Accel Pre-Calc Date: ____________

9.01 Review: Measures of Center and Spread

3 MEASURES OF CENTRAL TENDENCY		
Mean	Median	Mode
Denoted as $\bar{x}$, "x-bar" the average $\bar{x} = \frac{\sum x}{n}$	The number in the middle when the data is arranged in ascending order. If there are 2 numbers in the middle, then find their average.	The number which occurs most frequently. There does not have to be a mode. There can be more than one mode. Bimodal – 2 modes Trimodal – 3 modes

Example 1: Given scores from the latest test: 90, 89, 78, 81, 68, 100, 84, 83, 83, 74, 88, 80, 73, 89, 32

a) Find the measures of central tendency. Don't forget to put the data in ascending order!!!

n=15 *calculator instructions

Mean: 79.47 Median: 83 Mode: 83,89

5 NUMBER SUMMARY				
Minimum (Lower Extreme)	Lower (1st) Quartile Q_1	Median (2nd Quartile) Q_2	Upper (3rd) Quartile Q_3	Maximum (Upper Extreme)
Smallest number	The median of the lower half. If there are 2 numbers find their average.	Divides the data into a lower and upper half.	The median of the upper half. If there are 2 numbers find their average.	Largest number

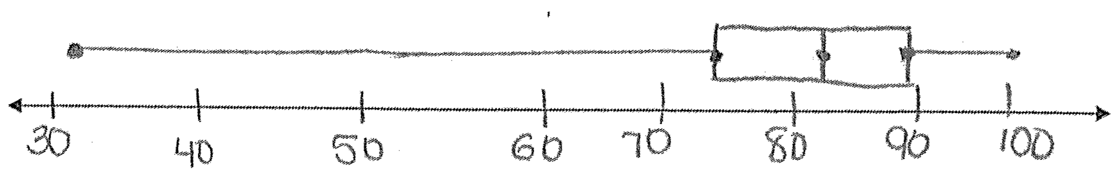
b) Find the 5-Number Summary of the test data above.

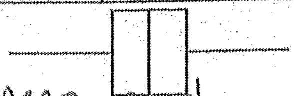
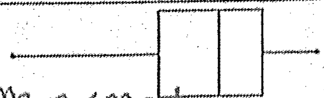
Min: 32 Q_1: 74 Median: 83 Q_3: 89 Max: 100

Box and Whisker Plot – A plot that displays the 5 number summary:

1. Draw a number line and scale it appropriately. Keep the minimum and maximum in mind.
2. Place points above the number line for each number in the 5 number summary.
3. Connect the minimum and Q_1 with a segment as well as Q_3 and the maximum.
4. Draw a box from Q_1 to Q_3.
5. Draw a vertical segment through the median.

c) Draw a box and whisker plot for the test data above.



SHAPE OF A BOX AND WHISKER PLOT		
Symmetric	Skewed Left	Skewed Right
 mean=med	 mean<med	 mean>med

MEASURES OF DISPERSION (SPREAD)			
Range	Interquartile Range	Mean Absolute Deviation (MAD)	Standard Deviation
The difference in the maximum and the minimum. (Max – Min)	The difference in the upper quartile and lower quartile. ($Q_3 - Q_1$)	$MAD = \frac{\sum \lvert x_i - \bar{x} \rvert}{n}$	$\sigma = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n}}$

d) Find the measures of spread for the given data set of test scores.

Range = 68 IQR = 15 MAD = 9.64 see back

Example 2: a) Collect the number of pets from 8 of your classmates. b) Calculate all measures of center, spread, and the 5 number summary for the data. c) Construct a box plot and describe the shape of the data.

a) The number of pets:

b) $\bar{x}$ = ________ Median: ________ Mode: ________

Min: ________ Q_1: ________ Median: ________ Q_3: ________ Max: ________

Range = ________ IQR = ________ MAD = ________

Shape: ________

BEST MEASURE OF CENTER AND SPREAD	
SYMMETRIC WITH NO OUTLIERS	SKEWED WITH OUTLIERS
Mean and Mean Absolute Deviation (MAD)	Median and Interquartile Range (IQR)

x	$\bar{x}$	$\lvert x-\bar{x}\rvert$	
90	79.47	10.53	
89	↓	9.53	
78		1.47	
81		1.53	
68		11.47	→ 34.53
100		20.53	
84		4.53	
83		3.53	
83		3.53	
74		5.47	
88		8.53	→ 80.65
80		0.53	
73		6.47	
89		9.53	
32		47.47	

$$\Sigma = \frac{144.65}{15}$$

MAD = 9.64

Accelerated Pre-Calculus

9.01: Review of Measures of Center and Spread

Date: Key

3 MEASURES OF CENTRAL TENDENCY		
Mean	Median	Mode
Denoted as $\bar{x}$, "x-bar" the average $\bar{x} = \frac{\sum x}{n}$	The number in the middle when the data is arranged in ascending order. If there are 2 numbers in the middle, then find their average.	The number which occurs most frequently. There does not have to be a mode. There can be more than one mode. Bimodal – 2 modes Trimodal – 3 modes

Example 1: Given scores from the latest test: ~~90, 89, 78, 81, 68, 100, 84, 83, 83, 74, 88, 80, 73, 89, 32~~

n=15

$\boxed{32}$ 68, 73, 74, 78, 80, 81 $\boxed{83}$ 83, 84, 88, 89, 89, 90, $\boxed{100}$

min Q1 Q2 Q3 max

a) Find the measures of central tendency. Don't forget to put the data in ascending order!!!

Mean: 79.467 Median: 83 Mode: 83, 89 (bimodal)

5 NUMBER SUMMARY				
Minimum (Lower Extreme)	Lower (1st) Quartile Q_1	Median (2nd Quartile) Q_2	Upper (3rd) Quartile Q_3	Maximum (Upper Extreme)
Smallest number	The median of the lower half. If there are 2 numbers find their average.	Divides the data into a lower and upper half.	The median of the upper half. If there are 2 numbers find their average.	Largest number

b) Find the 5-Number Summary of the test data above.

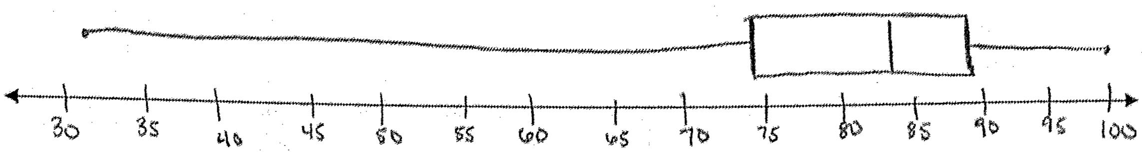
Min: 32 Q_1: 74 Median: 83 Q_3: 89 Max: 100

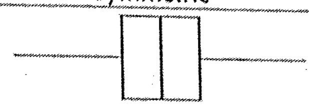
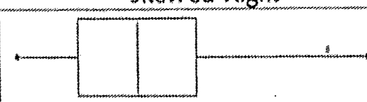
(no decimals). Restrict ans

Box and Whisker Plot – A plot that displays the 5 number summary:

1. Draw a number line and scale it appropriately. Keep the minimum and maximum in mind.
2. Place points above the number line for each number in the 5 number summary.
3. Connect the minimum and Q_1 with a segment as well as Q_3 and the maximum.
4. Draw a box from Q_1 to Q_3.
5. Draw a vertical segment through the median.

c) Draw a box and whisker plot for the previous test data.



SHAPE OF A BOX AND WHISKER PLOT		
Symmetric	Skewed Left	Skewed Right
		

MEASURES OF DISPERSION (SPREAD)		
Range	Interquartile Range	Mean Absolute Deviation
The difference in the <u>maximum</u> and the <u>minimum</u>. (Max – Min)	The difference in the <u>upper quartile</u> and <u>lower quartile</u>. ($Q_3 - Q_1$)	$\text{MAD} = \frac{\sum \lvert x_i - \bar{x} \rvert}{n}$

d) Find the measures of spread for the given data set of test scores.

Range = 68 IQR = 15 MAD = 9.644

Example 2:

a) List the number of pets from 8 of your classmates.

1 1 | 2 2 | 2 3 | 3 7

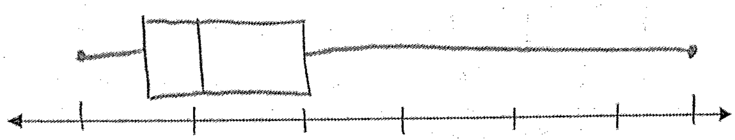
Q1 = 1.5 Q3 = 3

b) Calculate all measures of center, and the 5 number summary for the data.

$\bar{x} =$ 2.625 Median: 2 Mode: 2

Min: 1 Q_1: 1.5 Median: 2 Q_3: 3 Max: 7

c) Construct a box plot and describe the shape of the data.



Shape: right skewed

d) Calculate the measures of spread.

Range = 6 IQR = 1.5 MAD = 1.281

BEST MEASURE OF CENTER AND SPREAD	
SYMMETRIC WITH NO OUTLIERS	SKEWED or WITH OUTLIERS
Mean and Mean Absolute Deviation (MAD)	Median and Interquartile Range (IQR)

9.01 Homework: Statistics Review

Date: ______

1. Calculate all measures of center, spread, and the 5 number summary for the data provided. Construct a box plot and describe the shape of the data. Indicate if there are any outliers.

Homerun distances (in feet) to center field in 13 ballparks:

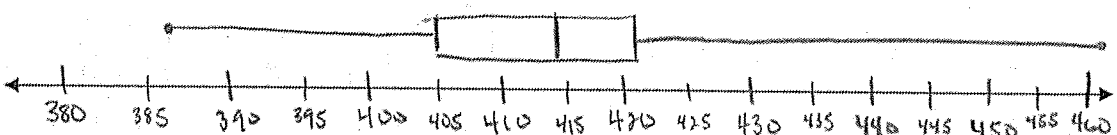
387, 400, 400, | 410, 410, 410, $\boxed{414,}$ 415, 420, 420 | 421, 457, 461

Q1 Q2 Q3

$\bar{x}$ = 417.308 Median: 414 Mode: 410

Min: 387 Q_1: 405 Median: 414 Q_3: 420.5 Max: 461

Range = 74 IQR = 15.5 MAD = 14.225



Describe the Shape/Skew/Outliers:

Skewed right
Outliers - 457, 461

2. Suppose that the numbers of unnecessary procedures recommended by five doctors in a 1-month period are 2, 2, 8, 10, and 18. If we ask a 6th doctor and find out that they recommend 35 procedures.

2 2 8 10 18 35

(a) How will the Median and Mean be affected?

Median is slightly affected (from 8 to 9)
Mean is significantly affected (from 8 to 12.5)

(b) How will the IQR and Mean Absolute Deviation be affected?

IQR increases slightly because there's more of a spread
(from 12 to 16)

MAD increases

3. Suppose the salaries (in dollars) of six employees are: 8000, 10000, 15000, 16000, 20000 and ~~39000~~. 39000

a. What are the Median and Mean salaries? Median \$ 15,500 Mean \$ 18000

b. Why are they such different numbers?

\$39000 is an outlier

Outliers have more effect on mean

c. Which measure of center is the better pick for this data? Why?

Median because there is an outlier skewing it right

d. Find the Mean Absolute Deviation.

MAD = 7666.667

X	$\bar{X}$	$X-\bar{X}$	$\lvert X-\bar{X}\rvert$
8000	18000	-10000	10000
10000	18000	-8000	8000
15000	18000	-3000	3000
16000	18000	-2000	2000
20000	18000	2000	2000
39000	1800	21000	21000

Σ 46000 $\div$ 6 = 7666.667

4. Based solely on the given mean and median, decide on the shape of each distribution:

(a) Mean = 100 Median = 98 Shape: Symmetric

(b) Mean = 20 Median = 41 Shape: Skewed left

(c) Mean = 934 Median = 850 Shape: skewed right

5. Give a set of numbers that would have a Mean Absolute Deviation of 0 units.

5 5 5 5 5 5

All values = mean

CCGPS Advanced Algebra
Worksheet: Stats Review

Name: Key
Date: ______ Period: ______

1. Calculate all measures of center, spread, and the 5 number summary for the data. Construct a box plot and describe the shape of the data. Indicate if there are any outliers.

Homerun distances (in feet) to center field in 13 ballparks:
387, 400, 400, 410, 410, 410, $\boxed{414}$, 415, 420, 420, 421, 457, 461

$n = 13$

$\Sigma x = 5425$

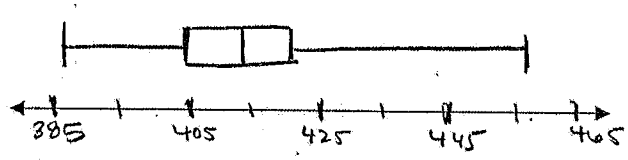
$|x_i - \bar{x}| =$ 30.31, 17.31, 17.31, 7.31, 7.31, 7.31, 3.31, 2.31, 2.69, 2.69, 3.69, 39.69, 43.69

$\Sigma |x_i - \bar{x}| = 184.92$

$\bar{x}$ = 417.31 ft Med: 414 ft Mode: 410

Min: 387 Q_1: 405 Med: 414 Q_3: 420.5 Max: 461

Range = 74 IQR = 15.5 MAD = 14.22 $\leftarrow \frac{\Sigma |x_i - \bar{x}|}{n}$



Describe the Shape/Skew/Outiers:

Skewed right with outliers of 457 and 461

2. Suppose that the numbers of unnecessary procedures recommended by five doctors in a 1-month period are 2, 2, 8, 10, and 18. If we ask a 6th doctor and find out that they recommend 35 procedures.

$\bar{x} = 8$ $M = 8$ $IQR = 8$ $MAD = 4.8$ $\bar{x} = 12.5$ $M = 10$ $IQR = 16$ $MAD = 9.1$

(a) How will the median and mean be affected?

The mean + median will both increase but the mean increases by a large amount due to the data being skewed to the right.

(b) How will the IQR and mean absolute deviation be affected?

The IQR and the MAD both increase; both almost double.

3. Suppose the salaries (in dollars) of six employees are: 8000, 10000, 15000 | 16000, 20000 and 39000.

a. What are the median and mean salaries?

$M = 15500 \qquad \bar{X} = 18000$

b. Why are they such different numbers?

The data is skewed to the right with an outlier of $39000.

c. Which measure of center is the better pick for this data? Why?

Median because the mean is skewed higher by the outlier of $39k.

d. Find the mean absolute deviation.

$$\sum |x_i - \bar{x}| = 10000 + 8000 + 3000 + 2000 + 2000 + 21000$$

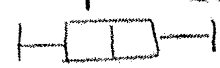
$$= 46000$$

$$\frac{\sum |x_i - \bar{x}|}{n} = \frac{46000}{6} = \$7666.67$$

4. Based solely on the given mean and median, decide on the shape of each distribution:

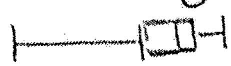
(a) Mean = 100 Median = 98 Shape: almost / no skew

$\bar{X} \approx M$



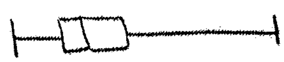
(b) Mean = 20 Median = 41 Shape: skewed left

$\bar{X} < M$



(c) Mean = 934 Median = 850 Shape: skewed right

$\bar{X} > M$



5. Give a set of numbers that would have a mean absolute deviation of 0 units.

7, 7, 7, 7, 7

Accelerated Precalculus
9.01 Practice: Statistics Review

TeleSchool Week 2

March 24, 2020

1. Calculate all measures of center, spread, and the 5 number summary for the data provided. Construct a box plot and describe the shape of the data. Indicate if there are any outliers.

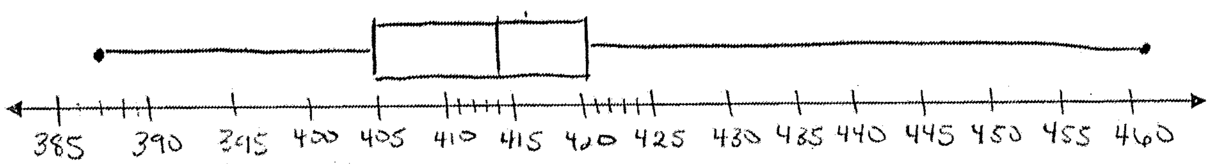
Homerun distances (in feet) to center field in 13 ballparks:
387, 400, 400, 410, 410, 410, 414, 415, 420, 420, 421, 457, 461

Q1 Q2 Q3

$\bar{x}$ = 417.308 Median: 414 Mode: 410

Min: 387 Q_1: 405 Median: 414 Q_3: 420.5 Max: 461

Range = 74 IQR = 15.5



Describe the Shape/Skew/Outliers:

Skewed right - 457 & 461 are outliers

2. Suppose that the numbers of unnecessary procedures recommended by five doctors in a 1-month period are 2, 2, 8, 10, and 18. If we ask a 6th doctor and find out that they recommend 35 procedures.

   a. How will the Median and Mean be affected (increase/decrease and which one would be more affected)? Both median & mean would increase. Mean would be more affected

   b. How will the IQR and Range be affected (increase/decrease and which one would be more affected)? Both IQR & Range would increase. Range would be more affected

3. Suppose the salaries (in dollars) of six employees are: 8000, 10000, 15000, 16000, 20000 and 39000.

Q2

a. What are the Median and Mean salaries?

Median: $15,500

Mean: 18,000

b. Why are they such different numbers?

$39,000 is an outlier that more significantly impacts the mean

c. Which measure of center is the better pick to describe this data? Why?

Median b/c it's less affected by the outlier

4. Based solely on the given mean and median, decide on the shape of each distribution (skewed left, skewed right, or approximately symmetric):

a. Mean = 100 Median = 98 Shape: approximately symmetric

close together

b. Mean = 20 Median = 41 Shape: Skewed left

mean significantly less

c. Mean = 934 Median = 850 Shape: Skewed right

mean significantly more

Accelerated Precalculus
9.02 Standard Deviation Practice

TeleSchool Week 2

March 24, 2020

Mrs. Ely has a problem and needs your input. She has to give one math award this year to a deserving student, but she can't make a decision. Here are the test grades for her two best students:

Emily: 90, 90, 80, 100, 99, 81, 98, 82 (90)

Jacob: 90, 90, 91, 89, 91, 89, 90, 90 (90)

1. Which of the two students should get the math award? Explain why he/she should be the recipient.

Jacob - he's the more consistent performer

2. Calculate the *variance* and *standard deviation* of Emily's distribution. The formulas are below. Fill out the table to help calculate by hand.

variance: $\sigma^2 = \frac{\Sigma (x_i - \bar{x})^2}{n}$ standard deviation: $\sigma = \sqrt{\frac{\Sigma (x_i - \bar{x})^2}{n}}$

a. The mean of Emily's test scores: 90

x_i for Emily	$x_i - \bar{x}$	$(x_i - \bar{x})^2$
90	0	0
90	0	0
80	-10	100
100	10	100
99	9	81
81	-9	81
98	8	64
82	-8	64

b. Variance for Emily (sum the last column and divide by n):

$\frac{490}{8} = 61.25$

c. Standard deviation for Emily (square root the variance):

7.826

d. What does standard deviation tell you about Emily's test scores?

While Emily performs well on tests, her scores vary a lot. She's not very consistent in her performance.

3. Calculate the *variance* and *standard deviation* of Jacob's distribution. The formulas are below. Fill out the table to help calculate by hand.

variance: $\sigma^2 = \frac{\sum (x_i - \bar{x})^2}{n}$ standard deviation: $\sigma = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n}}$

a. The mean of Jacob's test scores: 90

x_i for Jacob	$x_i - \bar{x}$	$(x_i - \bar{x})^2$
90	0	0
90	0	0
91	1	1
89	−1	1
91	1	1
89	−1	1
90	0	0
90	0	0

b. Variance for Jacob (the average of the last column):

$\frac{4}{8} = 0.5$

c. Standard deviation for Jacob (the square root of the variance):

$\sqrt{0.5} = 0.707$

d. What does standard deviation tell you about Jacob'x test scores?

Jacob is a consistent, high B/low A student.

4. Based on this information about the spread of test scores for Emily and Jacob, which of the two students should get the math award and discuss why he/she should be the one to receive it.

Jacob should receive the award because while they both have the same mean, Jacob is more consistent in his performance

9.03 Practice: Find the range and standard deviation of each data set.

1. 22, 18, 19, 25, 27, 21, 24

Range: 9
σ: 3.010

2. 38, 46, 37, 42, 39, 40, 48, 42

Range: 11
σ = 3.606

3. 8.4, 7.7, 8.6, 7.5, 8.9, 7.8, 8.6, 9.1, 8.0

Range: 1.6
σ = 0.530

4. 1.25, 3.69, 5.67, 4.89, 0.12, 4.35, 2.78

Range: 5.55
σ = 1.850

5. 515, 720, 635, 895, 585, 690, 770, 840

Range: 380
σ = 119.576

6. 116, 105, 117, 124, 107, 112, 117, 125, 110, 113

Range: 20
σ = 6.248

Find the mean, median, mode, range, and standard deviation of each data set.

7. Price (in dollars) of 7 different cordless phone models at an electronics store: 35, 50, 60, 60, 75, 65, 80

mean = 60.714
median = 60
mode = 60
range = 45
σ = 13.997

8. Number of homeruns for the 10 batters who hit the most homeruns during the 2014 MLB regular season: 32, 35, 40, 37, 35, 36, 32, 34, 37, 36

mean = 35.4
median = 35.5
mode = 32, 35, 36, 37
range = 8
σ = 2.289

9. Waiting times (in minutes) for several people at a Georgia Department of Driver Services office: 11, 7, 14, 2, 8, 13, 3, 6, 10, 3, 8, 4, 8, 4, 7

mean = 7.2
median = 7
mode = 8
range = 12
S = 3.668

10. Calories in a 1-ounce serving of several breakfast cereals: 135, 115, 120, 110, 110, 100, 105, 110, 125

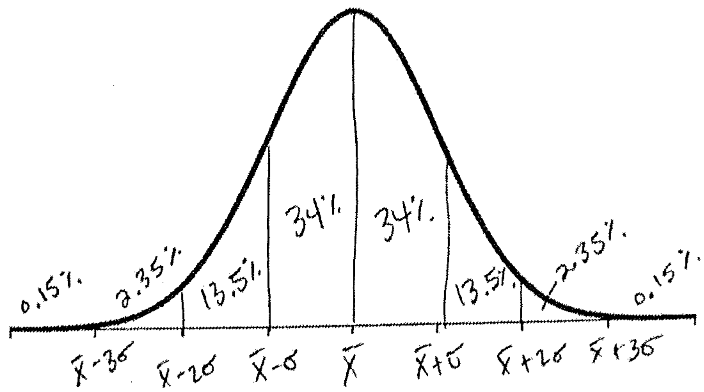
mean = 114.444
median = 110
mode = 110
range = 35
S = 10.737

Accelerated Precalculus TeleSchool, Week 3 March 31, 2020

9.04 Normal Distribution - The Empirical Rule

Normal Distribution is modeled by a normal (bell) curve and is symmetric about the mean.

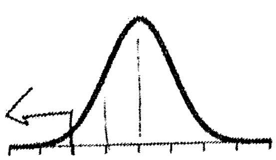
- It is formed using the mean and standard deviation.
- The total area under the curve is 100% because it represents all of the probability = 100 %
- Empirical Rule (68–95–99.7 Rule): the percent (probability) of the area under the curve for each standard deviation is shown on the graph below.



The normal distribution curve is used to find probability. It must be a normal distribution in order to use the above percentages (probabilities).

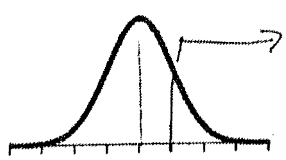
Example 1: For a normal curve, find the following probabilities of a randomly selected x-value from the distribution. If may be helpful to use a sketch.

a. $P(x \le \bar{x} - 2\sigma)$



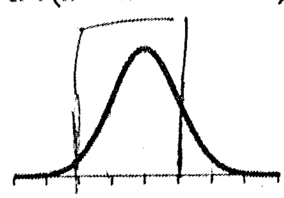
2.35% + 0.15%
2.5% → $\boxed{0.025}$

b. $P(x \ge \bar{x} + \sigma)$



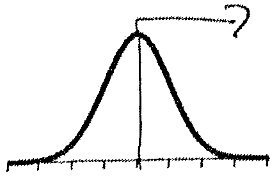
13.5 + 2.35 + 0.15 = 16%
$\boxed{0.16}$

c. $P(\bar{x} - 2\sigma \le x \le \bar{x} + \sigma)$



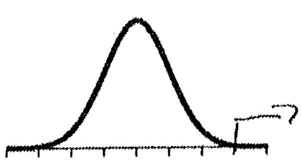
34 + 34 + 13.5 = 81.5%
$\boxed{0.815}$

d. $P(x \ge \bar{x})$



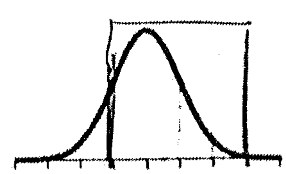
$\boxed{0.5}$

e. $P(x \ge \bar{x} + 3\sigma)$



0.15%
$\boxed{0.0015}$

f. $P(\bar{x} - \sigma \le x \le \bar{x} + 3\sigma)$



34 + 34 + 13.5 + 2.35
83.85% → $\boxed{0.8385}$

x ≤ means probability or percent to the left

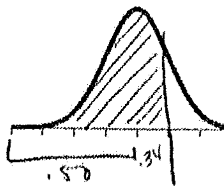
x ≥ means probability/percent to the right

≤ x ≤ means between

9.04 Homework – Normal Distribution & the Empirical Rule

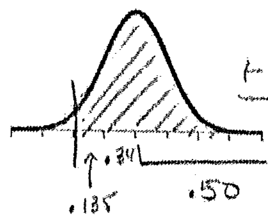
1. Find the indicated probability for a randomly selected x-value from the distribution. between

a. $P(x \le \bar{x} + \sigma)$



.50
+.34
$\boxed{.84}$

b. $P(x \ge \bar{x} - 2\sigma)$



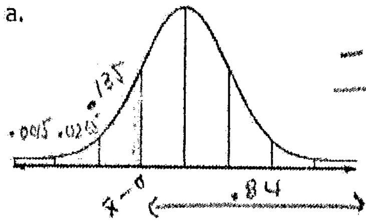
.50
.34
+.135
$\boxed{.975}$

c. $P(\bar{x} - \sigma \le x \le \bar{x} + \sigma)$

1 std dev away $\boxed{0.68}$

2. Give the percent of the area under the normal curve represented by the shaded region.

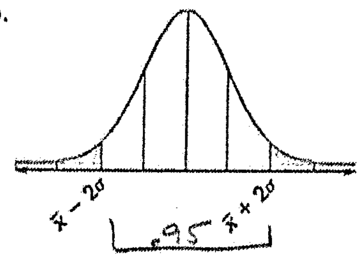
a.



1.00
− .84
$\boxed{.16}$

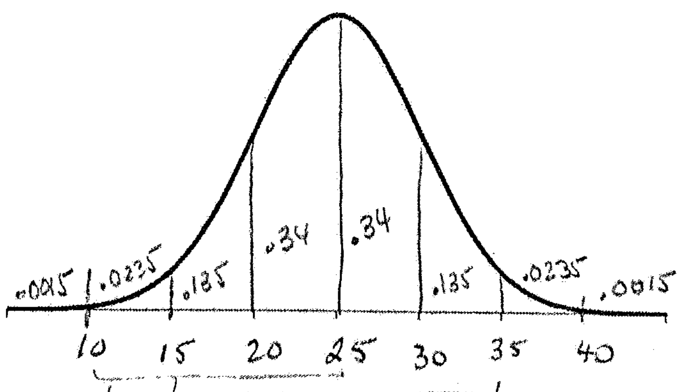
.0015
.0235
.135
.16

b.



1.00
− .95
$\boxed{.05}$

3. A normal distribution has a mean of 25 and a standard deviation of 5. Find the probability that a randomly selected x-value from the distribution is in the given interval. Label the x-axis and the probabilities under the curve below.



a. Between 20 and 30 upto 1 std away

$P(20 \le x \le 30) = \boxed{0.68}$

b. Between 10 and 25

$P(10 \le x \le 25) = .0235 + .135 + .34 = \boxed{0.4985}$

c. Between 15 and 35 up to 2 std dev away

$P(15 \le x \le 35) = \boxed{0.95}$

d. At least 20

$P(x \ge 20) = .34 + .50 = \boxed{.84}$

e. At least 35

$P(x \ge 35) = .0235 + .0015 = \boxed{.025}$

f. At most 30

$P(x \le 30) = .50 + .34 = \boxed{.84}$

CCGPS Advanced Algebra
More Practice with Standard Deviation

Name: Key
Date: ______________ Period: ____

Find the range and standard deviation of each data set.

1. 22, 18, 19, 25, 27, 21, 24

$\sigma = 3.010186787$

Range = 9

2. 38, 46, 37, 42, 39, 40, 48, 42

$\sigma = 3.605551275$

Range = 11

3. 8.4, 7.7, 8.6, 7.5, 8.9, 7.8, 8.6, 9.1, 8.0

$\sigma = 0.530082686$

Range = 1.6

4. 1.25, 3.69, 5.67, 4.89, 0.12, 4.35, 2.78

$\sigma = 1.8503822$

Range = 5.55

5. 515, 720, 635, 895, 585, 690, 770, 840

$\sigma = 119.5760741$

Range = 380

6. 116, 105, 117, 124, 107, 112, 117, 125, 110, 113

$\sigma = 6.248199741$

Range = 20

Find the mean, median, mode, range, and standard deviation of each data set.

7. Price (in dollars) of 7 different cordless phone models at an electronics store: 35, 50, 60, 60, 75, 65, 80

$\bar{x} = 60.71428571$

M = 60

mode = 60

Range = 45

$\sigma = 13.99708424$

$(S = 15.11857892)$

8. Number of homeruns for the 10 batters who hit the most homeruns during the 2014 MLB regular season:
32, 35, 40, 37, 35, 36, 32, 34, 37, 36

$\bar{x} = 35.4$

M = 35.5

mode = 32, 35, 36, 37

Range = 8

$\sigma = 2.289104628$

9. Waiting times (in minutes) for several people ← sample at a Georgia Department of Driver Services office:
11, 7, 14, 2, 8, 13, 3, 6, 10, 3, 8, 4, 8, 4, 7

$\bar{x} = 7.2$

M = 7

Mode = 8

Range = 12

$\sigma =$

$S = 3.668397786$

10. Calories in a 1-ounce serving of several ← sample breakfast cereals: 135, 115, 120, 110, 110, 100, 105, 110, 125

$\bar{x} = 114.4$

M = 110

Mode = 110

Range = 35

$\sigma =$

$S = 10.73674894$

CCGPS Advanced Algebra
Worksheet: Standard Deviation

Name Key
Date ________ Period: ______

Your teacher has a problem and needs your input. She has to give one math award this year to a deserving student, but she can't make a decision. Here are the test grades for her two best students:

Emily
Bryce: 90, 90, 80, 100, 99, 81, 98, 82

Jacob
Brianna: 90, 90, 91, 89, 91, 89, 90, 90

1. Write down which of the two students should get the math award and discuss why they should be the one to receive it.
   Bryce – higher max grade, more higher grades
   Brianna – higher min grade, more consistent grades

2. Calculate the *mean deviation*, *variance*, and *standard deviation* of Bryce's distribution. The formulas are below. Fill out the table to help you calculate them by hand.

Mean deviation: $\frac{\sum |x_i - \bar{x}|}{n}$ variance: $\sigma^2 = \frac{\sum (x_i - \bar{x})^2}{n}$ standard deviation: $\sigma = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n}}$

The mean of Bryce's test scores: 90

x_i for Bryce	deviations $x_i - \bar{x}$	MAD $\lvert x_i - \bar{x} \rvert$	σ $(x_i - \bar{x})^2$
90 -90	0	0	0
90 -90	0	0	0
80 -90	-10	10	100
100 -90	10	10	100
99 -90	9	9	81
81 -90	-9	9	81
98 -90	8	8	64
82 -90	-8	8	64
		54	490

a. Mean deviation for Bryce:
   $54 \div 8 = 6.75$

Emily

b. Variance for Bryce:
   $490 \div 8 = 61.25$

c. Standard deviation for Bryce:
   $\sqrt{61.25} \approx 7.826238$

d. What do these measures of spread tell you?
   Bryce's scores tend to be spread out away from the mean by 6.75 pts on average.

Jacob

3. Calculate the *mean deviation*, *variance*, and *standard deviation* of Brianna's distribution. The formulas are below. Fill out the table to help you calculate them by hand.

Mean deviation: $\frac{\Sigma |x_i - \bar{x}|}{n}$ variance: $\sigma^2 = \frac{\Sigma (x_i - \bar{x})^2}{n}$ standard deviation: $\sigma = \sqrt{\frac{\Sigma (x_i - \bar{x})^2}{n}}$

The mean of Brianna's test scores: 90

x_i for Brianna	$x_i - \bar{x}$	$\lvert x_i - \bar{x} \rvert$ MAD	$(x_i - \bar{x})^2$ σ
90 −90	0	0	0
90 −90	0	0	0
91 −90	1	1	1
89 −90	−1	1	1
91 −90	1	1	1
89 −90	−1	1	1
90 −90	0	0	0
90 −90	0	0	0
		4	4

a. Mean deviation for Brianna:

$4 \div 8 = 0.5$

Jacob

b. Variance for Brianna:

$4 \div 8 = 0.5$

c. Standard deviation for Brianna:

$\sqrt{0.5} = 0.7071068$

d. What do these measures of spread tell you?

Brianna's scores tend to be closer to the mean, varing away by 0.5 pts on average.

4. Based on this information about the spread of test scores for Bryce and Brianna, which of the two students should get the math award and discuss why they should be the one to receive it.

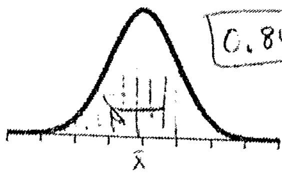
Bryce – has ability to do better than Brianna, but is inconsistent.

Brianna – is consistently a good student, but does not hit the highest marks.

9.04 Practice – Normal Distribution & the Empirical Rule

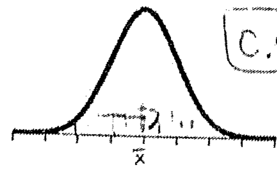
1. Find the indicated probability for a randomly selected x-value from the distribution.

a. $P(x \le \bar{x} + \sigma)$



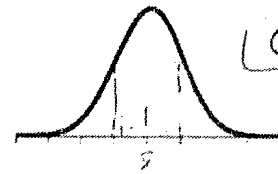
$\boxed{0.84}$

b. $P(x \ge \bar{x} - 2\sigma)$



$\boxed{0.975}$

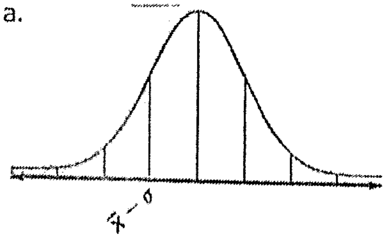
c. $P(\bar{x} - \sigma \le x \le \bar{x} + \sigma)$



$\boxed{0.68}$

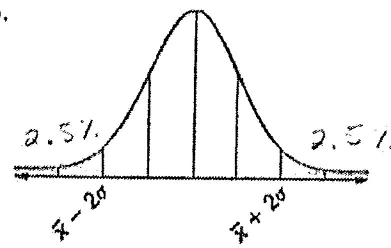
2. Give the percent of the area under the normal curve represented by the shaded region.

a.



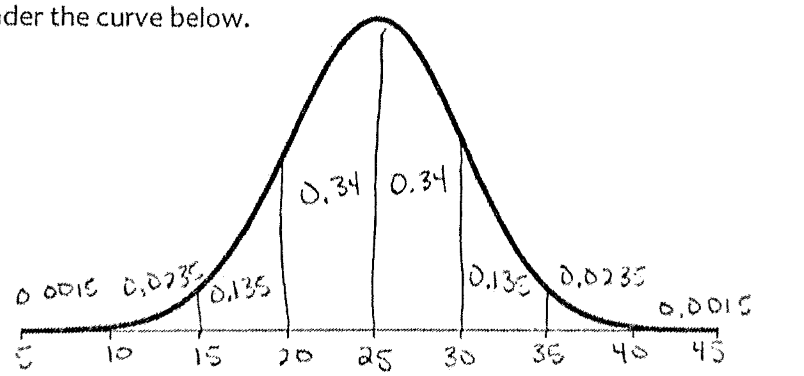
$\boxed{16\%}$

b.



$\boxed{5\%}$

3. A normal distribution has a mean of 25 and a standard deviation of 5. Find the probability that a randomly selected x-value from the distribution is in the given interval. Label the x-axis and the probabilities under the curve below.



a. Between 20 and 30

0.68

b. Between 10 and 25

0.4985

c. Between 15 and 35

0.95

d. At least 20

0.84

e. At least 35

0.025

f. At most 30

0.84

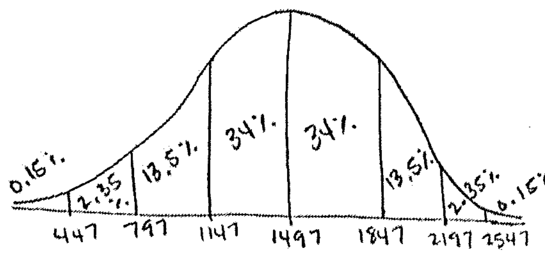
Accelerated Precalculus

9.05 Applications of the Empirical Rule   TeleSchool, Week 3   March 31, 2020

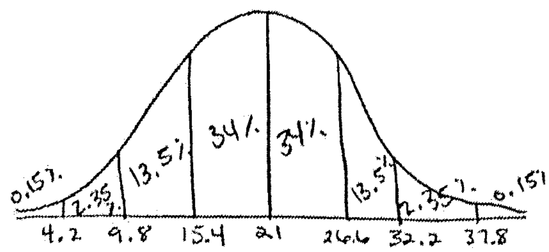
A) Comparing the SAT and ACT: college admissions offices need to compare scores of students who take the Scholastic Aptitude Test (SAT) with those who take the American College Test (ACT). Suppose that for recent college applicants who took the SAT, scores have a mean of 1497 (out of 2400) and a standard deviation of 350. Further, suppose that for recent college applicants who took the ACT, scores have a mean of 21 (out of 36) and a standard deviation of 5.6.

1. Sketch normal curves for both the SAT and ACT listing values for 1, 2, and 3 standard deviations on each side of the mean.

SAT



ACT



Apply the empirical rule to approximate the following:

2. About 95% of SAT takers score between what two values? (797, 2197)

(within $\pm 2\sigma$)

3. About 95% of ACT takers score between what two values? (9.8, 32.2)

(within $\pm 2\sigma$)

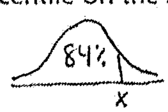
4. What is the proportion of students who score between 1147 and 1847 on the SAT? $\boxed{68\%}$

34% + 34%

5. What is the proportion of students who score between 15.4 and 32.2 on the ACT? $\boxed{81.5\%}$

34% + 34% + 13.5%

6. If John scored at the 84th percentile on the ACT, what score did he achieve? $\boxed{26.6}$



find .84 inside z table

$z = 1$ $\quad 1 = \frac{x-21}{5.6}$ $\quad 5.6 = x - 21$ $\quad x = 26.6$

7. College Board reports that 1,672,395 students took the SAT in 2014. About how many students achieved a score of at least 2197?

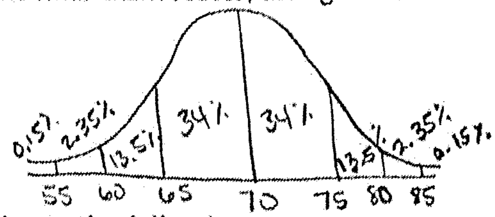
1672395 (0.025) ≈ 41810 students

8. ACT, Inc. reports that 1,845,787 students took the ACT in 2014. About how many students achieved a score of at most 21?

1845787 (0.5) ≈ 922894 students

B) Last spring, 250 students took the Algebra 2 final exam. The scores were distributed normally with a mean of 70 and a standard deviation of 5.

9. Sketch the normal curve for the final exam scores, listing values for 1, 2, and 3 standard deviations on each side of the mean.



Apply the empirical rule to approximate the following:

10. What percentage of scores is between scores 65 and 75? 68%

34% + 34%

11. What percentage of scores is between scores 60 and 70? 47.5%

13.5 + 34

12. What percentage of scores is between scores 60 and 85? 97.35%

13.5 + 34 + 34 + 13.5 + 2.35

13. What percentage of scores is less than a score of 55? 0.15%

14. What percentage of scores is at least a score of 80? 2.5%

2.35 + 0.15

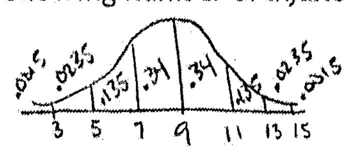
15. How many Algebra 2 students achieved a score between 70 and 80?

.34 + .135 = 0.475 250(0.475) ≈ 119 students

16. How many Algebra students achieved a score of at most 75?

.0015 + .0235 + .135 + .34 + .34 = 0.84 250(0.84) = 210 students

C) Statistics kept for NFL football teams regarding the number of injuries suffered by NFL players during their careers showed the distribution is approximately normal with the mean number of injuries per player to be 9 with a standard deviation of 2. If there are 1696 NFL players in the current season, determine how many players will have the following number of injuries:



17. Less than 9 injuries in their career.

1696(0.5) = 848 players

18. At least 7 injuries in their career.

1696(0.84) ≈ 1425 players

19. More than 5 but less than 11 injuries in their career.

1696(0.815) ≈ 1382 players

Accelerated Precalculus

TeleSchool, Week 3

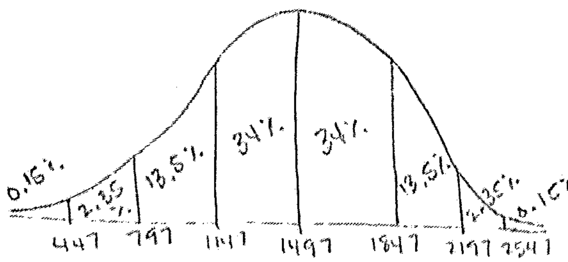
March 31, 2020

9.05 Applications of the Empirical Rule

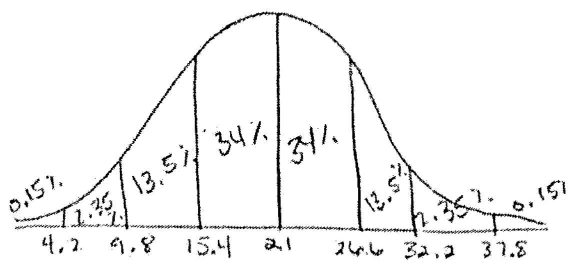
A) Comparing the SAT and ACT: college admissions offices need to compare scores of students who take the Scholastic Aptitude Test (SAT) with those who take the American College Test (ACT). Suppose that for recent college applicants who took the SAT, scores have a mean of 1497 (out of 2400) and a standard deviation of 350. Further, suppose that for recent college applicants who took the ACT, scores have a mean of 21 (out of 36) and a standard deviation of 5.6.

1. Sketch normal curves for both the SAT and ACT listing values for 1, 2, and 3 standard deviations on each side of the mean.

SAT



ACT



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6. If John scored at the 84th percentile on the ACT, what score did he achieve? 26.6

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B) Last spring, 250 students took the Algebra 2 final exam. The scores were distributed normally with a mean of 70 and a standard deviation of 5.

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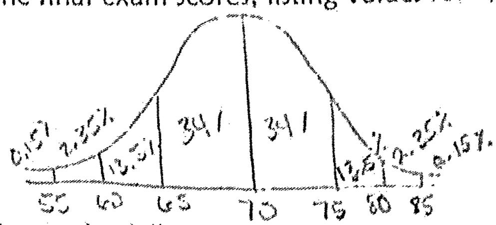

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12. What percentage of scores is between scores 60 and 85? 97.35%

13. What percentage of scores is less than a score of 55? 0.15%

14. What percentage of scores is at least a score of 80? 2.5%

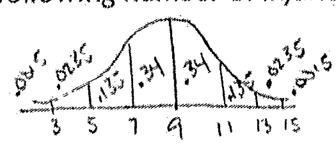
15. How many Algebra 2 students achieved a score between 70 and 80?

$250(0.475) \approx 119$ students

16. How many Algebra students achieved a score of at most 75?

$250(0.84) = 210$ students

C) Statistics kept for NFL football teams regarding the number of injuries suffered by NFL players during their careers showed the distribution is approximately normal with the mean number of injuries per player to be 9 with a standard deviation of 2. If there are 1696 NFL players in the current season, determine how many players will have the following number of injuries:



17. Less than 9 injuries in their career.

$1696(0.5) = 848$ players

18. At least 7 injuries in their career.

$1696(0.84) \approx 1425$ players

19. More than 5 but less than 11 injuries in their career.

$1696(0.815) \approx 1382$ players

Accelerated Precalculus
9.06 Standard Normal Distributions

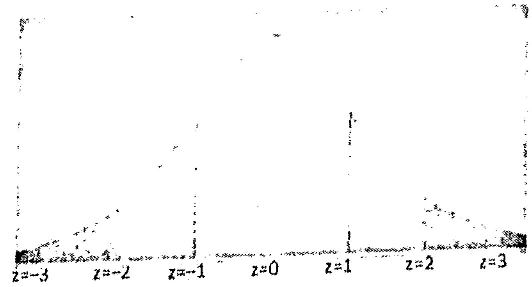
TeleSchool, Week 3

March 31, 2020

What do you do when you are looking for the probability of an x-value in a normal distribution, but that value does not fall on one of the standard deviations?

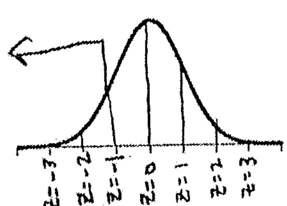
Standard Normal Distribution:

- Formed using a mean of 0 and a standard deviation of 1.
- Used when the x-value does not fall on a standard deviation.
- The Empirical Rule still applies to a standard normal distribution.
- To change an x-value from a normal distribution with mean $\bar{x}$ and standard deviation σ use the z-score formula: $\boxed{z = \frac{x-\bar{x}}{\sigma}}$
- The z-score is the number of standard deviations the x-value lies away from the mean.



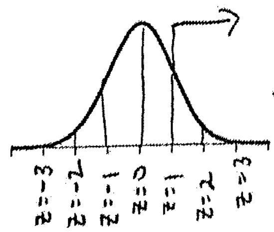
Example 1: For a standard normal distribution, find the following probabilities of a randomly selected value from the distribution. If may be helpful to draw a sketch.

a. $P(z \le -1)$ = 0.16



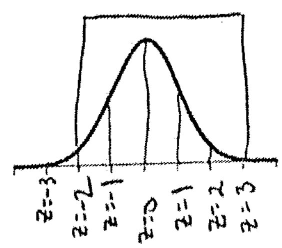
0.15
+ 2.35
+ 13.5
16%

b. $P(z \ge 1)$ = 0.16



13.5
+ 2.35
+ 0.15
16%

c. $P(-2 \le z \le 3)$ = 0.9735



13.5
+ 34
+ 34
+ 13.5
+ 2.35
97.35%

Converting to a z-score:
Example 2: Consider a normal distribution with a mean of 72 and a standard deviation of 3. Convert the following into z-scores.

a. x = 65 $z = \frac{65-72}{3}$
$z = -2.33$

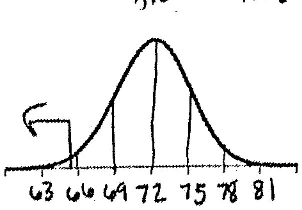
b. x = 81 $z = \frac{81-72}{3}$
$z = 3.00$

c. x = 76 $z = \frac{76-72}{3}$
$z = 1.33$

WHAT'S THE POINT???
Z-scores enable us to find any probability with a table of values. The probability given on the z-score table is the probability that is less than the z-score. BE CAREFUL! We are not always looking for less than!!!!
(Probability to the left)

Example 3: Consider a normal distribution with a mean of 72 and a standard deviation of 3. Sketch a graph and find the following:

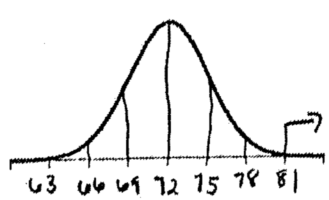
a. $P(x \le 65)$
≤ take the probability from table



$z = -2.33$

$P(x \le 65) = \boxed{0.0099}$

b. $P(x \ge 81)$
≥ 1 − probability from table

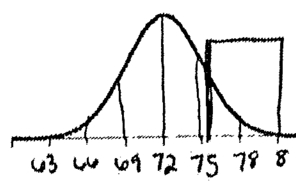


$z = 3.00$

$P(x \ge 81) = 1 - 0.9987$

$\boxed{0.0013}$

c. $P(76 \le x \le 81)$ ≤ x ≤
Subtract 2 probabilities from table



$z_{81} = 3.00$ $P(x \le 81) = 0.9987$

$z_{76} = 1.33$ $-P(x \le 76) = 0.9082$

$\boxed{0.0905}$

Accelerated Precalculus TeleSchool, Week 3 March 31, 2020

9.06 Standard Normal Distribution Homework

$Z = \frac{X - \bar{X}}{\sigma}$

1) A normal distribution has a mean of 70 and a standard deviation of 10. Calculate the z-score and use the z-score table to find the indicated probability.

a. P(x ≤ 65)

$Z = \frac{65-70}{10} = -0.5$

$P(x \le 65) = \boxed{0.3085}$

b. P(x ≥ 47)

$z = \frac{47-70}{10} = -2.3$

$1 - P(x \le 47)$

$1 - 0.0107$

$\boxed{0.9893}$

c. P(54 ≤ x ≤ 83)

$z_{54} = \frac{54-70}{10}$ $z_{83} = \frac{83-70}{10}$

$z_{54} = -1.6$ $z_{83} = 1.3$

$P(x \le 83) - P(x \le 54)$

$0.9032 - 0.0548 = \boxed{0.8484}$

d. P(x ≤ 91)

$z = \frac{91-70}{10} = 2.1$

$P(x \le 91) = \boxed{0.9821}$

e. P(x ≥ 39)

$z = \frac{39-70}{10} = -3.1$

$1 - P(x \le 39)$

$1 - 0.0010$

$\boxed{0.999}$

f. P(79 ≤ x ≤ 101)

$z_{79} = \frac{79-70}{10}$ $z_{101} = \frac{101-70}{10}$

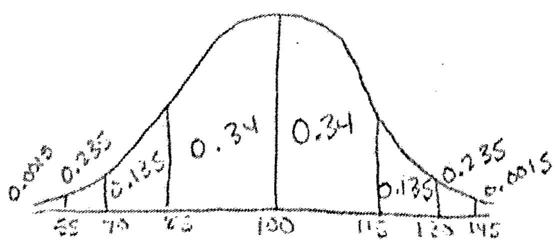
$z_{79} = 0.9$ $z_{101} = 3.1$

$P(x \le 101) - P(x \le 79)$

$0.9990 - 0.8159 = \boxed{0.1831}$

2) The scores on an Intelligence Quotient (IQ) test are normally distributed with a mean of 100 and a standard deviation of 15.

a. Draw and label the normal distribution.



b. What is the probability that a person will score at least a 121?

$P(x \ge 121)$

$z = \frac{121-100}{15} = 1.4$

$1 - 0.9192 = \boxed{0.0808}$

c. What is the probability that a person will score no more than 79?

$P(x \le 79)$

$z = \frac{79-100}{15} = -1.4$

$P(x \le 79) = \boxed{0.0808}$

d. What is the probability that a person will score between 92 and 111?

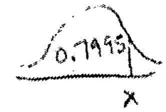
$P(92 \le x \le 111)$

$z_{111} = \frac{111-100}{15}$ $z_{92} = \frac{92-100}{15}$

$z_{111} = 0.73$ $z_{92} = -0.53$

$0.7673 - 0.2981 = \boxed{0.4692}$

e. What minimum score would someone need to score higher than 80% of those taking an IQ test?



← closest match on z-table

$z = 0.84$

$0.84 = \frac{x - 100}{15}$

$12.6 = x - 100$

$\boxed{x = 112.6}$

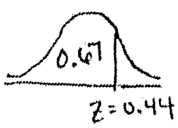
9.07 Standard Normal Distribution Applications

All data in the following exercises is normally distributed.

1. Jack took a standardized achievement test with a mean of 125 and a standard deviation of 15. Jack's score was 148.
   a. What is the percentile rank for Jack's score on this test? Percentile rank means that Jack scored as well or better than that percentage of students taking the test.

   $z = \frac{148-125}{15} = 1.53 \quad P(x \leq 148) = 0.9370 \quad \boxed{93.7\% \text{ ile}}$

   b. If Jill scored at the 67th percentile, what was her score on the test?

   $0.44 = \frac{x-125}{15} \quad \boxed{x = 131.6}$

   $6.6 = x - 125$

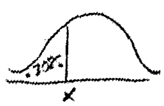
2. The average number of absences for 1st graders is 15 with a standard deviation of 6.
   a. What is the probability of a 1st grader having fewer than 6 absences?

   $z = \frac{6-15}{6} = -1.5 \quad P(x \leq 6) = \boxed{0.0668}$

   b. What is the probability of a 1st grader having more than 20 absences?

   $z = \frac{20-15}{6} = 0.83 \quad P(x \geq 20) = 1 - 0.7967 = \boxed{0.2033}$

   c. If a student is absent more often than 30.85% of other 1st graders, how many days did she miss?

   $z = -0.5 \quad -0.5 = \frac{x-15}{6} \quad \boxed{x = 12 \text{ days}}$

   $-3 = x - 15$

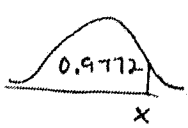
3. On the Scholastic Aptitude Test (SAT), scores have a mean of 500 and a standard deviation of 100.
   a. Mo scored 600 on the math section. What percentile did he achieve?

   $z = \frac{600-500}{100} = 1 \quad \boxed{84.13\% \text{ ile}}$ or $84\% \text{ ile}$

   b. Larry scored 750 on the math section. What percentile did he achieve?

   $z = \frac{750-500}{100} = 2.5 \quad \boxed{99.38\% \text{ ile}}$

   c. Curley scored better than 97.72% of students on the math section of the SAT. What was his score?

   $z = 2.0 \quad 2 = \frac{x-500}{100}$

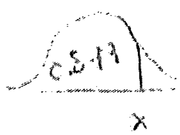
   $200 = x - 500$

   $\boxed{x = 700}$

4. A patient recently diagnosed with Alzheimer's disease took a cognitive abilities test. The mean score on this test is 52 with a standard deviation of 5.
   a. If the patient scored a 45 on the test, what is his percentile rank?

$z = \frac{45-52}{5} = -1.4$ $\boxed{8.08\%\text{ile}}$

   b. If a patient scored higher than 87.7% of other Alzheimer's patients, what was her score on the test?



$z = 1.16$ $1.16 = \frac{x-52}{5}$ $5.8 = x - 52$ $\boxed{x = 57.8}$

5. Mrs. Durand's Algebra 2 unit test scores have a mean of 82 with a standard deviation of 5.5.
   a. What is the probability that a student will score at least a 92?

$z = \frac{92-82}{5.5} = 1.82$ $P(x \geq 92) = 1 - 0.9656 = \boxed{0.0344}$

   b. What is the probability a student will get a B?

$z_{89.4} = \frac{89.4-82}{5.5} = 1.35$ $P(x \leq 89.4) = 0.9115$ $0.9115 - 0.3264$

$z_{79.5} = \frac{79.5-82}{5.5} = -0.45$ $P(x \leq 79.5) = 0.3264$ $\boxed{0.5851}$

   c. What is the probability a student will fail?

$z = \frac{69.4-82}{5.5} = -2.29$ $P(x \leq 69.4) = \boxed{0.0110}$

6. The diameter of maple trees in a Canadian forest have a mean of 10 inches and standard deviation of 3.2 inches. What is the percentage of trees with a diameter between 8 and 15 inches?

$z_{15} = \frac{15-10}{3.2} = 1.56$ $P(x \leq 15) = 0.9406$ $0.9406 - 0.2643 =$

$z_8 = \frac{8-10}{3.2} = -0.63$ $P(x \leq 8) = 0.2643$ $\boxed{0.6763}$

7. The weights of 1800 fish in a lake have a mean of 3 kilograms and standard deviation of .5 kilograms.
   a. What percentage of fish weigh at least 3.75 kilograms?

$z = \frac{3.75-3}{0.5}$ $P(x \geq 3.75) = 1 - 0.9332 = 0.0668$

$z = 1.5$ $\boxed{6.68\%}$

   b. Approximately how many fish in the lake weigh at least 3.75 kilograms?

$1800(0.0668) = 120.24$

$\boxed{120 \text{ fish}}$

APC 9.07 Applications KEY

Saturday, March 14, 2020 8:19 PM

CCGPS Accelerated Geometry
Standard Normal Distribution Applications

Name Key
Date ______ Period ____

Each distribution is normally distributed.

1. A fifth grader takes a standardized achievement test with a mean of 125 and a standard deviation of 15. The child scores a 148.
   a. What is the child's percentile rank when scoring 148? (Percentile ranks mean the student scored as well or better than that percentage of students taking the test.)

   $z = \frac{148-125}{15} = 1.5\overline{3}$ $P(x \le 148) = \boxed{93.7\%ile}$

   b. If Billy's scores at the 67th percentile, what was his score on the test?

   invNormal

   $z = 0.44 = \frac{x-125}{15}$

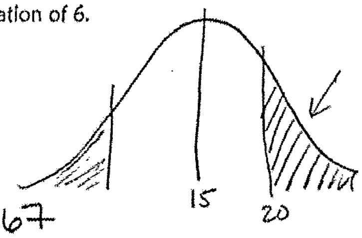
   $6.6 = x - 125$

   $\boxed{x = 131.6}$

2. The average number of absences for 1st graders is 15 with a standard deviation of 6.
   a. What is the probability of a 1st grader having fewer than 6 absences?

   $P(x < 6) = P(z < -1.5) = \boxed{0.0668}$

   $\frac{6-15}{6}$



   b. What is the probability of a 1st grader having more than 20 absences?

   $P(x > 20) = P(z > 0.8\overline{3}) = 1 - 0.7967$

   $= \boxed{0.2033}$

   c. If Nicole is absent more often than 30.85% of other 1st graders, how many days did she miss?

   invNormal
   area=0.3085

   $z = -0.50 = \frac{x-15}{6}$

   $-3 = x - 15$

   $12 = x$

   $\boxed{12 \text{ absences}}$

3. Scores on the SAT form a normal distribution with a mean of 500 and a standard deviation of 100.
   a. Hans receives a score of 600 on the math section. What percentile did he achieve?

   $P(x \le 600) = P(z \le 1) = 0.8413 = \boxed{84.13\%ile}$

   b. Claire receives a score of 750 on the math section. What percentile did she achieve?

   $P(x \le 750) = P(z \le 2.5) = 0.9938 = \boxed{99.38\ \%ile}$

   c. Stephanie scored better than 97.72% of students on the math section of the SAT. What score did she receive?

   invNormal
   area=97.72

   $z = 2 = \frac{x-500}{100}$

   $200 = x - 500$

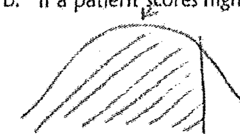
   $\boxed{700 = x}$

4. A patient recently diagnosed with Alzheimer's disease takes a cognitive abilities test. The mean score of a patient with Alzheimer's is 52 with a standard deviation of 5.
   a. If the patient scores a 45 on the test, what is his percentile rank?

$P(X \le 45) = P(Z \le -1.4) = 0.0808$

$Z = \frac{45-52}{5}$ $\boxed{8.08\%\text{ile}}$

   b. If a patient scores higher than 87.7% of other Alzheimer's patients, what was her score on the test?

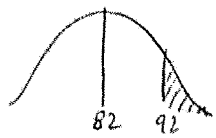


inv Normal
area = 87.7

$Z = 1.16 = \frac{X-52}{5}$

$5.8 = X - 52$ $\boxed{X = 57.8}$

5. Ms. Gonding's Advanced Algebra test has a mean of 82 with a standard deviation of 5.5.
   a. What is the probability you will score at least a 92?



$P(X \ge 92) = 1 - P(X \le 92) = 1 - P(Z \le 1.82)$

$Z = \frac{92-82}{5.5}$ $= 1 - 0.9656 = \boxed{0.0344}$

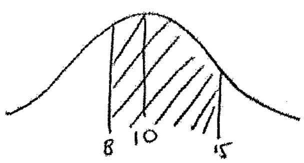
   b. What is the probability you will get a B?

$P(79.5 \le X \le 89.4) = P(X \le 89.4) - P(X \le 79.5)$
$= P(Z \le 1.35) - P(Z \le -0.45)$
$= 0.9115 - 0.3264$
$= \boxed{0.5851}$

   c. What is the probability you will fail?

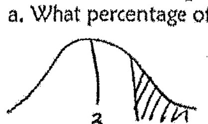
$P(X \le 69.5) = P(Z \le -2.27)$
$= \boxed{0.0116}$

6. Maple tree diameters in a forest area are normally distributed with mean 10 inches and standard deviation 3.2 inches. Find the proportion of trees having a diameter between 8 and 15 inches.



$P(8 \le X \le 15) = P(X \le 15) - P(X \le 8)$
$= P(Z \le 1.56) - P(Z \le -0.63)$
$= 0.9406 - 0.2643$
$= \boxed{0.6763}$

7. The weights of 1800 fish in a lake are normally distributed with mean of 3 kilograms and standard deviation of .5 kilograms.
   a. What percentage of fish has a weight greater than or equal to 3.75 kilograms?



$P(X \ge 3.75) = 1 - P(X \le 3.75) = 1 - P(Z \le 1.5)$
$= 1 - 0.9332 = 0.0668$ $\boxed{6.68\%}$

   b. Approximately how many fish in the lake weigh at least 3.75 kilograms?

$.0668(1800) = \boxed{120 \text{ fish}}$

Accel Pre-Calc Date ______________

9.07 Standard Normal Distribution Applications

All data in the following exercises is normally distributed.

$\bar{X}=125$
$\sigma=15$
$X=148$

1. Jack took a standardized achievement test with a mean of 125 and a standard deviation of 15. Jack's score was 148.
   a. What is the percentile rank for Jack's score on this test? Percentile rank means that Jack scored as well or better than that percentage of students taking the test.

   $Z=\frac{148-125}{15}=1.53 \quad P(X\leq 148)=93.7\%\text{tile}$

   b. If Jill scored at the 67th percentile, what was her score on the test?

   $Z=.44=\frac{X-125}{15} \quad \boxed{X=131.6}$

   $6.6=X-125$

2. The average number of absences for 1st graders is 15 with a standard deviation of 6.
   a. What is the probability of a 1st grader having fewer than 6 absences?

   $P(X<6)=P(Z<-1.5)=\boxed{0.0668}$

   $Z=\frac{6-15}{6}=-1.5$

   b. What is the probability of a 1st grader having more than 20 absences?

   $P(X>20)=P(Z>.83)=1-0.7967$

   $Z=\frac{20-15}{6}= \quad =\boxed{.2033}$

   c. If a student is absent more often than 30.85% of other 1st graders, how many days did she miss?

   area $=.3085 \quad Z=-.50=\frac{X-15}{6}$

   $X=12$ absences

3. On the Scholastic Aptitude Test (SAT), scores have a mean of 500 and a standard deviation of 100.
   a. Mo scored 600 on the math section. What percentile did he achieve?

   $Z=\frac{600-500}{100}=P(Z\leq 1)=.8413$

   $\boxed{84.13\%\text{tile}}$

   b. Larry scored 750 on the math section. What percentile did he achieve?

   $Z=\frac{750-500}{100}=P(Z\leq 2.5)=.9938$

   $\boxed{99.38\%\text{tile}}$

   c. Curley scored better than 97.72% of students on the math section of the SAT. What was his score?

   area $=.9772$

   $Z=2=\frac{X-500}{100}$

   $\boxed{X=700}$

4. A patient recently diagnosed with Alzheimer's disease took a cognitive abilities test. The mean score on this test is 52 with a standard deviation of 5.

   a. If the patient scored a 45 on the test, what is his percentile rank?

   $Z = \frac{45-52}{5} = -1.4$ $P(X \leq -1.4) = .0808$

   $\boxed{8.08\text{ \%tile}}$

   b. If a patient scored higher than 87.7% of other Alzheimer's patients, what was her score on the test?

   area $= .877$ $Z = 1.16 = \frac{X-52}{5}$

   $\boxed{X = 57.8}$

5. Mrs. Durand's Algebra 2 unit test scores have a mean of 82 with a standard deviation of 5.5.

   a. What is the probability that a student will score at least a 92?

   $P(X \geq 92) = 1 - P(Z \leq 1.82) = 1 - .9656 = \boxed{.0344}$

   $Z = \frac{92-82}{5.5} = 1.82$

   b. What is the probability a student will get a B?

   $P(79.5 \leq X \leq 89.4) = P(X \leq 89.4) - P(X \leq 79.5)$

   $Z = \frac{89.4-82}{5.5} = 1.35$ $= P(Z \leq 1.35) - P(X \leq -.45)$

   $= .9115 - .3264$

   $Z = \frac{79.5-82}{5.5} = .45$ $= \boxed{.5851}$

   c. What is the probability a student will fail?

   $P(X \leq 69.4) = P(Z \leq -2.29) = \boxed{.0110}$

   $Z = \frac{69.4-82}{5.5} = -2.29$

6. The diameter of maple trees in a Canadian forest have a mean of 10 inches and standard deviation of 3.2 inches. What is the percentage of trees with a diameter between 8 and 15 inches?

   $P(8 \leq X \leq 15) = P(X \leq 15) - P(X \leq 8)$

   $= P(Z \leq 1.56) - P(Z \leq -.63)$

   $= 0.9406 - 0.2643$

   $= \boxed{0.6763}$

7. The weights of 1800 fish in a lake have a mean of 3 kilograms and standard deviation of .5 kilograms.

   a. What percentage of fish weigh at least 3.75 kilograms?

   $P(X \geq 3.75) = 1 - P(X \leq 3.75) = 1 - P(Z \leq 1.5)$

   $= 1 - .9332$

   $= .0668$ $\boxed{6.68\%}$

   b. Approximately how many fish in the lake weigh at least 3.75 kilograms?

   $.0668 \times 1800 = \boxed{120 \text{ fish}}$

Accel Pre-Calc Date Key

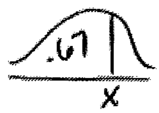
9.07 Standard Normal Distribution Applications

All data in the following exercises is normally distributed.

1. Jack took a standardized achievement test with a mean of 125 and a standard deviation of 15. Jack's score was 148.
   a. What is the percentile rank for Jack's score on this test? Percentile rank means that Jack scored as well or better than that percentage of students taking the test.

   $z = \frac{148-125}{15} = 1.53$ $P(x \le 148) = 0.9370$ $\boxed{93.7\%}$

   b. If Jill scored at the 67th percentile, what was her score on the test?

   $.6700 \rightarrow z = 0.44 = \frac{x-125}{15}$

   $6.6 = x - 125$

   $\boxed{x = 131.6}$

2. The average number of absences for 1st graders is 15 with a standard deviation of 6.
   a. What is the probability of a 1st grader having fewer than 6 absences?

   $z = \frac{6-15}{6}$

   $z = -1.5$

   $P(x \le 6) = \boxed{0.0668}$

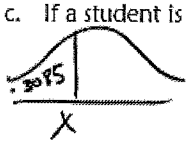
   b. What is the probability of a 1st grader having more than 20 absences?

   $z = \frac{20-15}{6}$

   $z = 0.83$

   $P(x \ge 20) = 1 - 0.7967 = \boxed{0.2033}$

   c. If a student is absent more often than 30.85% of other 1st graders, how many days did she miss?

   $.3085 \rightarrow z = -0.5 = \frac{x-15}{6}$

   $-3 = x - 15$

   $\boxed{x = 12}$

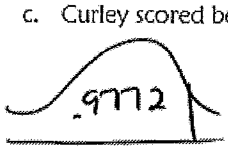
3. On the Scholastic Aptitude Test (SAT), scores have a mean of 500 and a standard deviation of 100.
   a. Mo scored 600 on the math section. What percentile did he achieve?

   $z = \frac{600-500}{100} = 1$ $P(x \le 600) = 0.8413$ $\boxed{84.13\%}$

   b. Larry scored 750 on the math section. What percentile did he achieve?

   $z = \frac{750-500}{100} = 2.5$ $P(x \le 750) = 0.9938$ $\boxed{99.38\%}$

   c. Curley scored better than 97.72% of students on the math section of the SAT. What was his score?

   $.9772 \rightarrow z = 2.00 = \frac{x-500}{100}$

   $200 = x - 500$

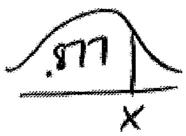
   $\boxed{x = 700}$

4. A patient recently diagnosed with Alzheimer's disease took a cognitive abilities test. The mean score on this test is 52 with a standard deviation of 5.

   a. If the patient scored a 45 on the test, what is his percentile rank?

   $z = \frac{45-52}{5} = -1.4$ $P(x \leq 45) = 0.0808$ $\boxed{8.08\%}$

   b. If a patient scored higher than 87.7% of other Alzheimer's patients, what was her score on the test?



   $0.877 \rightarrow z = 1.16 = \frac{x-52}{5}$ $\boxed{x = 57.8}$

   $5.8 = x - 52$

5. Mrs. Durand's Algebra 2 unit test scores have a mean of 82 with a standard deviation of 5.5.

   a. What is the probability that a student will score at least a 92?

   $z = \frac{92-82}{5.5} = 1.82$ $P(x \geq 92) = 1 - 0.9656 = \boxed{0.0344}$

   b. What is the probability a student will get a B? $(79.5, 89.4)$

   $z_{89.4} = \frac{89.4-82}{5.5} = 1.35$ $P(x \leq 89.4) = 0.9115$ $.9115 - .3264$

   $z_{79.5} = \frac{79.5-82}{5.5} = -0.45$ $P(x \leq 79.5) = 0.3264$ $= \boxed{0.5851}$

   c. What is the probability a student will fail? ≤ 69.4

   $z = \frac{69.4-82}{5.5} = -2.29$ $P(x \leq 69.4) = \boxed{0.0110}$

6. The diameter of maple trees in a Canadian forest have a mean of 10 inches and standard deviation of 3.2 inches. What is the percentage of trees with a diameter between 8 and 15 inches?

   $z_{15} = \frac{15-10}{3.2} = 1.56$ $P(x \leq 15) = 0.9406$ $0.9406 - 0.2643$

   $= 0.6763$

   $z_8 = \frac{8-10}{3.2} = -0.63$ $P(x \leq 8) = 0.2643$ $\boxed{67.63\%}$

7. The weights of 1800 fish in a lake have a mean of 3 kilograms and standard deviation of .5 kilograms.

   a. What percentage of fish weigh at least 3.75 kilograms?

   $z = \frac{3.75-3}{0.5} = 1.5$ $P(x \geq 3.75) = 1 - 0.9332 = 0.0668$

   $\boxed{6.68\%}$

   b. Approximately how many fish in the lake weigh at least 3.75 kilograms?

   $1800(0.0668) = 120.24$ $\boxed{120 \text{ fish}}$

CCGPS Advanced Algebra

Notes: Standard Normal Distributions

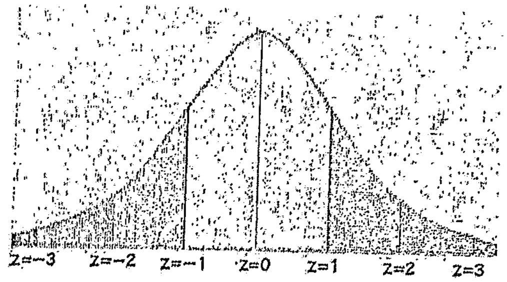
Name Key

Date ______ Period ______

ESSENTIAL QUESTION: What happens when you are looking for the probability of something that does not fall on one of the standard deviations of a normal distribution?

Standard Normal Distribution:

- Formed using a mean of 0 and a standard deviation of 1.
- Used when the given X value does not fall on a standard deviation.
- The Empirical Rule still applies to a standard normal distribution.
- To change an x-value from a normal distribution with mean $\bar{x}$ and standard deviation σ use the z-score

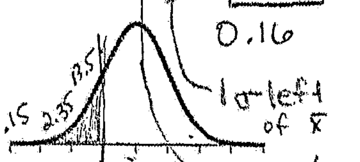


memorize → formula: $\boxed{z = \frac{x-\bar{x}}{\sigma}}$ $\frac{\text{target} - \text{mean}}{\text{std. dev}}$

Key idea → ✗ The z-score is the number of standard deviations the x-value lies away from the mean.

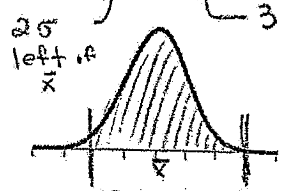
Example 1: For a standard normal distribution, find the following probabilities of a randomly selected x-value from the distribution. It may be helpful to draw a sketch.

a. $P(z \le -1) = \boxed{16\%}$ 0.16



b. $P(z \ge 1) = \boxed{16\%}$ 0.16

c. $P(-2 \le z \le 3)$ shade between



13.5 + 34 + 34 + 13.5 + 2.35 = $\boxed{97.35}$

Converting to a z-score:

Example 2: Consider a normal distribution with a mean of 72 ($\bar{x}$) and a standard deviation of 3 (σ). Convert the following into z-scores.

a. x = 65

$z = \frac{65-72}{3}$

$z = \frac{-7}{3} = -2.33$

b. x = 81

$z = \frac{81-72}{3}$

$z = 3$

c. x = 76

$z = \frac{76-72}{3}$

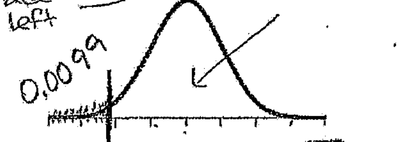
$z = 1.33$

WHAT'S THE POINT??? Z-scores will allow you to find any probability ... provided you have a table of values. The probability given on the z-score table is the probability that is less than the z-score. BE CAREFUL!!! You are not always looking for less than!!!!

Example 3: Consider a normal distribution with a mean of 72 and a standard deviation of 3. Sketch a graph and find the following:

a. $P(x \le 65)$

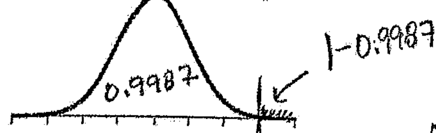
$P(z \le -2.33)$ shade left



$\boxed{0.0099}$

b. $P(x \ge 81)$

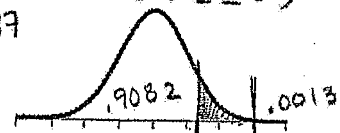
$P(z \ge 3)$ shade right



$\boxed{0.0013}$

c. $P(76 \le x \le 81)$ shade between

$P(1.33 \le z \le 3)$



.9987
− .9082

$\boxed{0.0905}$

Classwork:

1) A normal distribution has a mean of 70 ($\bar{x}$) and a standard deviation of 10 (σ). Calculate the z-score and use the table to find the indicated probability. $z = \frac{x - \bar{x}}{\sigma}$

a. $P(x \le 65) = P(z \le -0.5)$

$z = \frac{65-70}{10} = -0.5$

$\boxed{P = 0.3085}$ table + calculator

b. $P(x \ge 47) = P(z \ge -2.3)$

$z = \frac{47-70}{10} = -2.3$

calc + table 2

$P = 1 - 0.0107 = \boxed{0.9893}$

c. $P(54 \le x \le 83)$

$z = \frac{54-70}{10} = -1.6$

$z = \frac{83-70}{10} = 1.3$

table + calc

$P(-1.6 \le z \le 1.3)$

$= 0.9032 - 0.0548$

d. $P(x \le 91) = P(z \le 2.1)$

$z = \frac{91-70}{10} = 2.1$

$P = \boxed{0.9821}$

table + calc

e. $P(x \ge 39) = P(z \ge -3.1)$

$z = \frac{39-70}{10} = -3.1$

$P = 1 - 0.0010 = \boxed{0.9990}$

table + calc

f. $P(79 \le x \le 101)$ $\boxed{0.8484}$

$z = \frac{79-70}{10} = 0.9$

$z = \frac{101-70}{10} = 3.1$

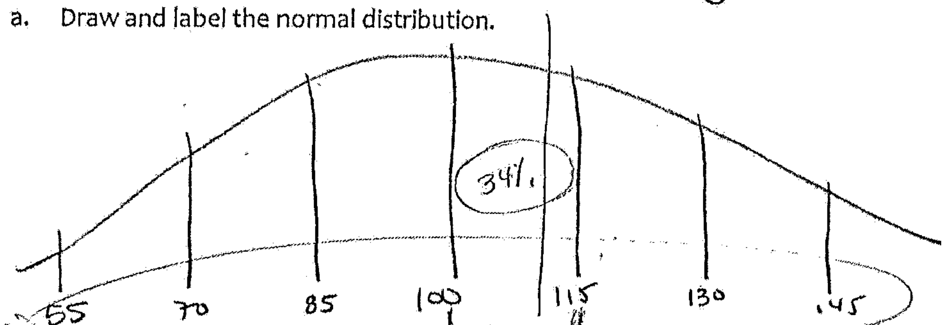
$P(0.9 \le z \le 3.1)$ table + calc

$= 0.9990 - 0.8159$

$= \boxed{0.1831}$

2) Suppose scores on an Intelligence Quotient (IQ) test are normally distributed with a mean of 100 ($\bar{x}$) and a standard deviation of 15 (σ). $z = \frac{x - \bar{x}}{\sigma}$

a. Draw and label the normal distribution.



b. What is the probability that a person will score at least a 121? ← 121 and up

$P(x \ge 121)$

$= P(z \ge 1.4)$

$= 1 - 0.9192$

$= \boxed{0.0808}$

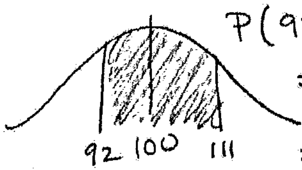
c. What is the probability that a person will score no more than 79? ← up to 79

$P(x \le 79)$

$= P(z \le -1.4)$

$= \boxed{0.0808}$

d. What is the probability that a person will score between 92 and 111?



$P(92 \le x \le 111)$

$= P(-0.53 \le z \le 0.73)$

$= 0.7673 - 0.2981$

$= \boxed{0.4692}$

or Calculator: 0.4714 ← more accurate!

e. What minimum score would someone need to make if they wanted to score higher than 80% of those taking an IQ test?

$P(x \le \#) = 0.8000$

table says $P = 0.8000$ @

$z \approx 0.84 = \frac{x - 100}{15}$

$12.6 = x - 100$

$\boxed{112.6 = x}$ ← minimum score

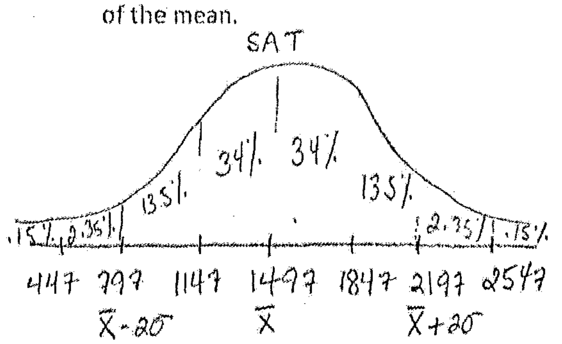
use invNormal on calculator
set "area" = 0.8000

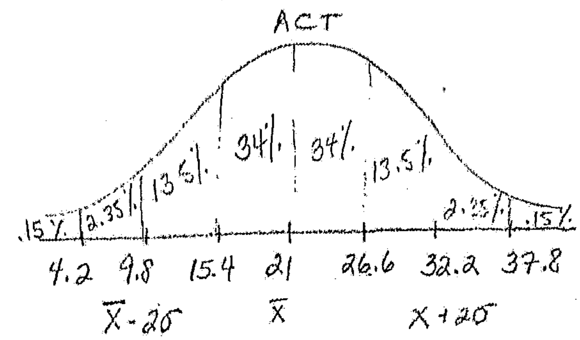
CCGPS Advanced Algebra Name: Key - 48.04

More Practice: Applications of the Normal Distribution and the Empirical Rule

A) Comparing the SAT and ACT: college admissions offices need to compare scores of students who take the Scholastic Aptitude Test (SAT) with those who take the American College Test (ACT). Suppose that among the college applicants who take the SAT, scores have a mean of 1497 (out of 2400) and a standard deviation of 350. Further, suppose that among the college applicants who take the ACT, the scores have a mean of 21 (out of 36) and a standard deviation of 5.6.

1. Sketch normal curves for both the SAT and ACT listing values for 1, 2, and 3 standard deviations on each side of the mean.





Apply the empirical rule to approximate.

2. About 95% of SAT takers score between what two values?

(797, 2197)

3. About 95% of ACT takers score between what two values?

(9.8, 32.2)

4. What will be the proportion of students who score between 1147 and 1847 on the SAT?

$P(1147 \le x \le 1847) = 34 + 34 = 68\%$

5. What will be the proportion of students who score between 15.4 and 32.2 on the ACT?

$P(15.4 \le x \le 32.2) = 34 + 34 + 13.5 = 81.5\%$

6. If John scored at the 84th percentile on the ACT, what score did he achieve?

26.6

7. College Board reports that 1,672,395 students took the SAT in 2014. About how many students achieved a score of at least 2197?

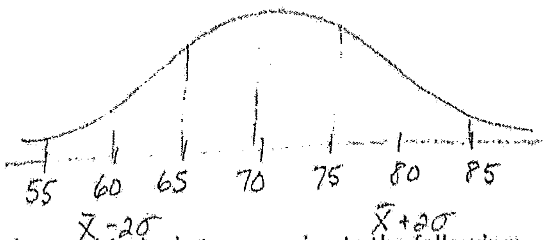
$P(x \ge 2197) = 2.5\% = 0.025 \times 1{,}672{,}395 = 41{,}810$

8. ACT, Inc. reports that 1,845,787 students took the ACT in 2014. About how many students achieved a score of at most 21?

$P(x \le 21) = 50\% = 0.5 \times 1{,}845{,}787 = 922{,}893$

B) 250 students took the advanced algebra final exam. The scores were distributed normally with a mean of 70 and a standard deviation of 5.

9. Sketch the normal curve for both the final exam scores, listing values for 1, 2, and 3 standard deviations on each side of the mean.



Apply the empirical rule to approximate the following:

10. What percentage of scores is between scores 65 and 75?

$P(65 \le x \le 75) = 34 + 34 = 68\%$

11. What percentage of scores is between scores 60 and 70?

$P(60 \le x \le 70) = 13.5 + 34 = 47.5\%$

12. What percentage of scores is between scores 60 and 85?

$P(60 \le x \le 85) = 13.5 + 34 + 34 + 13.5 + 2.35 = 97.35\%$

13. What percentage of scores is less than a score of 55?

$P(x \le 55) = .15\%$

14. What percentage of scores is at least a score of 80?

$P(x \ge 80) = 2.35 + .15 = 2.5\%$

15. How many advanced algebra students achieved a score between 70 and 80?

$P(70 \le x \le 80) = 34 + 13.5 = 47.5\%$ $\quad 0.475 \times 250 = 119$ students

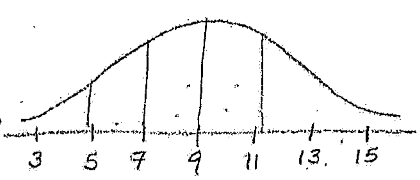
16. How many advanced algebra students achieved a score of at most 75?

$P(x \le 75) = 50 + 34 = 84\%$ $\quad 0.84 \times 250 = 210$ students

C) Statistics kept for NFL football teams regarding the number of injuries suffered by NFL players during their careers showed the distribution is approximately normal with the mean number of injuries per player to be 9 with a standard deviation of 2. If there are 1696 NFL players in the current season, determine how many players will have the following number of injuries:

17. Less than 9 injuries in their career.

$P(x \le 9) = 50\%$ $\quad .50\ (1696) = 848$ players



18. At least 7 injuries in their career.

$P(x \ge 7) = 50 + 34 = 84\%$ $\quad .84\ (1696) = 1425$ players

19. More than 5 but less than 11 injuries in their career.

$P(5 \le x \le 11) = 13.5 + 34 + 34 = 81.5\%$ $\quad .815\ (1696) = 1382$ players

9.09 Confidence Intervals Yay! Date: 2019

Opener: We plan to meet Saturday morning for a fun day at Six Flags. If I tell you that I will be there at 10:30, what time do you expect me to arrive? 10:30 Would any other times also be reasonable? If so, what are they? 10:25, 10:35, 10:15 10:30 ± ? minutes

Would you be more confident that I will arrive "on time" if you make my window of arrival times wider or narrower? Why? Because there are more options for the reasonable arrival times.

Population vs. Sample:

Population: includes all elements of a set of data

example: all U.S. adults

Sample: includes a portion of a set of data

example: 3500 adults called randomly

Parameter: a number relating to the population

example: N = 252,063,800 U.S. adults
p = 30.11% of U.S. adults have a college degree

Statistic: a number relating to the sample

example: n = 3500 ← # called
$\hat{p}$ = 28% of adults called have a college degree.

Identify the population, sample, and statistic for each of the following scenarios:

A survey of 1300 American households found that 32% of those households have basements.

Population: all American households Sample: 1300 American households surveyed Statistic: $\hat{p}$ = .32 have basements

The average bill from every 6th person getting food at Chipotle within a 3-hour period was $19.61.

Population: all Chipotle customers Sample: Every 6th Chipotle customer in that 3-hr period Statistic: $\bar{x}$ = $19.61 avg meal cost

Confidence Intervals are intervals of plausible values for estimating a parameter, with a given percent confidence. We use a sample mean to estimate the population mean. We use a sample proportion to estimate a population proportion.

Consider this: The Milton Parks and Recreation Department wants to build a new park in Crabapple. To allocate funds to build the park, they need to determine if residents in the area want one. They mail a survey to residents within 1 mile of the proposed location and find that 78% of residents who responded are in favor of building the new park. They'll find confidence intervals to project what all residents in the area may think of the new park.

Confidence Interval for Proportion:	Confidence Interval for Mean:
$\hat{p} \pm z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$	$\bar{x} \pm z\frac{\sigma}{\sqrt{n}}$
$\hat{p}$ = sample proportion in decimal/fraction	$\bar{x}$ = sample mean
z = z score for probability of $\frac{1-C\%}{2}$	σ = population standard deviation
n = sample size	z = z score for probability of $\frac{1-C\%}{2}$
Margin of error = $z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$ doubled makes interval width	n = sample size
	Margin of error = $z\frac{\sigma}{\sqrt{n}}$ doubled makes interval width
p = true population proportion	μ = true population mean

← what we are estimating →

Examples: $n = 1150$ $\hat{p} = .846$

1. A survey of 1150 people found that 84.6% of respondents believed a toilet paper roll should roll over (not under). Construct the following confidence intervals and state the margin of error for each. $\frac{1-.90}{2} = \frac{.10}{2}$

90%

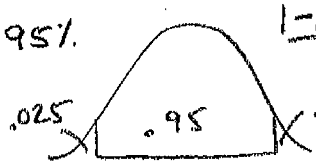
look in chart for .05 $z = 1.65$

ME

$$CI = .846 \pm 1.65\sqrt{\frac{.846(1-.846)}{1150}} = .846 \pm .018$$

$$= \boxed{(.828, .864)}$$

95% $\frac{1-.95}{2} = \frac{.05}{2}$ look in chart for .025



$z = -1.96$

ME

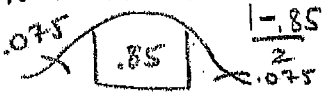
$$CI = .846 \pm 1.96\sqrt{\frac{.846(1-.846)}{1150}} = .846 \pm .021$$

$$= \boxed{(.825, .867)}$$

2. In a sample of 2500 people, 770 people separate their Skittle's by color before eating them. Construct an 85% confidence interval ~~and state the margin of error.~~

$n = 2500$

$\hat{p} = \frac{770}{2500}$



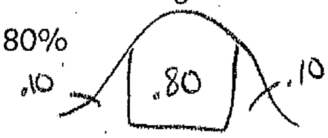
$\frac{1-.85}{2} = \frac{.15}{2}$

$$CI = \frac{770}{2500} \pm 1.44\sqrt{\frac{\frac{770}{2500}\left(\frac{1730}{2500}\right)}{2500}}$$

look in chart for .075 $z = -1.44$ $= \boxed{(.299, 0.317)}$

3. A recent survey of 133 Milton students found their average daily screen time is 5.402 hours. If the population standard deviation is 1.565, construct the following confidence intervals and state the margin of error for each. $n = 133$ $\bar{x} = 5.402$ $\sigma = 1.565$

80%



$\frac{1-.80}{2}$ chart @ .10 $\Rightarrow z = -1.28$

ME

$$CI = 5.402 \pm 1.28\left(\frac{1.565}{\sqrt{133}}\right) = 5.402 \pm 0.174$$

~~90%~~

$$= \boxed{(5.228 \text{ hrs}, 5.576 \text{ hrs})}$$

99%

$\frac{1-.99}{2}$ chart @ .005 $\Rightarrow z = -2.58$

ME

$$CI = 5.402 \pm 2.58\left(\frac{1.565}{\sqrt{133}}\right) = 5.402 \pm 0.350$$

$$= \boxed{(5.052 \text{ hrs}, 5.752 \text{ hrs})}$$

4. A recent survey found that Milton students get an average of 6.303 hours of sleep each night. Given the sample size of 540 students and population standard deviation of 0.926, construct an 88% confidence interval. $\bar{x} = 6.303$ hrs, $n = 540$ $\sigma = .926$

$\frac{1-.88}{2}$ chart @ .06 = -1.56

$$CI = 6.303 \pm 1.56\ \frac{0.926}{\sqrt{540}} = \boxed{(6.241 \text{ hrs}; 6.365 \text{ hrs})}$$

9.09 Homework: Key

1. High school students who take the SAT Mathematics exam a second time generally score higher than on their first try. The change in score has a normal distribution with standard deviation $\sigma = 50$. A random sample of 1000 students gain an average of 22 points on their second try. $\leftarrow \bar{x}$

   a) Construct a 95% confidence interval for the mean score gain μ in the population.

   $95\%\ CI = \bar{x} \pm z \frac{\sigma}{\sqrt{n}} = 22 \pm (1.96)\frac{50}{\sqrt{1000}} = \boxed{(18.901, 25.099)}$

   .025 .95 .025 z = look for .025 in chart (or .975)

   b) Construct a 90% confidence interval for μ.

   $90\%\ CI = 22 \pm (1.65)\frac{50}{\sqrt{1000}} = \boxed{(19.391, 24.609)}$

   .05 .90 .05 z = look for .05 in chart (or .95)

   c) Construct a 99% confidence interval for μ.

   $99\%\ CI = 22 \pm (2.58)\frac{50}{\sqrt{1000}} = \boxed{(17.921, 26.079)}$

   .005 .99 .005 z = look for .005 in chart (or .995)

   d) What is the margin of error for each of the confidence intervals calculated above?

   in part a) $\pm 1.96\left(\frac{50}{\sqrt{1000}}\right)$ or $22 - 18.901 = \boxed{\pm 3.099}$

   in part b) $\pm 1.65 \cdot \frac{50}{\sqrt{1000}}$ or $22 - 19.391 = \boxed{\pm 2.609}$

   in part c) $\pm 2.58 \cdot \frac{50}{\sqrt{1000}}$ or $22 - 17.921 = \boxed{\pm 4.079}$

2. The National Survey of Student Engagement found that 87% of students report that their peers at least "sometimes" copy information from the Internet in their reports without citing the source. Assume that the sample size is 400. Construct an 88% confidence interval and find the margin of error.

   $\hat{p} = .87 \leftarrow$ proportion from a sample

   $n = 400$

   $88\%\ CI = \hat{p} \pm z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = .87 \pm 1.55\sqrt{\frac{.87(.13)}{400}} = .87 \pm .026$

   .06 .88 .06

   z = look for .06 in chart (or .94)

   $\boxed{(.844, .896)}$

   $1.00 - .88 = .12 \div 2$

   $\boxed{\text{Margin of Error: } \pm .026}$

3. A recent survey of 1366 adults found that 1127 of those respondents like hot sauce on their eggs. Construct an 80% confidence interval.

n2

$\hat{p} = \frac{1127}{1366}$ .10 .80 .10 z = look for .10 on chart (or .90)

$$80\%\,CI:\ \frac{1127}{1366} \pm 1.28\sqrt{\frac{\frac{1127}{1366}\left(\frac{239}{1366}\right)}{1366}} = .825 \pm .013 = \boxed{(.812, .838)}$$

4. A national opinion poll found that 44% of all American adults agree that parents should be given vouchers good for education at any public or private school of their choice. The result was based on a small sample. How large of a random sample is required to obtain a margin of error of 0.03 in a 95% confidence interval?

$$ME = z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

.025 .95 .025 $z = 1.96$

$$\frac{.03}{1.96} = \frac{1.96}{1.96}\sqrt{\frac{.44(.56)}{n}} \Rightarrow \left(\frac{.03}{1.96}\right)^2 = \left(\sqrt{\frac{.44(.56)}{n}}\right)^{2} \Rightarrow \left(\frac{.03}{1.96}\right)^2 = \frac{.44(.56)}{n}$$

$$n = \frac{.44(.56)}{\left(\frac{.03}{1.96}\right)^2} \text{ or } \left(\frac{1.96}{.03}\right)^2(.44)(.56) = 1051.745$$

$\boxed{\text{sample 1052 adults}}$

5. A radio talk show invites listeners to enter a dispute about a proposed pay increase for city council members. "What yearly pay do you think council members should get? Call us with your number." In all, 958 people call. The mean pay they suggest is \$8740 per year, and the standard deviation of the responses is \$1125. Calculate a 90% confidence interval for the mean pay μ that all citizens would propose for council members.

$n = 958$ $\bar{x} = \$8740$ $\sigma = \$1125$

.05 .90 .05

z @ .05 probability is 1.65

$$90\%\,CI = \bar{x} \pm z\frac{\sigma}{\sqrt{n}} = 8740 \pm 1.65\left(\frac{1125}{\sqrt{958}}\right)$$

$$= 8740 \pm 59.973$$

$$= \boxed{(\$8680.03, \$8799.97)}$$

9.09 Homework:

1. Students who take the SAT Mathematics exam a second time generally score higher than their first try. The change in score has a Normal distribution with standard deviation $\sigma = 50$. A random sample of $\underset{n}{\underline{1000}}$ students gain an average of $\bar{x} = 22$ points on their second try.

   a) Construct a 95% confidence interval for the mean score gain μ in the population.

   $z = 1.96$ $\quad 22 \pm 1.96\left(\frac{50}{\sqrt{1000}}\right) = 22 \pm 3.099 = (18.901, 25.099)$

   b) Construct a 90% confidence interval for μ.

   $z = 1.645$ $\quad 22 \pm 1.645\left(\frac{50}{\sqrt{1000}}\right) = 22 \pm 2.601 = (19.399, 24.601)$

   c) Construct a 99% confidence interval for μ.

   $z = 2.575$ $\quad 22 \pm 2.575\left(\frac{50}{\sqrt{1000}}\right) = 22 \pm 4.071 = (17.929, 26.071)$

   d) What is the margin of error for each of the confidence intervals calculated above?

   in part a) ± 3.099

   in part b) ± 2.601

   in part c) ± 4.071

2. The National Survey of Student Engagement found that 87% of 400 students reported their peers at least "sometimes" copy information from the Internet in their reports without citing the source. Construct an 88% confidence interval for the proportion of students who would report the issue and find the margin of error.

   $\hat{p} = 0.87$
   $n = 400$
   $z = 1.555$

   $$0.87 \pm 1.555\sqrt{\frac{0.87(1-0.87)}{400}}$$

   $$0.87 \pm \underset{\text{m.o.e.}}{0.026} = (0.844, 0.896)$$

3. A recent survey of 1366 adults found that 1127 of those respondents like hot sauce on their eggs. Construct an 80% confidence interval for the proportion of adults that prefer spicy eggs.

$n = 1366$

$\hat{p} = \frac{1127}{1366}$

$z = 1.28$

$$\frac{1127}{1366} \pm 1.28\sqrt{\frac{\frac{1127}{1366}\left(1 - \frac{1127}{1366}\right)}{1366}}$$

$$\frac{1127}{1366} \pm \underset{\text{m.o.e.}}{0.013} = (0.812, 0.838)$$

4. A national opinion poll found that 44% of all American adults agree that parents should be given vouchers good for education at any public or private school of their choice. The result was based on a small sample. How large of a random sample is required to obtain a margin of error of 0.03 in a 95% confidence interval for the proportion of American adults who agree?

$\hat{p} = 0.44$

$z = 1.96$

m.o.e. $= 0.03$

$$0.03 = 1.96\sqrt{\frac{0.44(1-0.44)}{n}}$$

$$\frac{0.03}{1.96} = \sqrt{\frac{0.44(1-0.44)}{n}}$$

$$\left(\frac{0.03}{1.96}\right)^2 = \frac{0.44(0.56)}{n}$$

$$n = \frac{0.44(0.56)}{\left(\frac{0.03}{1.96}\right)^2}$$

$n = 1051.745$

1052 people

5. A radio talk show invites listeners to enter a dispute about a proposed pay increase for city council members. "What yearly pay do you think council members should get? Call us with your number." In all, 958 people call. The mean pay they suggest is \$8740 per year. Assume that the standard deviation of all citizens is \$1125. Calculate a 90% confidence interval for the mean pay μ that all citizens would propose for council members.

$n = 958$

$\bar{x} = 8740$

$\sigma = 1125$

$z = 1.645$

$$8740 \pm 1.645\left(\frac{1125}{\sqrt{958}}\right)$$

$$8740 \pm 59.791$$

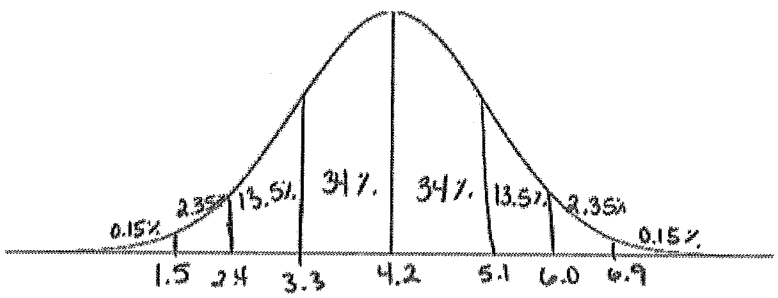
$$(8680.209, 8799.791)$$

$$(\$8680.21, \$8799.79)$$

9.09 Are You Normal? Date Key

Part 1: The Empirical Rule

a. The time a typical American teenager spends doing homework each week is normally distributed with a mean of 4.2 hours and standard deviation of 0.9 hours. Label the normal distribution curve below with 3 standard deviations above and below the mean.



b. Use the Empirical Rule (68-95-99.7) to find the **probability** that a randomly selected teenager spends less than 5.1 hours each week doing homework.

0.15+2.35+13.5+34+34 = 84% $\boxed{0.84}$

c. What is the **probability** that a teenager will spend more than 2.4 hours each week doing homework?

1 − (0.15+2.35) = 97.5% $\boxed{0.975}$

d. What is the **percentage** of teenagers spend between 1.5 and 6.0 hours each week doing homework?

2.35+13.5+34+34+13.5 = 97.35% $\boxed{0.9735}$

e. $P(x \leq 6.9)$ = 0.9985. Explain what this means.

99.85% of students spend no more than 6.9 hours doing homework each week.

f. Use the Empirical Rule to *estimate* the **percentage** of students who spend less than 1.6 hours each week doing homework.

0.17%

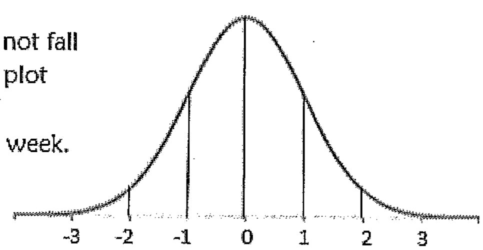
slightly more than 0.15%

g. Why does the Empirical Rule give an estimate, rather than an exact answer, for part **f**?

Because the bell curve is a curve. If we partition any section, we can't calculate the exact area.

Part 2: Z-scores

h. Patrick spends 1.6 hours on homework each week which does not fall exactly on a standard deviation from the mean. Calculate and plot Patrick's z-score on the right. Shade to show the percentage of teenagers that spend less time than Patrick on homework each week.



$z = \frac{1.6-4.2}{0.9} = -2.89$

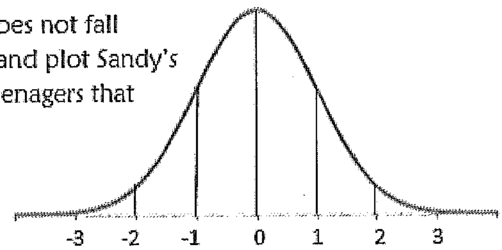
i. What does Patrick's z-score mean? (Use the phrase "standard deviations" and a direction.)

Patrick's hw time is 2.89 standard deviations below the mean

j. Use the z-score table to find the **percentage** of teenagers that spend less time on homework than Patrick.

0.19%

k. Sandy spends 5.8 hours on homework each week which does not fall exactly on a standard deviation from the mean. Calculate and plot Sandy's z-score on the right. Shade to show the percentage of of teenagers that spend less time than Sandy on homework each week.



$z = \frac{5.8-4.2}{0.9} = 1.78$

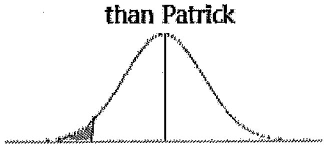
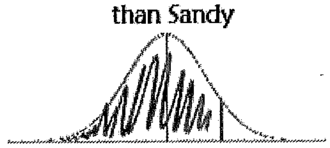
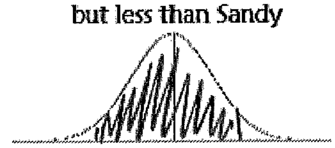
l. What does Sandy's z-score mean? (Use the phrase "standard deviations" and a direction.)

Sandy's hw time is 1.78 standard deviations above the mean.

m. Use the z-score table to find the **percentage** of teenagers that spend more time on homework than Sandy.

$1 - .9625 = 0.0375$ 3.75%

n. Label and shade each scenario described below to compare Patrick and Sandy.

Less homework than Patrick	Less homework than Sandy	More than Patrick but less than Sandy
		
P(less hw than Patrick) =	P(less hw than Sandy) =	P(Patrick ≤ x ≤ Sandy) =
0.0019	0.9625	.9625 - .0019 = 0.9606

o. Using the graphs above, find the **percentage** of teenagers that spend more time than Patrick, but less time than Sandy on homework each week.

$0.9625 - 0.0019$ 0.9606 96.06%

p. Out of 1500 randomly selected teenagers, **how many** spend more time than Patrick, but less time than Sandy on homework each week?

$1500(.9606) = 1440.9$ 1440 or 1441

q. Estimate how much time you spend on homework each week & calculate the z-score for your estimate.

r. What is the **percentage** of teenagers that spend more time than you do on homework each week?

9.10 More Confidence Intervals Practice Date: 2019

1. A town takes a poll of its residents to find out how many people would be willing to pay a new tax to repair the town's sidewalks. Out of 1100 people polled, only 130 said that they would be willing to pay.

n x

(a) Find a 90% confidence interval for the proportion of the whole town that would be willing to pay the extra tax.

$\hat{p} = \frac{130}{1100}$ $CI = \hat{p} \pm z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = .118 \pm 1.65\sqrt{\frac{.118(1-.118)}{1100}}$

look in chart for $-z$ .0500 .90 .05 $-z = -1.65$

$\boxed{(.1020, .1340)}$

(b) If 15,000 people live in this town, then we are 90% confident that between 1529 and 2011 will be willing to pay this tax. (Fill in with numbers of people.)

$.1020(15000) = 1529.26$ $.1340(15000) = 2010.74$

2. A company that produces white bread is concerned about the distribution of the amount of sodium in its bread. The company takes a simple random sample of 100 slices of bread and computes the sample mean to be 103 milligrams of sodium per slice.

n

(a) Construct a 99% confidence interval for the unknown mean sodium level assuming that the population standard deviation is 10 milligrams.

$n = 100$ slices
$\bar{x} = 103$ mg
$\sigma = 10$ mg

$CI = \bar{x} \pm z\frac{\sigma}{\sqrt{n}} = 103 \pm 2.58 \frac{10}{\sqrt{100}} = \boxed{(100.42 \text{ mg}, 105.58 \text{ mg})}$

.0050 .99 .005 $-z$ $-z = 2.58$

(b) Interpret the 99% confidence interval found in (a).

We are 99% confident that the mean sodium level per bread slice is between 100.42 mg and 105.58 mg.

3. You work for a consumer advocate agency and want to find the mean repair cost of a washing machine. In the past, the standard deviation of the cost of repairs for washing machines has been \$17.50. As part of your study, you randomly select 40 repair costs and find the mean to be \$100.00.

(a) Calculate a 85% confidence interval for the population mean.

$\sigma = 17.50$
$n = 40$ repairs
$\bar{x} = \$100.00$

$CI = \bar{x} \pm z\frac{\sigma}{\sqrt{n}} = \$100 \pm 1.44 \frac{17.50}{\sqrt{40}} = \boxed{(\$96.02, \$103.98)}$

.0750 .85 .0750 $-z$ $-z = -1.44$

(b) Interpret the interval found in (a).

We are 85% confident that the average repair cost for a washing machine is between \$96.02 and \$103.98.

4. You want to estimate the mean fuel efficiency for all Ford Focus cars with 99% confidence and a margin of error of no more than 1 mile per gallon. Preliminary data suggests that $\sigma = 2.4$ miles per gallon is a reasonable standard deviation for all cars of this make and model. How large a sample do you need?

$ME \leq 1$ mpg
$\sigma = 2.4$ mpg
$n = ?$
$z_{99} = 2.58$ (see #2 above)

$z \cdot \frac{\sigma}{\sqrt{n}} = ME$
$2.58 \cdot \frac{2.4}{\sqrt{n}} \leq 1$
$\left(2.58 \cdot \frac{2.4}{1}\right)^2 \leq (\sqrt{n})^2$
$(6.192)^2 \leq n$

Sample at least 39 cars of this make + model

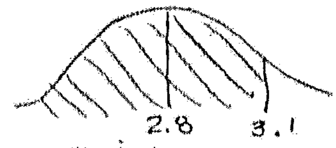
$N(\mu, \sigma) = N(2.8, 0.24)$

5. The actual time it takes to cook a 10-pound turkey is a Normal random variable with a mean of 2.8 hours and a standard deviation of 0.24 hours. Suppose that a random sample of 35 10-pound turkeys is taken. $n = 35$

(a) What is the probability that a randomly selected 10-pound turkey will take less than 3.1 hours to cook?

$$P(x < 3.1) = P(z < 1.25) = \boxed{.8944}$$

$$z = \frac{3.1 - 2.8}{0.24} = 1.25$$



(b) What is the probability that the average cooking time of a 10-pound turkey will take between 2.7 and 2.95 hours to cook?

$$P(2.7 \le x \le 2.95) = P(-.42 \le z \le 0.63) = P(z \le 0.63) - P(z \le -.42)$$

$$z = \frac{2.7 - 2.8}{.24} = -0.42 \qquad z = \frac{2.95 - 2.8}{.24} = 0.63 \qquad .7357 - .3372 = \boxed{.3985}$$

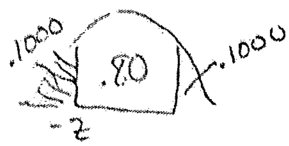
(c) Given that an average of 2.9 hours was found for a sample of 35 turkeys, calculate an 80% confidence interval for the average cooking time of a 10-pound turkey.

$\bar{X} = 2.9$

$\sigma = 0.24$

$n = 35$

$$CI = \bar{x} \pm z \frac{\sigma}{\sqrt{n}} = 2.9 \pm 1.28\left(\frac{0.24}{\sqrt{35}}\right) = \boxed{(2.848, 2.952) \text{ hrs hrs}}$$



$-z = -1.28$

6. Weight Watchers takes a poll of 250 members and finds that 95 of them include exercise with their diet program, while the rest do not. Find a 99% confidence interval for the proportion of all members that do exercise.

$X = 95$

$n = 250$

$\hat{p} = \frac{95}{250} = .38$

$$CI = \hat{p} \pm z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = .38 \pm 2.58\sqrt{\frac{.38(1-.38)}{250}}$$

$$= \boxed{(0.301, 0.459)}$$

$z_{99} = 2.58$ (see #2)

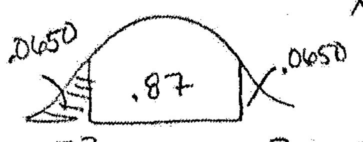
7. A magazine polls 395 readers and finds that 95 of them bought the magazine in the store, while the rest had a subscription. Find an 87% confidence interval for the proportion of all readers who have a subscription.

$X = 95$

$n = 395$

$\hat{p} = \frac{95}{395} = .2405$

$$CI = .2405 \pm 1.51\sqrt{\frac{.2405(1-.2405)}{395}} =$$



$-z = -1.51$

$$\boxed{(0.2080, 0.2730)}$$

8. Fill in the blanks with one of the following for how the margin of error is impacted: *increases*, *decreases*, or *stays the same* where $ME = z\left(\frac{\sigma}{\sqrt{n}}\right)$.

As the sample size (n) increases, the margin of error (ME) decreases (divide by large #)

As the confidence level (C%) increases, the margin of error (ME) increases (wider window, z↑)

As the standard deviation (σ) increases, the margin of error (ME) increases (more variability in data)

9.10 Confidence Intervals

Date: Key

Opener: We plan to meet Saturday morning for a fun day at Six Flags. If I tell you that I will be there at 10:30, what time do you expect me to arrive? 10:30 Would any other times also be reasonable? If so, what are they? 10:20 – 10:40

Would you be more confident that I will arrive "on time" if you make my window of arrival times wider or narrower? Why? wider because it gives them a larger margin of error

Population vs. Sample:

Population: includes all elements of a set of data
example: All HS students

Sample: includes a portion of a set of data
example: Ms. Latimer's students

Parameter: a number relating to the population
example: average screen time for all HS students

Statistic: a number relating to the sample
example: average screen time for Ms. Latimer's students

Identify the population, sample, and statistic for each of the following scenarios:

A survey of 1300 American households found that 32% of those households have basements.

Population: American households

Sample: 1300 American households

Statistic: 32% have basements

The average bill from every 6th person getting food at Chipotle within a 3-hour period was $19.61.

Population: Chipotle customers

Sample: those customers surveyed

Statistic: $19.61 = average bill

Confidence Intervals are intervals of plausible values for estimating a parameter, with a given percent confidence. We use a sample mean to estimate the population mean. We use a sample proportion to estimate a population proportion.

Consider this: The Milton Parks and Recreation Department wants to build a new park in Crabapple. To allocate funds to build the park, they need to determine if residents in the area want one. They mail a survey to residents within 1 mile of the proposed location and find that 78% of residents who responded are in favor of building the new park. They'll find confidence intervals to project what all residents in the area may think of the new park.

Confidence Interval for Proportion:	Confidence Interval for Mean:
$\hat{p} \pm z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$	$\bar{x} \pm z\frac{\sigma}{\sqrt{n}}$
$\hat{p}$ = proportion (sample)	$\bar{x}$ = mean (sample)
z = z-score	σ = standard deviation (population)
n = sample size	z = z-score
Margin of error = $z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$	n = sample size
	Margin of error = $z\left(\frac{\sigma}{\sqrt{n}}\right)$
p = proportion (population)	μ = mean (population)

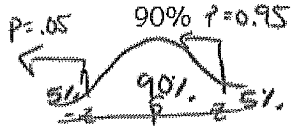
mu

Examples:

1. A survey of 1150 people found that 84.6% of respondents believed a toilet paper roll should roll over (not under). Construct the following confidence intervals for the proportion of people whose toilet paper rolls over and state the margin of error for each.

n $\quad \hat{p} = 0.846$

90% $\rho = 0.95$

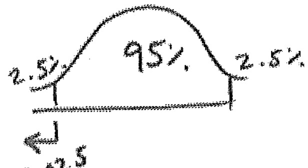


$z = 1.645$

$$0.846 \pm 1.645\sqrt{\frac{0.846(1-0.846)}{1150}} = 0.846 \pm \underbrace{0.018}_{m.o.e}$$

$(0.828, 0.864)$

95%



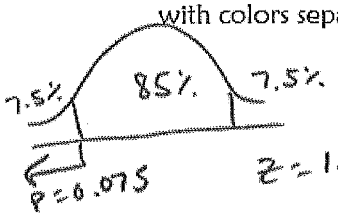
$z = 1.96$

$$0.846 \pm 1.96\sqrt{\frac{0.846(1-0.846)}{1150}}$$

$$0.846 \pm \underbrace{0.021}_{m.o.e.} \qquad (0.825, 0.867)$$

2. In a sample of 2500 people, 770 people separate their Skittle's by color before eating them. Construct an 85% confidence interval for the proportion of people who "taste the rainbow" with colors separated.

n $\quad \hat{p} = \frac{770}{2500} = 0.308$



$z = 1.44$

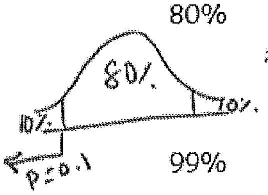
$$0.308 \pm 1.44\sqrt{\frac{0.308(1-0.308)}{2500}} \qquad (0.295, 0.321)$$

$$0.308 \pm 0.013$$

3. A recent survey of 133 Milton students found their average daily screen time is 5.402 hours. If the population standard deviation is 1.565, construct the following confidence intervals for the average daily screen time for all Milton students and state the margin of error for each.

$n \quad \bar{x} \quad \sigma$

80%

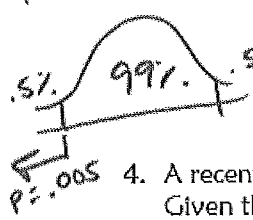


$z = 1.28$

$$5.402 \pm 1.28\left(\frac{1.565}{\sqrt{133}}\right)$$

$$5.402 \pm 0.174 = (5.228, 5.576)$$

99%



$z = 2.575$

$$5.402 \pm 2.575\left(\frac{1.565}{\sqrt{133}}\right) \qquad (5.053, 5.751)$$

$$5.402 \pm 0.349 =$$

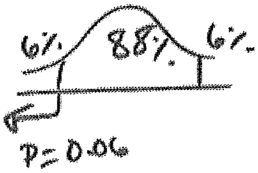
4. A recent survey found that Milton students get an average of 6.303 hours of sleep each night. Given the sample size of 540 students and population standard deviation of 0.926, construct an 88% confidence interval for the average amount of sleep by Milton students.

$\bar{x} = 6.303$
$n = 540$
$\sigma = 0.926$
$z = 1.555$

$$6.303 \pm 1.555\left(\frac{0.926}{\sqrt{540}}\right)$$

$$6.303 \pm 0.062$$

$$(6.241, 6.365)$$



10 More Confidence Intervals Practice

Date: ______________

1. A town takes a poll of its residents to find out how many people would be willing to pay a new tax to repair the town's sidewalks. Out of 1100 people polled, only 130 said that they would be willing to pay.
   a) Find a 90% confidence interval for the proportion of the whole town that would be willing to pay the extra tax.

$\hat{p} = \frac{130}{1100} = 0.118$

$n = 1100$

$z = 1.645$

$0.118 \pm 1.645\sqrt{\frac{.118(1-.118)}{1100}}$

$0.118 \pm 0.016 = \boxed{(0.102, 0.134)}$

   b) If 15,000 people live in this town, then we are 90% confident that between __1530__ and __2010__ will be willing to pay this tax. (Fill in with numbers of people.)

population = 15,000

2. A company that produces white bread is concerned about the distribution of the amount of sodium in its bread. The company takes a simple random sample of 100 slices of bread and computes the sample mean to be 103 milligrams of sodium per slice.
   (a) Construct a 99% confidence interval for the unknown mean sodium level assuming that the population standard deviation is 10 milligrams.

$\bar{x} = 103$

$z = 2.575$ $n = 100$

$\sigma = 10$

$103 \pm 2.575\left(\frac{10}{\sqrt{100}}\right)$

$103 \pm 2.575 = \boxed{(100.425, 105.575)}$

   (b) Interpret the 99% confidence interval found in (a).

We are 99% confident the mean amount of sodium per slice for all slices of bread lies between 100.425 & 105.575 mg.

3. You work for a consumer advocate agency and want to find the mean repair cost of a washing machine. In the past, the standard deviation of the cost of repairs for washing machines has been $17.50. As part of your study, you randomly select 40 repair costs and find the mean to be $100.00.
   (a) Calculate a 85% confidence interval for the population mean.

$\bar{x} = 100$

$z = 1.44$ $n = 40$

$\sigma = 17.5$

$100 \pm 1.44\left(\frac{17.5}{\sqrt{40}}\right) = 100 \pm 3.984$

$\boxed{(96.016, 103.984)}$

   (b) Interpret the interval found in (a).

We are 85% confident the average cost of all washer repairs costs between $96.02 & $103.98.

4. You want to estimate the mean fuel efficiency for all Ford Focus cars with 99% confidence and a margin of error of no more than 1 mile per gallon. Preliminary data suggests that $\sigma = 2.4$ miles per gallon is a reasonable standard deviation for all cars of this make and model. How large a sample do you need?

m.o.e. = 1

m.o.e. = $z\left(\frac{\sigma}{\sqrt{n}}\right)$

$\sigma = 2.4$

$z = 2.575$

$1 = 2.575\left(\frac{2.4}{\sqrt{n}}\right)$

$\frac{1}{2.575} = \frac{2.4}{\sqrt{n}}$

$\sqrt{n} = 2.575(2.4)$

$n = 6.18^2$

$\boxed{n \approx 38 \text{ cars}}$

$n = 38.192$

5. The actual time it takes to cook a 10–pound turkey is a Normal random variable with a mean of 2.8 hours and a standard deviation of 0.24 hours. Suppose that a random sample of 35 10–pound turkeys is taken.

(a) What is the probability that a randomly selected 10–pound turkey will take less than 3.1 hours to cook?

$P(x \leq 3.1)$ $\quad z = \frac{3.1 - 2.8}{.24} = 1.25 \quad P(X \leq 3.1) = \boxed{0.8944}$

(b) What is the probability that the average cooking time of a 10–pound turkey will take between 2.7 and 2.95 hours to cook?

$z(2.95) = \frac{2.95 - 2.8}{.24} = 0.63 \quad P(x \leq 2.95) = 0.7357$

$z(2.7) = \frac{2.7 - 2.8}{.24} = -0.42 \quad P(x \leq 2.7) = 0.3372$

$.7357 - .3372$ $\quad \boxed{0.3985}$

(c) Given that an average of 2.9 hours was found for a sample of 35 turkeys, calculate an 80% confidence interval for the average cooking time of a 10–pound turkey.

$z = 1.28$ $\quad \bar{X} = 2.9$, $\sigma = 0.24$, $n = 35$

$2.9 \pm 1.28\left(\frac{0.24}{\sqrt{35}}\right)$

2.9 ± 0.052

$\boxed{(2.848, 2.952)}$

6. Weight Watchers takes a poll of 250 members and finds that 95 of them include exercise with their diet program, while the rest do not. Find a 99% confidence interval for the proportion of all members that do exercise.

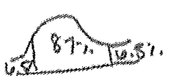
$\hat{p} = \frac{95}{250} = 0.38$ $\quad z = 2.575$ $\quad n = 250$

$0.38 \pm 2.575\sqrt{\frac{.38(1-.38)}{250}}$

0.38 ± 0.079

$\boxed{(0.301, 0.459)}$

7. A magazine polls 395 readers and finds that 95 of them bought the magazine in the store, while the rest had a subscription. Find an 87% confidence interval for the proportion of all readers who have a subscription.



$\hat{p} = \frac{300}{395} = 0.759$ $\quad n = 395$ $\quad z = 1.51$

$0.759 \pm 1.51\sqrt{\frac{.759(1-.759)}{395}}$

$0.759 \pm 0.032 = \boxed{(0.727, 0.791)}$

8. Fill in the blanks with one of the following for how the margin of error is impacted: *increases, decreases,* or *stays the same* where $ME = z\left(\frac{\sigma}{\sqrt{n}}\right)$.

As the sample size (n) increases, the margin of error (ME) decreases.
As the confidence level (C%) increases, the margin of error (ME) increases.
As the standard deviation (σ) increases, the margin of error (ME) increases.

9.12 Review

Date: Key

1) A sample of 16 students finds that the average age is 22 years. All student ages have a standard deviation of 6 years. Construct a 95% confidence interval for the average age of students.

$n=16$
$\bar{x}=22$
$\sigma=6$

$z=1.96$

$22 \pm 1.96\left(\frac{6}{\sqrt{16}}\right)$

22 ± 2.94

$(19.06, 24.94)$

2) Construct a 99% confidence interval for the population mean lifetime of fluorescent lightbulbs. Assume the population has a Normal distribution with a standard deviation of 31 hours. A sample of 16 fluorescent light bulbs have a mean life of 645 hours.

$z=2.575$
$n=16$
$\bar{x}=645$
$\sigma=31$

$645 \pm 2.575\left(\frac{31}{\sqrt{16}}\right)$

645 ± 19.956

$(625.043, 664.956)$

3) A sample of 100 bean cans showed an average weight of 13 ounces. If all been cans have a standard deviation of 0.8 ounces, construct an 85% confidence interval for the mean weight of the population.

$n=100$
$\bar{x}=13$
$\sigma=0.8$
$z=1.44$

$13 \pm 1.44\left(\frac{0.8}{\sqrt{100}}\right)$

13 ± 0.1152

$(12.885, 13.115)$

4) A researcher wants to know the percentage of Columbus residents who would favor a two cent increase in the gasoline tax to fund road repairs. A random sample of 900 residents finds 278 favor the increase.

a) Specify the parameter and statistic for this problem.

Parameter - the unknown proportion of all Columbus residents in favor

Statistic - $278/900$ (0.309)

b) Find an 80% confidence interval for the parameter

$\hat{p}=\frac{278}{900}$

$z=1.28$

$\frac{278}{900} \pm 1.28\sqrt{\frac{\frac{278}{900}\left(1-\frac{278}{900}\right)}{900}}$

$\frac{278}{900} \pm 0.0197$

$(0.289, 0.329)$

5) A random sample of female college students has a mean height of **64.5** inches, which is greater than the **63**-inch mean height of all adult American women. Determine if each bold-faced number is a parameter or a statistic.

64.5 inches: statistic

63 inches: parameter

6) In a certain Normal distribution of scores, the mean is 20 and the standard deviation is 3.

a. Find the z-score corresponding to a score of 24. $z = \frac{24-20}{3} = 1.33$

b. Find the percentile for a score of 24. $P(x \leq 24) = 0.9082$ 90.82%

7) The Jackson triplets, Jenny, John, and James are in different math classes at City High. On their final exams, Jenny scored 82 on a test with a mean of 76 and a standard deviation of 7.5; John scored 77 on a test with a mean of 72 and a standard deviation of 10.5; and James scored 78 on a test with a mean of 66 and a standard deviation of 10.5. Who had the best z-score <u>and</u> what does this say about that triplet's test score in relation to their peers?

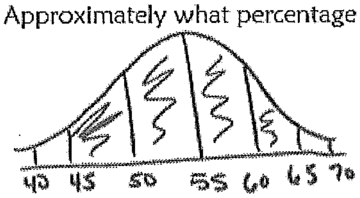
Jenny $z = \frac{82-76}{7.5} = 0.8$

John $z = \frac{77-72}{10.5} = 0.48$

James $z = \frac{78-66}{10.5} = 1.2$

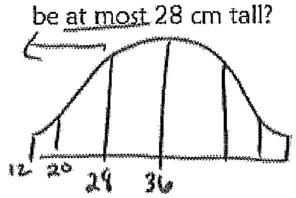
James; highest z-score means highest percentile

8) Some test scores were Normally distributed with a mean of 55 and a standard deviation of 5. Approximately what percentage of the scores lie between 45 and 65?



$\pm 2\ \sigma$'s 95%

9) The heights of a certain group of adult parrots were found to be Normally distributed. The mean height is 36 cm with a standard deviation of 8 cm. In a group of 1000 of these birds, how many would be <u>at most</u> 28 cm tall?



16% $0.16(1000) = 160$ birds

10) The life expectancy (in hours) of a fluorescent tube is normally distributed with a mean of 5000 and a standard deviation of 500. Find the probability that a tube lasts for <u>at least</u> 5650 hours.

$z = \frac{5650-5000}{500} = 1.3$ $P(x \geq 5650) = 1 - 0.9032$

0.0968

11) A potato chip company sells a small bag of chips. The volume of the snack bag is Normally distributed with a mean of 1.75 ounces and a standard deviation of 0.15 ounces. What is the probability that a bag contains between 1.63 and 1.84 ounces?

$z_{1.84} = \frac{1.84-1.75}{0.15} = 0.6$ $P(x \leq 1.84) = 0.7257$

$z_{1.63} = \frac{1.63-1.75}{0.15} = -0.8$ $P(x \leq 1.63) = 0.2119$

$$\begin{array}{r} 0.7257 \\ -\ 0.2119 \\ \hline \boxed{0.5138} \end{array}$$

9.12 Review Date: Key

1) A sample of 16 students finds that the average age is 22 years. All student ages have a standard deviation of 6 years. Construct a 95% confidence interval for the average age of students.

$\bar{x} = 22$ yrs

$n = 16$ students

$\sigma = 6$ yrs

$$CI = \bar{x} \pm z \frac{\sigma}{\sqrt{n}} = 22 \pm 1.96 \frac{6}{\sqrt{16}} = \boxed{(19.06 \text{ yrs}, 24.94 \text{ yrs})}$$

$-z = -1.96$

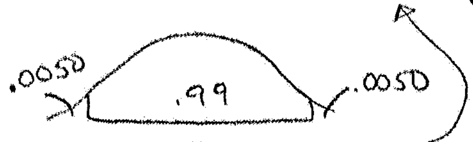
2) Construct a 99% confidence interval for the population mean lifetime of fluorescent lightbulbs. Assume the population has a Normal distribution with a standard deviation of 31 hours. A sample of 16 fluorescent light bulbs have a mean life of 645 hours.

$\sigma = 31$ hrs

$n = 16$ bulbs

$\bar{x} = 645$ hrs

$$CI = 645 \pm 2.58 \frac{31}{\sqrt{16}} = \boxed{(625.005 \text{ hrs}, 664.995 \text{ hrs})}$$



$-z = -2.58$

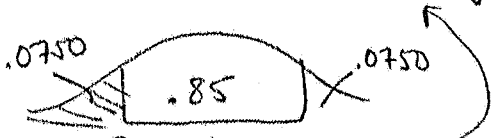
3) A sample of 100 bean cans showed an average weight of 13 ounces. If all been cans have a standard deviation of 0.8 ounces, construct an 85% confidence interval for the mean weight of the population.

$n = 100$ bean cans

$\bar{x} = 13$ oz

$\sigma = 0.8$ oz

$$CI = 13 \pm 1.44 \frac{0.8}{\sqrt{100}} = \boxed{(12.885 \text{ oz}, 13.115 \text{ oz})}$$



$-z = -1.44$

4) A researcher wants to know the percentage of Columbus residents who would favor a two cent increase in the gasoline tax to fund road repairs. A random sample of 900 residents finds 278 favor the increase.

n x

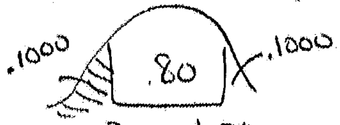
a) Specify the parameter and statistic for this problem.

Parameter: Proportion of all residents in favor of paying 2¢ increase. p

Statistic: $\hat{p}$ $\frac{278}{900} = .309$ proportion of sample willing to pay 2¢ increase.

b) Find an 80% confidence interval for the parameter.

$$CI = \hat{p} \pm z \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = .309 \pm 1.28 \sqrt{\frac{.309(1-.309)}{900}}$$



$$= \boxed{(0.289, 0.329)}$$

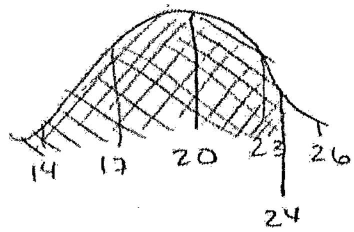
$-z = -1.28$

5) A random sample of female college students has a mean height of **64.5** inches, which is greater than the **63**-inch mean height of all adult American women. Determine if each bold-faced number is a parameter or a statistic.

$\bar{x} = 64.5''$ is a statistic because it came from a sample of female college students

$\mu = 63''$ is a parameter because it describes all American women.

6) In a certain Normal distribution of scores, the mean is 20 and the standard deviation is 3. $N(20,3)$



a. Find the z-score corresponding to a score of 24.

$$z = \frac{24-20}{3} = \frac{4}{3} = 1.33$$

b. Find the percentile for a score of 24.

$$P(x \le 24) = P(z \le 1.33) = .9082$$

$\boxed{90.82\text{ \%ile}}$

7) The Jackson triplets, Jenny, John, and James are in different math classes at City High. On their final exams, Jenny scored 82 on a test with a mean of 76 and a standard deviation of 7.5; John scored 77 on a test with a mean of 72 and a standard deviation of 10.5; and James scored 78 on a test with a mean of 66 and a standard deviation of 10.5. Who had the best z-score and what does this say about that triplet in relation to their test score in relation to their peers?

Jenny $= \frac{82-76}{7.5} = \frac{6}{7.5} = 0.8$ stdev above class avg.

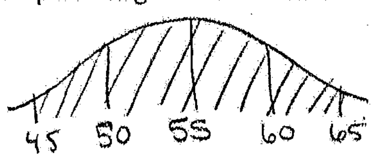
John $= \frac{77-72}{10.5} = \frac{5}{10.5} = 0.476$ stdev above class avg

James $= \frac{78-66}{10.5} = \frac{12}{10.5} = 1.143$ std dev above class avg

James has the highest z-score meaning he did better than his class peers.

8) Some test scores were Normally distributed with a mean of 55 and a standard deviation of 5. Approximately what percentage of the scores lie between 45 and 65?

$N(55,5)$



According to the empirical rule, about $\boxed{95\%}$.

.9544 so 95.44% according to z chart.

9) The heights of a certain group of adult parrots were found to be Normally distributed. The mean height is 36 cm with a standard deviation of 8 cm. In a group of 1000 of these birds, how many would be more than 28 cm tall?

$N(36,8)$ $\quad P(x > 28) = 50\% + 34\% = 84\%$

$n = 1000$

84% of 1000 = $\boxed{840 \text{ birds}}$

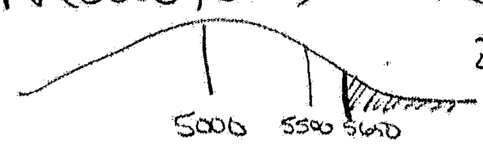
10) The life expectancy (in hours) of a fluorescent tube is normally distributed with a mean of 5000 and a standard deviation of 500. Find the probability that a tube lasts for at least 5650 hours.

$N(5000, 500)$ $\quad P(x \ge 5650) = P(z \ge 1.30) = 1 - P(z \le 1.30)$

$= 1 - .9032$

$= \boxed{.0968}$

$$z = \frac{5650-5000}{500}$$



11) A potato chip company sells a small bag of chips that has a mean volume of 1.75 ounces with a standard deviation of 0.15 ounces. What is the probability that a bag contains at most 1.84 ounces?

Normal

$N(1.75, 0.15)$ $\quad P(x \le 1.84\text{ oz}) = P(z \le 0.60) = \boxed{0.7257}$

$$z = \frac{1.84-1.75}{0.15} = 0.60$$