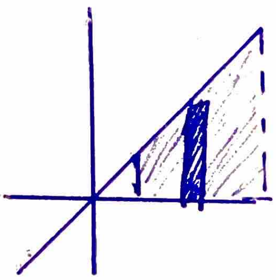
Ch. 8 Unit Review AP Practice Problems (p.633)

Key (51)

1. The base of a solid is the region in the first quadrant bounded by the lines $y = 2x$, $x = 1$, and $x = 4$. Every cross section of the solid perpendicular to the x-axis is a semicircle. What is the volume of the solid?

$\boxed{\text{(A)}}$ $\frac{21}{2}\pi$ (B) $\frac{32}{3}\pi$ (C) 21π (D) 42π



base $= 2x - 0$

Area $= \frac{\pi}{8}(\text{base})^2$

$V = \frac{\pi}{8}\int_1^4 [2x]^2\,dx$

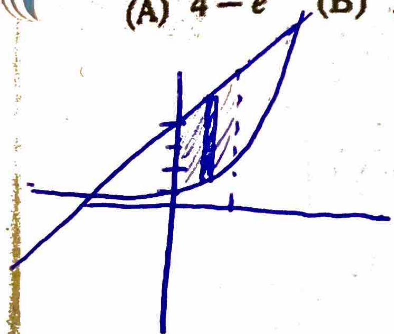
$V = \frac{\pi}{8}\int 4x^2\,dx$

$\frac{\pi}{8}\cdot\frac{4x^3}{3}\Big]_1^4$

$\frac{\pi}{6}x^3\Big]_1^4 = \frac{(4)^3\pi}{6} - \frac{\pi(1)^3}{6} = \frac{63\pi}{6} = \boxed{\frac{21\pi}{2}}$

2. The area of the region bounded by the graphs of $y = e^x$ and $y = 2x + 4$, $0 \le x \le 1$, is

(A) $4 - e$ (B) $5 - e$ $\boxed{\text{(C)}}$ $6 - e$ (D) $e - 4$



Area $= \int_0^1 2x+4-(e^x)\,dx$

$\int_0^1 2x+4-e^x\,dx$

$\frac{2x^2}{2}+4x-e^x\Big]_0^1 = 1^2+4-e^1-(0+0-e^0)$

$5-e+1 = \boxed{6-e}$

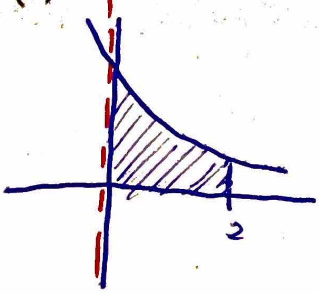
$\boxed{3.}$ What is the volume of the solid of revolution generated when the region bounded by the graph of $y = 4e^{-x^2}$, the x-axis, and the lines $x = 0$ and $x = 2$ is revolved about the y-axis?

(A) $2\pi(1 - e^{-4})$ (B) $4\pi e^{-4}$

(C) $4\pi(e^2 - 1)$ (D) $4\pi(1 - e^{-4})$

(Skip)

(*Shell Method)

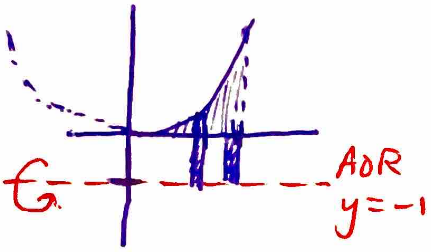


52

4. The region in the first quadrant bounded by the x-axis, the graph of $y = x^2$, and the line $x = 2$ is revolved about the line $y = -1$. The volume of the resulting solid of revolution is

(A) $\frac{22}{5}\pi$ (B) $\frac{52}{5}\pi$ $\boxed{\text{(C)}}$ $\frac{176}{15}\pi$ (D) $\frac{232}{5}\pi$

$\boxed{V = \frac{176}{15}\pi}$



Washer Method

$R(x) = x^2 - (-1) = x^2 + 1$

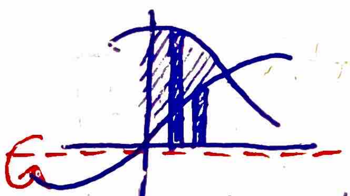
$r(x) = 0 - (-1) = 1$

$V = \pi \int_0^2 [x^2+1]^2 - [1]^2 \, dx$

15. A solid of revolution is formed by revolving the region bounded by the graphs of $y = \sin x$, $y = \cos x$, $0 \le x \le \frac{\pi}{4}$, and the line $x = 0$ about the x-axis. The volume of the solid is

(A) $\frac{1}{4}\pi^2$ $\boxed{\text{(B)}}$ $\frac{1}{2}\pi$ (C) $\frac{\sqrt{2}}{4}\pi$ (D) π

$*\cos 2x = \cos^2 x - \sin^2 x$



Washer Method

$R(x) = \cos x - 0$

$r(x) = \sin x - 0$

$V = \pi \int_0^{\pi/4} (\cos x)^2 - (\sin x)^2 \, dx$

$V = \pi \int \cos(2x) \, dx$

$\pi \int \cos u \cdot \frac{du}{2}$

$\frac{\pi}{2} \sin(2x) \Big]_0^{\pi/4}$

$\frac{\pi}{2} \sin(\pi/2) - \frac{\pi}{2} \sin(0)$

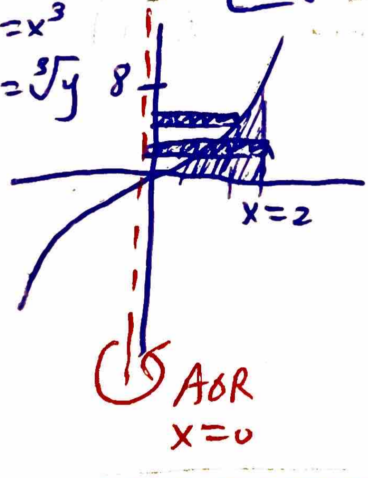
$= \boxed{\frac{\pi}{2}}$

16. The region in the first quadrant bounded by the graph of $y = x^3$, the line $x = 2$, and the x-axis, $\boxed{\text{is revolved about the y-axis}}$. The volume of the solid of revolution is

(A) $\frac{32\pi}{5}$ $\boxed{\text{(B)}}$ $\frac{64\pi}{5}$ (C) $\frac{256}{7}\pi$ (D) $\frac{65,536}{5}\pi$

$y = x^3$

$x = \sqrt[3]{y}$



*washer Method (Right - Left)

$R(y) = 2 - 0$

$r(y) = \sqrt[3]{y} - 0$

$V = \pi \int_0^8 (2)^2 - (y^{1/3})^2 \, dy$

$V = \pi \int 4 - y^{2/3} \, dy$

$4y - \frac{y^{5/3}}{5/3} \Big]_0^8$

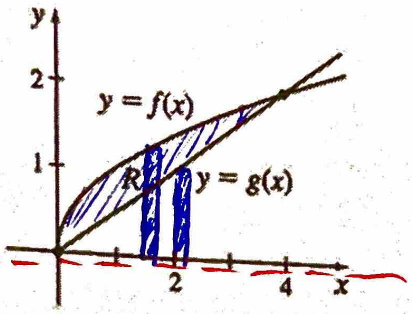
$32 - \frac{3}{5}(8)^{5/3} - (0 - 0)$

$32 - \frac{3}{5}(2)^5 = 32 - \frac{3}{5}(32)$

$\boxed{V = \frac{64}{5}\pi}$

7 (a, b)

7. The graphs of the functions $f(x) = \sqrt{x}$ and $g(x) = \frac{1}{2}x$ are shown in the figure below.



b) G AOR: $y=0$

(a) Find the area of the region R bounded by the graphs of f and g.

(b) Find the volume of the solid of revolution generated by revolving the region R about the x-axis.

a) $\text{Area} = \int_0^4 \sqrt{x} - \frac{1}{2}x\,dx = \int x^{1/2} - \frac{1}{2}x\,dx = \left.\frac{x^{3/2}}{3/2} - \frac{1}{2}\left(\frac{x^2}{2}\right)\right]_0^4$

$\frac{2}{3}(4)^{3/2} - \frac{(4)^2}{4} - (0-0) = \frac{2}{3}(8) - \frac{16}{4} = \boxed{\frac{4}{3}}$

b) * Washer Method

$R(x) = \sqrt{x} - 0$

$r(x) = \frac{1}{2}x - 0$

$V = \pi \int_0^4 [x^{1/2}]^2 - [\frac{1}{2}x]^2\,dx$

$\int x - \frac{1}{4}x^2\,dx$

$\left.\frac{x^2}{2} - \frac{1}{4}\cdot\frac{x^3}{3}\right]_0^4$

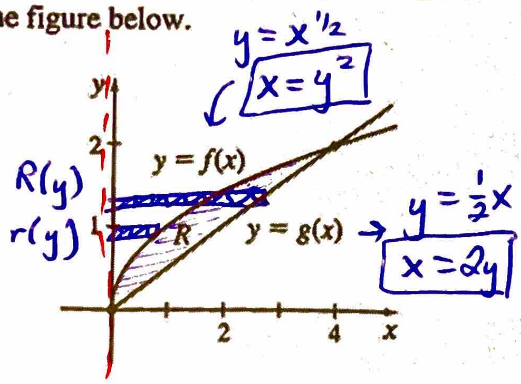
$= \frac{4^2}{2} - \frac{(4)^3}{12} - (0-0)$

$= \frac{16}{2} - \frac{64}{12} = \boxed{\frac{8}{3}\pi}$

(54) 7 (c, d)

7. The graphs of the functions $f(x) = \sqrt{x}$ and $g(x) = \frac{1}{2}x$ are shown in the figure below.

c)



(c) Find the volume of the solid of revolution generated by revolving the region R about the y-axis.

(d) The region R is the base of a solid. Find the volume of the solid if cross sections perpendicular to the base along the x-axis are squares.

c) $R(y) = 2y - 0$

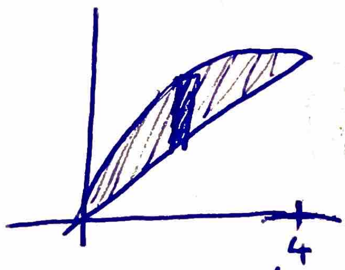
$r(y) = y^2 - 0$

$$V = \pi\int_0^2 [2y]^2 - [y^2]^2\,dy$$

$$= \int 4y^2 - y^4\,dy = \frac{4y^3}{3} - \frac{y^5}{5}\Bigg]_0^2$$

$$= \frac{4(2)^3}{3} - \frac{2^5}{5} - (0-0) = \frac{32}{3} - \frac{32}{5} = \boxed{\frac{64}{15}\pi}$$

d) *cross section (Squares)



base $= \sqrt{x} - \frac{1}{2}x$

Area(square) = (base)^2

$$V = \int_0^4 \left(\sqrt{x} - \tfrac{1}{2}x\right]^2 dx = \int x - x(x^{1/2}) + \tfrac{1}{4}x^2\,dx$$

$$\frac{x^2}{2} - \frac{x^{5/2}}{5/2} + \frac{1}{4}\cdot\frac{x^3}{3}\Bigg]_0^4$$

$$\frac{4^2}{2} - \frac{2}{5}(4)^{5/2} + \frac{1}{12}(4)^3 - (0 - 0 + 0)$$

$$8 - \frac{2}{5}(32) + \frac{64}{12} = \boxed{\frac{8}{15}}$$