pm \frac{1}{3} x$



✓gcp

(or divide 5 from entire equation)

★ x is 2nd variable. factor −5 out carefully!

6. $-5x^2 + 5y^2 - 20x + 10y - 60 = 0$

C: $5y^2 + 10y - 5x^2 - 20x = 60$

A: $5(y^2 + 2y + \underline{1}) - 5(x^2 + 4x + \underline{4}) = 60$ $+5(\underline{1})$ $-5(\underline{4})$

B: $\left(\frac{2}{2}\right)^2 \nearrow$ $\left(\frac{4}{2}\right)^2 \nearrow$

factor: $\frac{5(y+1)^2}{45} - \frac{5(x+2)^2}{45} = \frac{45}{45}$

divide:

Standard Form: $\frac{(y+1)^2}{9} - \frac{(x+2)^2}{9} = 1$

a = 3 ↕

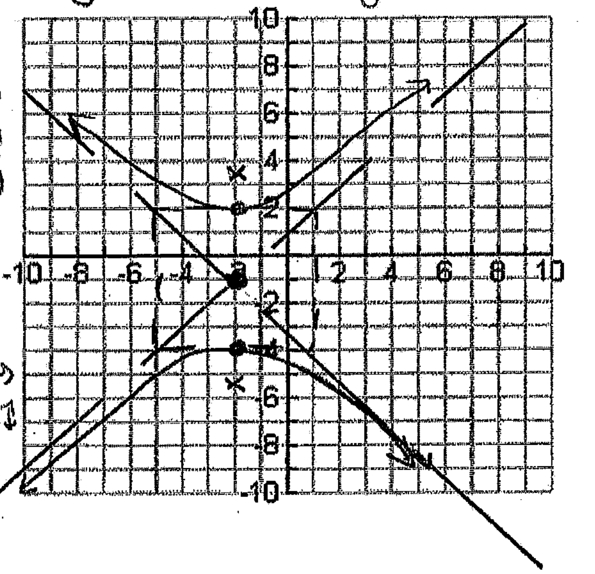
b = 3 ↔

c = $3\sqrt{2}$ ↕

★ Center: (−2, −1) Vertices: (−2, 2) (−2, −4)

Eccentricity = $\frac{3\sqrt{2}}{3} = \sqrt{2}$ Foci: $(-2, -1 \pm 3\sqrt{2})$

Asymptotes: $y + 1 = \pm 1 (x + 2)$



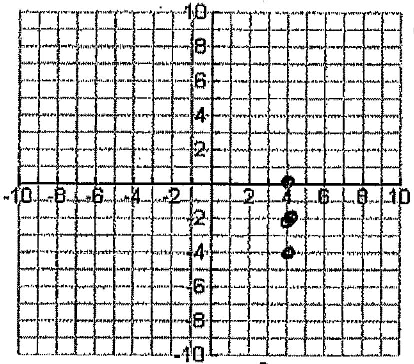
Accelerated Pre-Calculus Name: Key

8.10 Hyperbolas – Day 3 Date: ____________

Use the given information to write the standard form equation of each hyperbola. Use the grid if it helps.

1. The center is at (4, -2), a = 2, b = 3, and the transverse axis is vertical.

a



Equation: $\frac{(y+2)^2}{4} - \frac{(x-4)^2}{9} = 1$

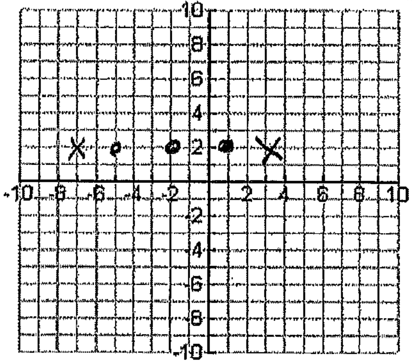
2. The length of the transverse axis is 6 units and the foci are at (3, 2) and (-7, 2).

$2a=6$
$a=3$

$2c=10$) (
$c=5$

$3^2+b^2=5^2$
$9+b^2=25$
$b^2=16$ ↕

center @ (-2, 2)



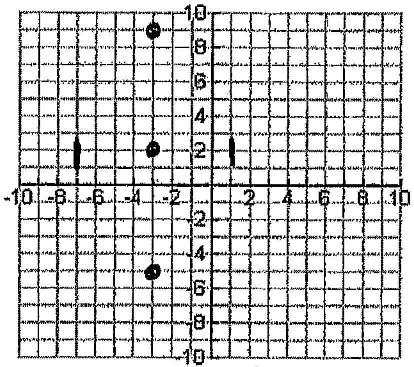
Equation: $\frac{(x+2)^2}{9} - \frac{(y-2)^2}{16} = 1$

3. The length of the conjugate axis is 8 units and the vertices are (-3, 9) and (-3, -5)

$2b=8$
$b=4$

$2a=14$
$a=7$ ↕

center @ (-3, 2)



Equation: $\frac{(y-2)^2}{49} - \frac{(x+3)^2}{16} = 1$

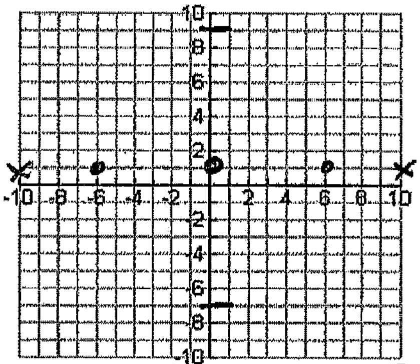
4. The center is at (0, 1), one focus is at (10, 1) and the eccentricity is $\frac{5}{3}$

$c=10$ ↔

$\frac{c}{a} = \frac{5^{\cdot 2}}{3_{\cdot 2}} = \frac{10}{c}$

$a=6$ ↔

$36+b^2=100$
$b^2=64$ ↕



Equation: $\frac{x^2}{36} - \frac{(y-1)^2}{64} = 1$

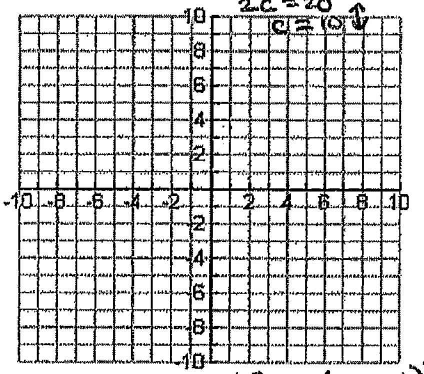
5. Endpoints of transverse axis: (-1, 14) (-1, -4)
Foci: (-1, 15) (-1, -5)

vertices $2a=18$
$a=9$ ↕

$2c=20$
$c=10$ ↕

$81+b^2=100$
$b^2=19$
↔

center @ (-1, 5)



Equation: $\frac{y-5)^2}{81} - \frac{(x+1)^2}{19} = 1$

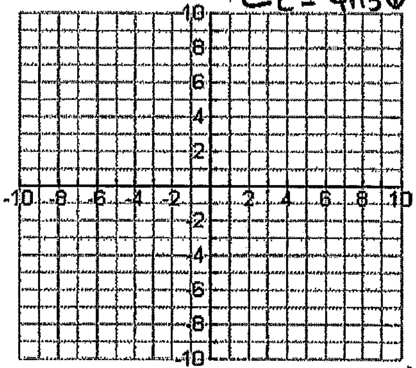
6. Endpoints of the conjugate axis: (16, -2) (-8, -2)
Foci: (4, -2 ± 4$\sqrt{13}$)

$c=4\sqrt{13}$ ↕

$2b=24$
$b=12$ ↔

$a^2+144=208$
$a^2=64$ ↕

center @ (4, -2)



Equation: $\frac{(y+2)^2}{64} - \frac{(x-4)^2}{144} = 1$

center @ (-2,-5)

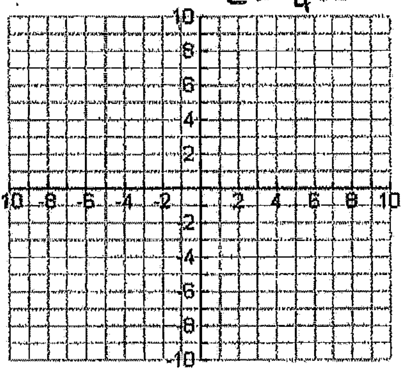
7. Endpoints of conjugate axis: (-2, 1) (-2, -11)

$2b=12$

$b=6$ ↕

Eccentricity = $\frac{5}{4}$ $\frac{c}{a}$

$c=\frac{5}{4}a$



$a^2+36=c^2$

$a^2+36=\left(\frac{5}{4}a\right)^2$

$a^2+36=\frac{25a^2}{16}$

$36=\frac{9}{16}a^2$

$64=a^2$ ⟷

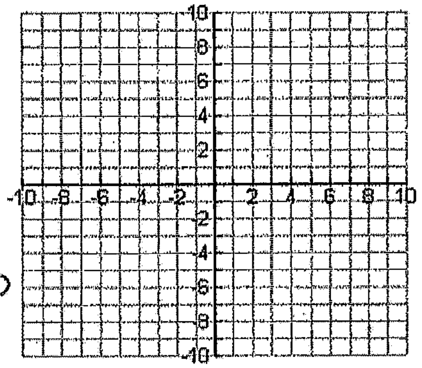
Equation: $\frac{(x+2)^2}{64}-\frac{(y+5)^2}{36}=1$

8. Vertices: (8, 5) (8, -3)

$2a=8$

$a=4$ ↕

Eccentricity = $\frac{\sqrt{10}}{2}$



$\frac{c}{a}=\frac{\sqrt{10}\cdot 2}{2\cdot 2}=\frac{2\sqrt{10}}{4}$

$c=2\sqrt{10}$ ↕

$16+b^2=40$

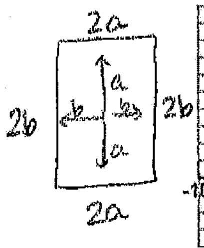
$b^2=24$ ⟷

center @ (8,1)

Equation: $\frac{(y-1)^2}{16}-\frac{(x-8)^2}{24}=1$

9. Vertices: (-3, 6) (-3, -4) $2a=10$ $a=5$ ↕

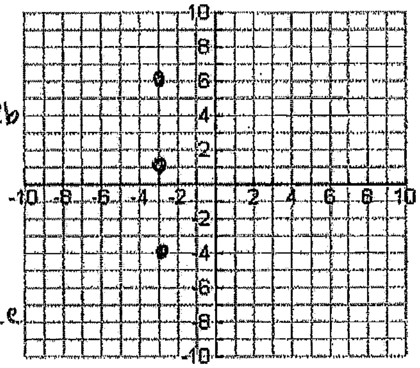
Perimeter of Central Rectangle = 72



Perimeter is the distance around all 4 sides.

$2(2a)+2(2b)$

$4a+4b$



$P=4a+4b$

$72=4(5)+4b$

$72=20+4b$

$52=4b$

$13=b$

center @ (-3,1)

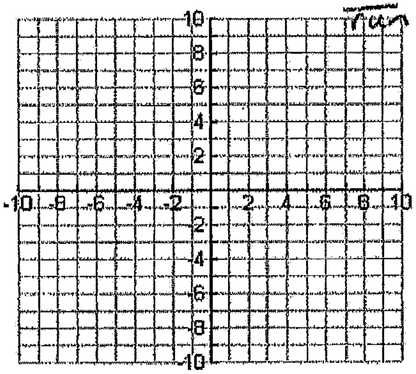
Equation: $\frac{(y-1)^2}{25}-\frac{(x+3)^2}{169}=1$

10. Vertices: (2, -4) (-14, -4) $2a=16; a=8$ ⟷ run

Asymptotes: $y=\frac{9}{8}x+\frac{11}{4}$ and $y=-\frac{9}{8}x-\frac{43}{3}$

$\frac{\text{rise}}{\text{run}}$ $m=\pm\frac{9}{8}=\frac{b}{a}$

$b=9$ ↕ rise

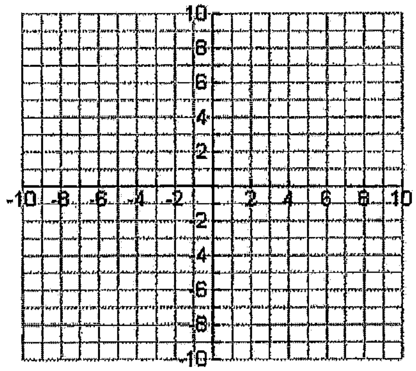


center @ (-6,-4)

Equation: $\frac{(x+6)^2}{64}-\frac{(y+4)^2}{81}=1$

11. Vertices: (7, 5) (7, -15) $2a=20$ $a=10$ ↕

Asymptotes: $y=\frac{5}{3}x-\frac{50}{3}$ and $y=-\frac{5}{3}x+\frac{20}{3}$



$m=\pm\frac{5}{3}$ $\frac{\text{rise}}{\text{run}}$ $\frac{a}{b}$

$\frac{a}{b}=\frac{5\cdot 2}{3\cdot 2}=\frac{10}{6}$

$b=6$ ⟷

center @ (7,-5)

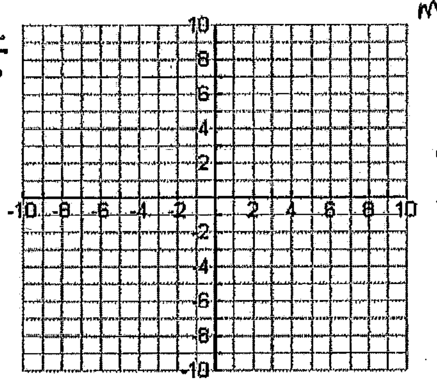
Equation: $\frac{(y+5)^2}{100}-\frac{(x-7)^2}{36}=1$

12. Foci: $(7\pm7\sqrt{2}, -3)$ $c=7\sqrt{2}$ ⟷

Asymptotes: y = x – 10 and y = -x + 4

$m=\frac{\text{rise}}{\text{run}}=\frac{b}{a}=\frac{1}{1}$

$a=b$



$a^2+b^2=c^2$

$a^2+a^2=98$

$2a^2=98$

$a^2=49$

$a=7$

$b=7$

center @ (7,-3)

Equation: $\frac{(x-7)^2}{49}-\frac{(y+3)^2}{49}=1$

Accel Algebra 2

10.06 – Completing the Square – Hyperbolas and Ellipses

Write the standard form of the equation and identify the type of conic.

1. $49x^2 + 16y^2 + 160y - 384 = 0$

16·25

$$49x^2+16(y^2+10y\underline{+25}) = 384+\underline{400}$$

$$\frac{49x^2}{784}+\frac{16(y+5)^2}{784}=\frac{784}{784}$$

Type of Conic: Ellipse

Standard Form: $\frac{x^2}{16}+\frac{(y+5)^2}{49}=1$

2. $36x^2 - 49y^2 - 72x - 294y - 2169 = 0$

$$36x^2-72x-49y^2-294y=2169$$

36·1 -49·9

$$36(x^2-2x\underline{+1})-49(y^2+6y\underline{+9}) = 2169\underline{+36}\underline{-441}$$

$$\frac{36(x-1)^2}{1764}-\frac{49(y+3)^2}{1764}=\frac{1764}{1764}$$

Type of Conic: hyperbola

Standard Form: $\frac{(x-1)^2}{49}-\frac{(y+3)^2}{36}=1$

3. $3x^2 + y^2 + 18x - 2y + 4 = 0$

$$3x^2+18x+y^2-2y=-4$$

3·9

$$3(x^2+6x\underline{+9})+y^2-2y+\underline{1} = -4+\underline{27}+\underline{1}$$

$$\frac{3(x+3)^2}{24}+\frac{(y-1)^2}{24}=\frac{24}{24}$$

Type of Conic: ellipse

Standard Form: $\frac{(x+3)^2}{8}+\frac{(y-1)^2}{24}=1$

4. $$\frac{3x^2 - 18x - 18y + 36}{3} = \frac{0}{3}$$

$$x^2-6x-6y+12=0$$

$$x^2-6x\ \underline{+9} = 6y-12\ \underline{+9}$$

$$(x-3)^2 = 6y-3$$

Type of Conic: parabola

Standard Form: $(x-3)^2 = 6(y-1/2)$

5. $x^2 - 9y^2 - 72y - 153 = 0$

$$x^2-9y^2-72y=153$$

$$x^2-9(y^2+8y\ \underline{+16}) = 153\ \underset{}{\overset{-9\cdot 16}{\underline{-144}}}$$

$$\frac{x^2}{9} - \frac{9(y+4)^2}{9} = \frac{9}{9}$$

Type of Conic: hyperbola

Standard Form: $\frac{x^2}{9} - (y+4)^2 = 1$

6. $-5x^2 + 5y^2 - 20x + 10y - 60 = 0$

$$5y^2+10y-5x^2-20x = 60$$

$$5(y^2+2y\ \underline{+1}) - 5(x^2+4x\ \underline{+4}) = 60\ \overset{5\cdot 1}{\underline{+5}}\ \overset{-5\cdot 4}{\underline{-20}}$$

$$\frac{5(y+1)^2}{45} - \frac{5(x+2)^2}{45} = \frac{45}{45}$$

Type of Conic: hyperbola

Standard Form: $\frac{(y+1)^2}{9} - \frac{(x+2)^2}{9} = 1$

10.06 Conics Extra Practice

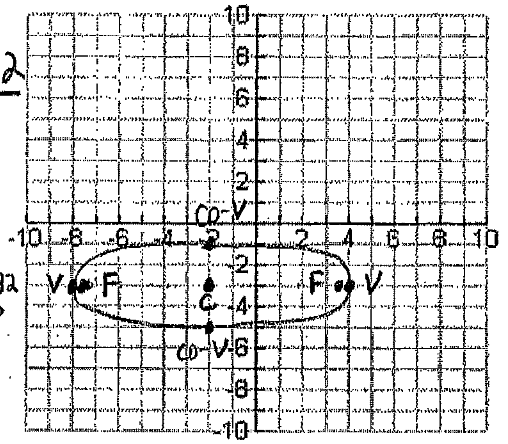
Identify the type of conic. Write the standard form of the equation of each conic and then graph the equation. List the coordinates of the center, foci, and the major and minor axis vertices. State the eccentricity of the conic.

1. $2x^2 + 18y^2 + 8x + 108y + 99 = 1$

$$2x^2+8x+18y^2+108y=-98$$

$$2(x^2+4x\underline{+4})+18(y^2+6y\underline{+9})=-98\underline{+8}\underline{+162}$$

$$\frac{2(x+2)^2}{72}+\frac{18(y+3)^2}{72}=\frac{72}{72}$$



Type of Conic: ellipre

Ⓗ Standard Form: $\frac{(x+2)^2}{36}+\frac{(y+3)^2}{4}=1$ $a=6 \leftrightarrow$ $b=2 \updownarrow$ $c^2=36-4=32$ $c=4\sqrt{2} \leftrightarrow$

Center: $(-2,-3)$ Vertices: $(-8,-3)(4,-3)$

Eccentricity = $\frac{4\sqrt{2}}{6}=\frac{2\sqrt{2}}{3}$ Foci: $(-2\pm4\sqrt{2},-3)$

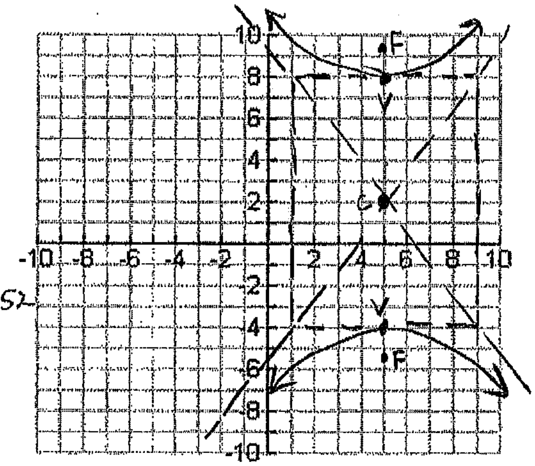
Asymptotes/Directrix: none

2. $-9x^2 + 4y^2 + 90x - 16y - 353 = 0$

$$4y^2-16y-9x^2+90x=353$$

$$4(y^2-4y\underline{+4})-9(x^2-10x\underline{+25})=353\underline{+16}\underline{-225}$$

$$\frac{4(y-2)^2}{144}-\frac{9(x-5)^2}{144}=\frac{144}{144}$$



Type of Conic: hyperbola

Ⓥ Standard Form: $\frac{(y-2)^2}{36}-\frac{(x-5)^2}{16}=1$ $a=6 \updownarrow$ $b=4 \leftrightarrow$ $c^2=36+16=52$ $c=2\sqrt{13}$

Center: $(5,2)$ Vertices: $(5,-4)(5,8)$

Eccentricity = $\frac{2\sqrt{13}}{6}=\frac{\sqrt{13}}{3}$ Foci: $(5,2\pm2\sqrt{13})$

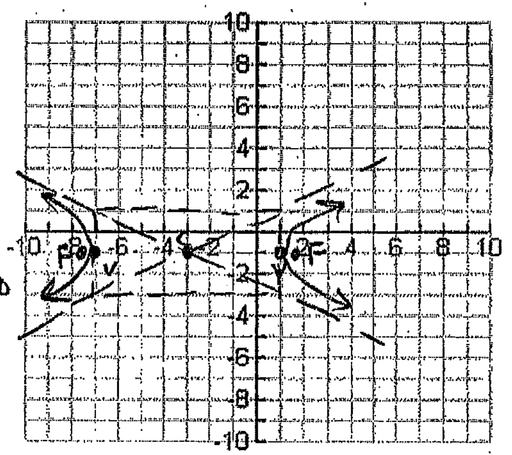
[✓ Asymptotes]/Directrix: $y-2=\pm\frac{3}{2}(x-5)$

3. $x^2 - 4y^2 + 6x - 8y - 11 = 0$

$$x^2+6x-4y^2-8y=11$$

$$x^2+6x\underline{+9}-4(y^2+2y\underline{+1})=11\underline{+9}\underline{-4}$$

$$\frac{(x+3)^2}{16}-\frac{4(y+1)^2}{16}=\frac{16}{16}$$



Type of Conic: hyperbola

Ⓗ Standard Form: $\frac{(x+3)^2}{16}-\frac{(y+1)^2}{4}=1$ $a=4 \leftrightarrow$ $b=2 \updownarrow$ $c^2=16+4=20$ $c=2\sqrt{5}$

Center: $(-3,-1)$ Vertices: $(-7,-1)(1,-1)$

Eccentricity = $\frac{2\sqrt{5}}{4}=\frac{\sqrt{5}}{2}$ Foci: $(-3\pm2\sqrt{5},-1)$

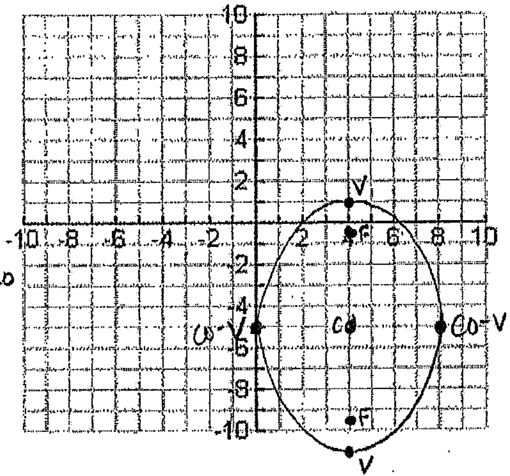
[✓ Asymptotes]/Directrix: $y+1=\pm 1/2(x+3)$

4. $9x^2 + 4y^2 - 72x + 40y + 100 = 0$

$9x^2 - 72x + 4y^2 + 40y = -100$

$9(x^2 - 8x \underline{+16}) + 4(y^2 + 10y + 25) = -100 \underline{+144} \underline{+100}$

$\frac{9(x-4)^2}{144} + \frac{4(y+5)^2}{144} = \frac{144}{144}$



Type of Conic: ellipse

(V) Standard Form: $\frac{(x-4)^2}{16} + \frac{(y+5)^2}{36} = 1$ $a = 6 \updownarrow$ $b = 4 \leftrightarrow$ $c^2 = 36 - 16 = 20$ $c = 2\sqrt{5}$

Center: $(4, -5)$ Vertices: $(4, -11)(4, 1)$

Eccentricity = $\frac{2\sqrt{5}}{6} = \frac{\sqrt{5}}{3}$ Foci: $(4, -5 \pm 2\sqrt{5})$

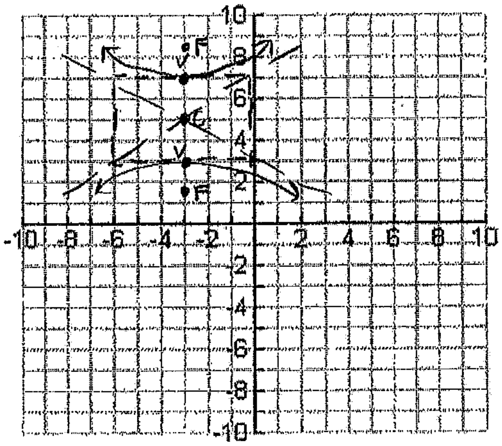
Asymptotes/Directrix: none

5. $-4x^2 + 9y^2 - 24x - 90y + 153 = 0$

$9y^2 - 90y - 4x^2 - 24x = -153$

$9(y^2 - 10y \underline{+25}) - 4(x^2 + 6x \underline{+9}) = -153 \underline{+225} \underline{-36}$

$\frac{9(y-5)^2}{36} - \frac{4(x+3)^2}{36} = \frac{36}{36}$



Type of Conic: hyperbola

(V) Standard Form: $\frac{(y-5)^2}{4} - \frac{(x+3)^2}{9} = 1$ $a = 2 \updownarrow$ $b = 3 \leftrightarrow$ $c^2 = 4 + 9 = 13$ $c = \sqrt{13}$

Center: $(-3, 5)$ Vertices: $(-3, 3)(-3, 7)$

Eccentricity = $\sqrt{13}/2$ Foci: $(-3, 5 \pm \sqrt{13})$

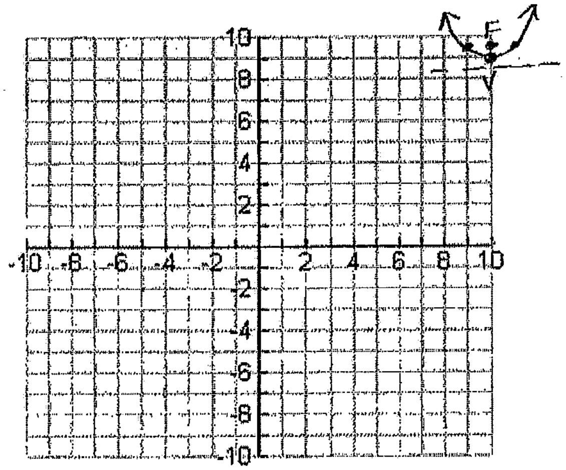
Asymptotes/Directrix: $y - 5 = \pm \, ^2/_3(x+3)$

6. $\frac{-x^2 + 20x + 2y - 118}{-1} = \frac{0}{-1}$

$x^2 - 20x - 2y + 118 = 0$

$x^2 - 20x \underline{+100} = 2y - 118 \underline{+100}$

$(x-10)^2 = 2y - 18$



Type of Conic: parabola

(V) Standard Form: $(x-10)^2 = 2(y-9)$ $4p = 2$ $p = 1/2$

FW=2

Center: none Vertices: $(10, 9)$

Eccentricity = none Foci: $(10, 9\frac{1}{2})$

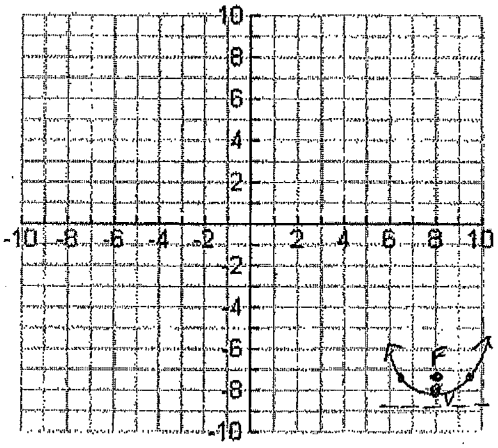
~~Asymptotes~~/Directrix: $y = 8\frac{1}{2}$

7. $-x^2 + 16x + 3y - 40 = 0$

$x^2 - 16x - 3y + 40 = 0$

$x^2 - 16x \underline{+64} = 3y - 40 \underline{+64}$

$(x-8)^2 = 3y + 24$



Type of Conic: parabola

Ⓥ Standard Form: $(x-8)^2 = 3(y+8)$ $4p = 3$ $p = 3/4$ FW=3

Center: none Vertices: (8, -8)

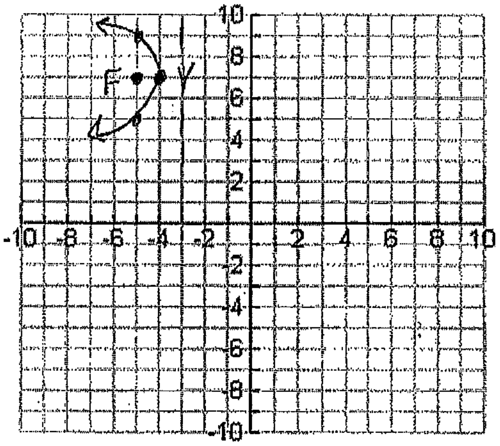
Eccentricity = none Foci: (8, -7 1/4)

~~Asymptotes~~/Directrix: $y = -8\,3/4$

8. $4y^2 - 56y$ ~~$+ y + 201$~~ $= 0$ $4x + 65$

$y^2 - 14y + 49 = -4x - 65 \underline{+49}$

$(y-7)^2 = -4x - 16$



Type of Conic: parabola

Ⓗ Standard Form: $(y-7)^2 = -4(x+4)$ $4p = -4$ $p = -1$ FW=4

Center: none Vertices: (-4, 7)

Eccentricity = none Foci: (-5, 7)

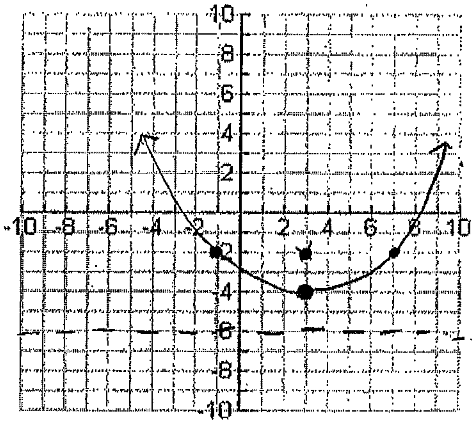
~~Asymptotes~~/Directrix: $x = -3$

Accelerated Algebra 2 Name: Key

10.07 Conics Review: Parabolas, Ellipses & Hyperbolas Date: ____________

Graph the equation and identify the characteristics.

1. $(x-3)^2 = 8(y+4)$ $4p=8$ $p=2$



Vertex: (3,-4)

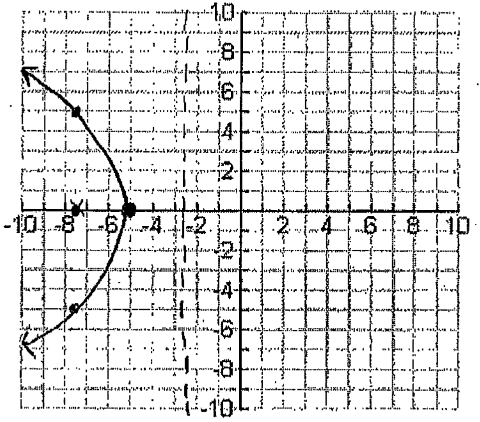
Axis of Symmetry: x=3

Focus: (3,-2)

Directrix: y=-6

Focal Width: 8

2. $y^2 = -10(x+5)$ $4p=-10$ $p=-5/2$



Vertex: (-5,0)

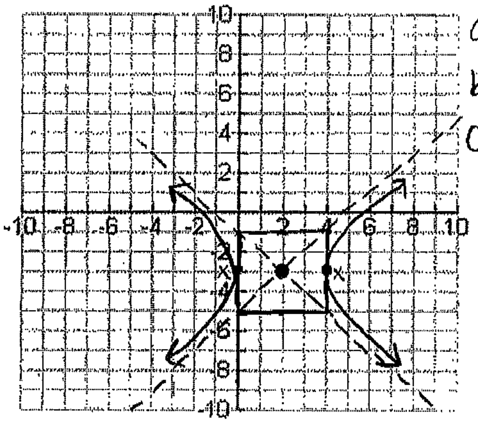
Axis of Symmetry: y=0

Focus: (-7.5,0)

Directrix: x=-2.5

Focal Width: 10

3. $\frac{(x-2)^2}{4} - \frac{(y+3)^2}{4} = 1$



$a=2$

$b=2$

$c=2\sqrt{2}$

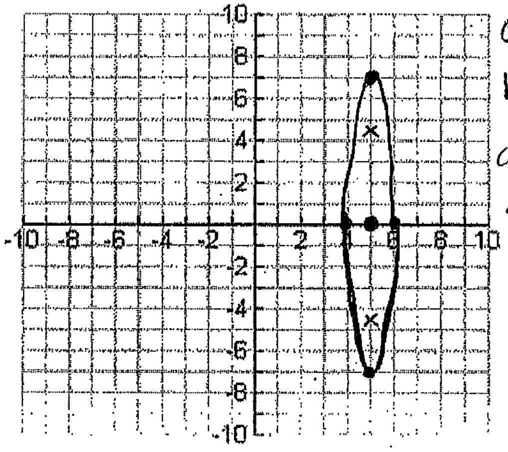
$a^2+b^2=c^2$

$4+4=c^2$

$\sqrt{8}=\sqrt{c^2}$

$c=2\sqrt{2}$

4. $(x-5)^2 + \frac{y^2}{49} = 1$



$a=7$ ⇕

$b=1$ ↔

$a^2-b^2=c^2$

$49-1=c^2$

$\sqrt{48}=\sqrt{c^2}$

$c=4\sqrt{3}$

Center: (2,-3) Foci: $(2\pm2\sqrt{2},-3)$

Vertices: (0,-3) (4,-3) e = $\frac{2\sqrt{2}}{2}=\sqrt{2}$

Asymptotes: $y+3=\pm1(x-2)$

Conjugate axis = 4 Transverse axis = 4

2b 2a

Center: (5,0) Foci: $(5,\pm4\sqrt{3})$

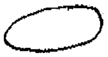
Vertices: (5,7) (5,-7) e = $\frac{4\sqrt{3}}{7}$

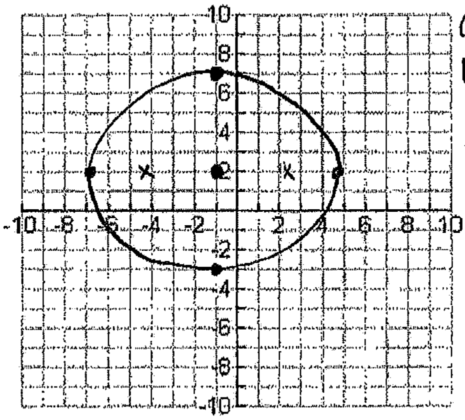
Co-Vertices: (4,0) (6,0)

Major axis = 14 Minor axis = 2

2a 2b

5. $\frac{(x+1)^2}{35}+\frac{(y-2)^2}{25}=1$





$a=\sqrt{35}$ ↔

$b=5$ ↕

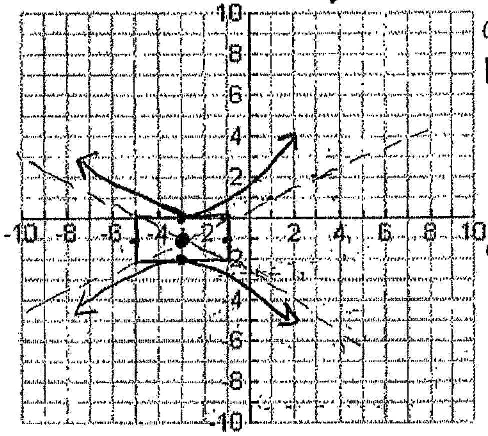
$\sqrt{35}^2-5^2=c^2$

$35-25=c^2$

$10=c^2$

$c=\sqrt{10}$

6. $(y+1)^2-\frac{(x+3)^2}{4}=1$



$a=1$

$b=2$

$1^2+2^2=c^2$

$5=c^2$

$c=\sqrt{5}$

Center: $(-1,2)$ Foci: $(-1\pm\sqrt{10}\,,2)$

Vertices: $(-1\pm\sqrt{35}\,,2)$ e = $\frac{\sqrt{10}}{\sqrt{35}}=\frac{\sqrt{2}}{\sqrt{7}}=\frac{\sqrt{14}}{7}$

Co-Vertices: $(-1,7)$ $(-1,-3)$

Minor axis = 10 Major axis = $2\sqrt{35}$

$2b$ $2a$

Center: $(-3,-1)$ Foci: $(-3,-1\pm\sqrt{5})$

Vertices: $(-3,0)$ $(-3,-2)$ e = $\sqrt{5}$

Asymptotes: $y+1=\pm\frac{1}{2}(x+3)$

Transverse axis = 2 Conjugate axis = 4

$2a$ $2b$

Classify each conic section as a *parabola, ellipse* or *hyperbola.* Write each equation in standard form.

7. $9x^2+49y^2-294y=0$ $49\cdot 9$

$9x^2+49(y^2-6y+\underline{9})=0+\underline{441}$

$\frac{9x^2}{441}+\frac{49(y-3)^2}{441}=\frac{441}{441}$

$\boxed{\frac{x^2}{49}+\frac{(y-3)^2}{9}=1}$

Ellipse

8. $x^2-4y^2+2x-16y-31=0$

$x^2+2x-4y^2-16y=31$

$(x^2+2x+\underline{1})-4(y^2+4y+\underline{4})=31+\underline{1}-\underline{16}$

$\frac{(x+1)^2}{16}-\frac{4(y+2)^2}{16}=\frac{16}{16}$

$\boxed{\frac{(x+1)^2}{16}-\frac{(y+2)^2}{4}=1}$

hyperbola

9. $x^2+10x+14y=-11$

$x^2+10x+\underline{25}=-14y-11+\underline{25}$

$(x+5)^2=-14y+14$

$\boxed{(x+5)^2=-14(y-1)}$

Parabola

10. $-9x^2+4y^2-36x-8y-68=0$

$4y^2-8y-9x^2-36x=68$

$4(y^2-2y+\underline{1})-9(x^2+4x+\underline{4})=68+\underline{4}-\underline{36}$

$\frac{4(y-1)^2}{36}-\frac{9(x+2)^2}{36}=\frac{36}{36}$

$\boxed{\frac{(y-1)^2}{9}-\frac{(x+2)^2}{4}=1}$

hyperbola

11. $16x^2 + y^2 - 128x + 4y + 244 = 0$

$16x^2 - 128x + y^2 + 4y = -244$

$16(x^2 - 8x + \underline{16}) + (y^2 + 4y + 4) = -244 + \underline{256} + \underline{4}$

$\frac{16(x-4)^2}{16} + \frac{(y+2)^2}{16} = \frac{16}{16}$

$\boxed{(x-4)^2 + \frac{(y+2)^2}{16} = 1}$

Ellipse

12. $2y^2 - 8y + 12x + 56 = 0$

$\frac{2y^2 - 8y = -12x - 56}{2}$

$y^2 - 4y + \underline{4} = -6x - 28 + \underline{4}$

$(y-2)^2 = -6x - 24$

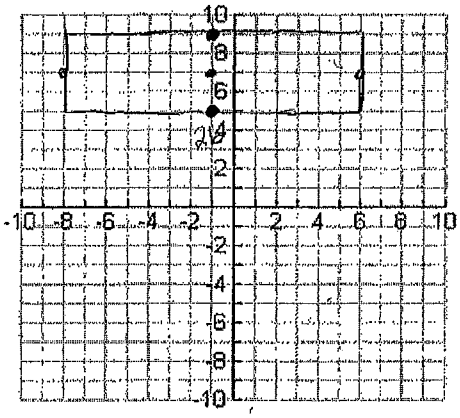
$\boxed{(y-2)^2 = -6(x+4)}$

Parabola

Use the information provided to write the standard form equation of each shape described.

13. Vertices: (-1, 9), (-1, 5)
Perimeter of Central Rectangle = 36

↑ hyperbola



$P = 4a + 4b$

$36 = 4(2) + 4b$

$28 = 4b$

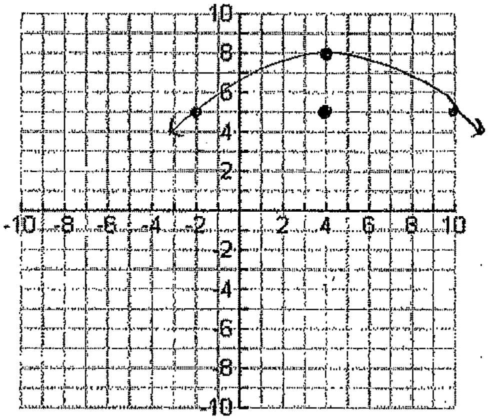
$b = 7$

Center: (-1, 7)

Equation: $\frac{(y-7)^2}{4} - \frac{(x+1)^2}{49} = 1$

14. Focus: (4,5) Focal Width: 12
Opens Down

Parabola



$4p = -12$

$p = -3$

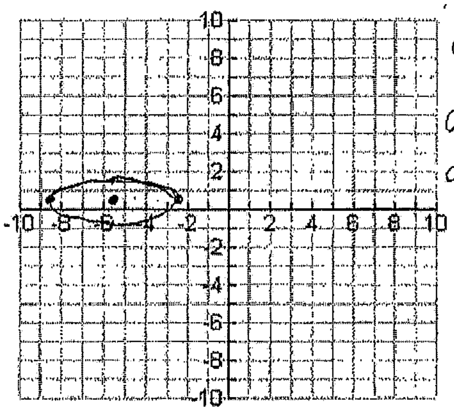
vertex: (4, 8)

Equation: $(x-4)^2 = -12(y-8)$

15. Center: $\left(-\frac{11}{2},\frac{1}{2}\right)$

Vertex: $\left(-\frac{17}{2},\frac{1}{2}\right)$ Eccentricity = $\frac{\sqrt{7}}{3}$ $\frac{c}{a}$

* c is smaller, so ellipse



$a=3$

$a^2-b^2=c^2$

$9-b^2=7$

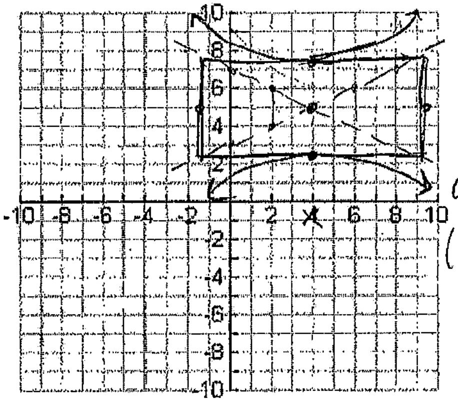
$-b^2=-2$

$b^2=2$

Equation: $\frac{(x+\frac{11}{2})^2}{9}+\frac{(y-\frac{1}{2})^2}{2}=1$

16. Foci: $(\overset{h}{4},\overset{k}{5}\pm\overset{c}{\sqrt{35}})$

Slope of the asymptotes = $\pm\frac{1}{2}$ $\frac{a}{b}$



$\frac{a}{b}=1/2$

$a=\frac{1}{2}b$

$a^2+b^2=c^2$

$(\frac{1}{2}b)^2+b^2=(\sqrt{35})^2$

$\frac{1}{4}b^2+b^2=35$

$\frac{5}{4}b^2=35$

$b^2=28$

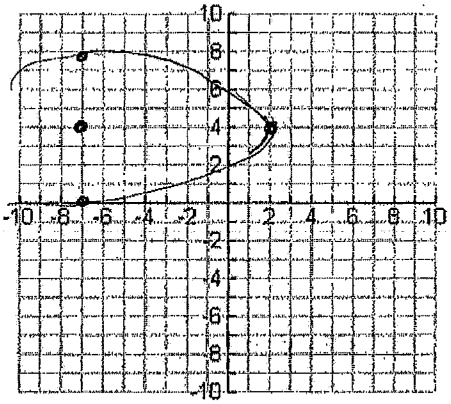
$a^2=7$

Center: (4,5)

Equation: $\frac{(y-5)^2}{7}-\frac{(x-4)^2}{28}=1$

17. Co-Vertices: $(\overset{h}{-7},\overset{k}{4}\pm\overset{c}{\sqrt{15}})$

Major Axis is 18 units long → $a=9$



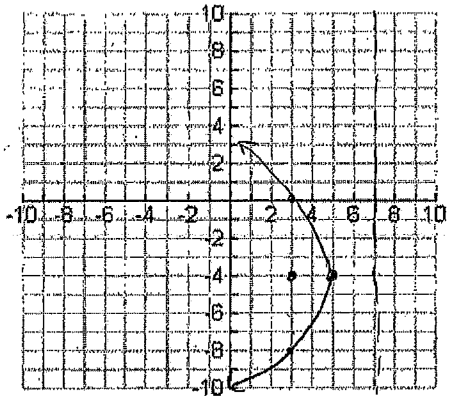
$81-b^2=15$

$-b^2=-66$

$b^2=66$

Equation: $\frac{(x+7)^2}{81}+\frac{(y-4)^2}{66}=1$

18. Focus: (3,-4) Directrix: $x=7$



Center: (5,-4) $p=-2$

Equation: $(y+4)^2=-8(x-5)$

Accelerated Pre-Calculus

Name: Key

8.13 Qu'est Review: Ellipses & Hyperbolas

Date: ____________

Classify each conic section as *ellipse* or *hyperbola*. Write each equation in standard form.

1. $9x^2 + 49y^2 - 294y = 0$

$9x^2+49(y^2-6y+\underline{9})=0+49(\underline{9})$

$\frac{9x^2}{441}+\frac{49(y-3)^2}{441}=\frac{441}{441}$

$\boxed{\frac{x^2}{49}+\frac{(y-3)^2}{9}=1}$

Ellipse has plus

2. $x^2 - 4y^2 + 2x - 16y - 31 = 0$

$x^2+2x-4y^2-16y=31$

$(x^2+2x+\underline{1})-4(y^2+4y+\underline{4})=31+1-4(4)$

$\frac{(x+1)^2}{16}-\frac{4(y+2)^2}{16}=\frac{16}{16}$

$\boxed{\frac{(x+1)^2}{16}-\frac{(y+2)^2}{4}=1}$

Hyperbola has minus

3. $16x^2 + y^2 - 128x + 4y + 244 = 0$

$16x^2-128x+y^2+4y=-244$

$16(x^2-8x+\underline{16})+(y^2+4y+\underline{4})=-244$
$+16(16)$
$+1(4)$

$\frac{16(x-4)^2}{16}+\frac{(y+2)^2}{16}=\frac{16}{16}$

$\boxed{(x-4)^2+\frac{(y+2)^2}{16}=1}$ Ellipse

4. $-9x^2 + 4y^2 - 36x - 8y - 68 = 0$

$4y^2-8y-9x^2-36x=68$

$4(y^2-2y+\underline{1})-9(x^2+4x+\underline{4})=68+4(1)$
$-9(4)$

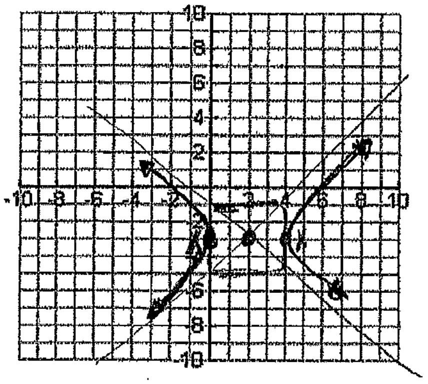
$\frac{4(y-1)^2}{36}-\frac{9(x+2)^2}{36}=\frac{36}{36}$

$\boxed{\frac{(y-1)^2}{9}-\frac{(x+2)^2}{4}=1}$

Hyperbola

Graph the equation and identify center, foci, vertices, co-vertices, eccentricity, asymptotes, and lengths of axes (as appropriate).

5. $\frac{(x-2)^2}{\underset{a^2}{4}} - \frac{(y+3)^2}{\underset{b^2}{4}} = 1$



$a=2 \longleftrightarrow$

$b=2 \updownarrow$

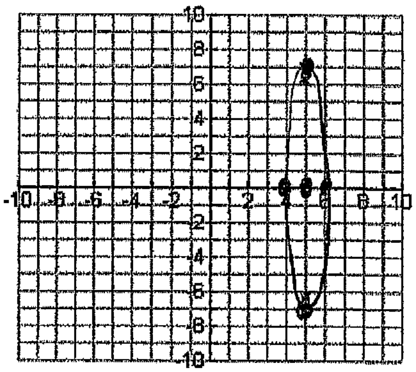
$c=2\sqrt{2} \longleftrightarrow$

$a^2+b^2=c^2$

$4+4=c^2$

$8=c^2$

6. $\frac{(x-5)^2}{1} + \frac{y^2}{49} = 1$



$a=7 \updownarrow$

$b=1 \longleftrightarrow$

$c=4\sqrt{3} \updownarrow$

$a^2-b^2=c^2$

$49-1=c^2$

$48=c^2$

Center: $(2,-3)$ Foci: $(2\pm2\sqrt{2},-3)$

Vertices: $(0,-3)$ $(4,-3)$ e = $\frac{2\sqrt{2}}{2}=\sqrt{2}$

Asymptotes: $y+3=\pm1(x-2)$

Conjugate axis = 4 Transverse axis = 4
2b 2a

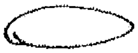
Center: $(5,0)$ Foci: $(5,\pm4\sqrt{3})$

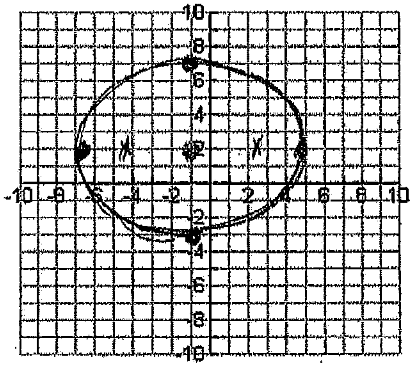
Vertices: $(5,7)$ $(5,-7)$ e = $\frac{4\sqrt{3}}{7}$

Co-Vertices: $(4,0)$ $(6,0)$

Major axis = 14 Minor axis = 2
2a 2b

7. $\frac{(x+1)^2}{35}+\frac{(y-2)^2}{25}=1$





$a=\sqrt{35} \longleftrightarrow$

$b=5 \updownarrow$

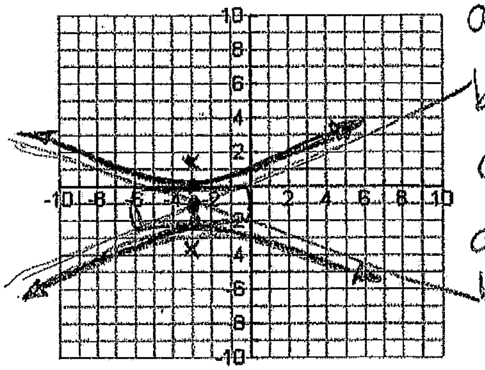
$c=\sqrt{10} \longleftrightarrow$

$a^2-b^2=c^2$

$35-25=c^2$

$10=c^2$

8. $\frac{(y+1)^2}{1}-\frac{(x+3)^2}{4}=1$



$a=1 \updownarrow$

$b=2 \longleftrightarrow$

$c=\sqrt{5} \updownarrow$

$a^2+b^2=c^2$

$1+4=c^2$

$5=c^2$

Center: $(-1,2)$ Foci: $(-1\pm\sqrt{10},2)$

Vertices: $(-1\pm\sqrt{35}, 2)$ e = $\frac{\sqrt{10}}{\sqrt{35}}=\frac{\sqrt{2}}{\sqrt{7}}=\frac{\sqrt{14}}{7}$

Co-Vertices: $(-1,7)$ $(-1,-3)$

Minor axis = 10 Major axis = $2\sqrt{35}$

2b 2a

Center: $(-3,-1)$ Foci: $(-3,-1\pm\sqrt{5})$

Vertices: $(-3,0)$ $(-3,-2)$ e = $\sqrt{5}$

Asymptotes: $y+1=\pm\frac{1}{2}(x+3)$

Transverse axis = 2 Conjugate axis = 4

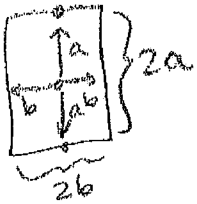
2a 2b

Use the information provided to write the standard form equation of each shape described.

center @ $(-8,7)$

9. Vertices: (-8, 9), (-8, 5) $2a=4$, $a=2 \updownarrow$

Perimeter of Central Rectangle = 36 ← Hyperbola



Perimeter = 4a+4b

$36=4(2)+4b$

$36=8+4b$

$28=4b$

$7=b \longleftrightarrow$

$\frac{(y-7)^2}{4}-\frac{(x+8)^2}{49}=1$

Equation

Center @ $(-7,4)$ ← $b=\sqrt{15} \updownarrow$

10. Co-Vertices: $(-7, 4\pm\sqrt{15})$

Major Axis is 18 units long $2a=18$

↑ Ellipse $a=9 \longleftrightarrow$

$\frac{(x+7)^2}{81}+\frac{(y-4)^2}{15}=1$

Equation

11. Center: $\left(-\frac{11}{2},\frac{1}{2}\right)$

Vertex: $\left(-\frac{35}{2},\frac{1}{2}\right)$ $a=\frac{24}{2}=12 \longleftrightarrow$

Eccentricity = $\frac{\sqrt{7}}{4}=\frac{c}{a}=\frac{?}{12}$

e<1, Ellipse $c=3\sqrt{7} \longleftrightarrow$

$a^2-b^2=c^2$

$b^2=144-63$

$b^2=81$

$\frac{(x+11/2)^2}{144}-\frac{(y-1/2)^2}{81}=1$

Equation

12. Foci: $(4, 5\pm 7\sqrt{5})$ ← $c=7\sqrt{5} \updownarrow$

Slope of the asymptotes = $\pm\frac{1}{2}$

↑ Hyperbola

a ← rise

$m=\frac{1}{2}=\frac{a}{b}$ $a=\frac{1}{2}b$

$a^2+b^2=c^2$

$(\frac{1}{2}b)^2+b^2=245$

$\frac{1}{4}b^2+b^2=245$

$\frac{5}{4}b^2=245$

$\frac{(y-5)^2}{49}-\frac{(x-4)^2}{196}=1$

Equation

$b^2=196$

$b=14 \rightarrow a=7 \rightarrow a^2=49 \updownarrow$