IX. Verify solutions to Differential Equation

22) The function $y = e^{3x} - 5x + 7$ is a solution to which of the following differential equations?

(A) $y'' - 3y' - 15 = 0$

(B) $y'' - 3y' + 15 = 0$

(C) $y'' - y' - 5 = 0$

(D) $y'' - y' + 5 = 0$

23) A curve has slope $2x + 3$ at each point (x, y) on the curve. Which of the following is an equation for this curve if it passes through the point $(1, 2)$?

(A) $y = 5x - 3$

(B) $y = x^2 + 1$

(C) $y = x^2 + 3x$

(D) $y = x^2 + 3x - 2$

(E) $y = 2x^2 + 3x - 3$

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24) Of the following, which are solutions to the differential equation $y'' - 10y' + 9y = 0$?

I. $y = 2\sin(3x)$
II. $y = 5e^x$
III. $y = Ce^{9x}$, where C is a constant.

(A) I only

(B) II only

(C) III only

(D) II and III only

25)

For what value of k, if any, is $y = e^{2x} + ke^{-3x}$ a solution to the differential equation $4y - y'' = 10e^{-3x}$?

(A) -2

(B) $\frac{10}{3}$

(C) 10

(D) There is no such value of k.

26)

Of the following, which are solutions to the differential equation $y'' - 6y' + 8y = 0$?

I. $y = 2\sin(4x)$
II. $y = 3e^{2x}$
III. $y = Ce^{4x}$, where C is a constant.

(A) I only

(B) II only

(C) III only

(D) II and III only

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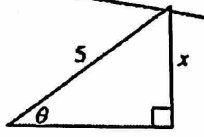
27) ~~Review Topic: Trig Related Rates~~



In the triangle shown above, if θ increases at a constant rate of 3 radians per minute, at what rate is x increasing in units per minute when x equals 3 units?

(A) 3 (B) $\frac{15}{4}$ (C) 4 (D) 9 (E) 12

IX. Verify solutions to Differential Equation

*Recall $\frac{d}{dx}e^u = e^u \cdot u'$

22) The function $y = e^{3x} - 5x + 7$ is a solution to which of the following differential equations?

(A) $y'' - 3y' - 15 = 0$
(B) $y'' - 3y' + 15 = 0$
(C) $y'' - y' - 5 = 0$
(D) $y'' - y' + 5 = 0$

*Since all answer choices involve y' and y'', let's find those derivative equations

$y' = e^{3x} \cdot 3 - 5 \rightarrow 3e^{3x} - 5$

$y'' = 3e^{3x} \cdot 3 - 0 \rightarrow 9e^{3x}$

*Test answer choices through substitution to find the match for the original equation.

$y'' - 3y' - 15 = 0$

$(9e^{3x}) - 3(3e^{3x} - 5) - 15 = 0$

$9e^{3x} - 9e^{3x} + 15 - 15 = 0$

$0 + 0 = 0$ ✓

23) A curve has slope $2x + 3$ at each point (x, y) on the curve. Which of the following is an equation for this curve if it passes through the point $(1, 2)$?

(A) $y = 5x - 3$
(B) $y = x^2 + 1$
(C) $y = x^2 + 3x$
(D) $y = x^2 + 3x - 2$
(E) $y = 2x^2 + 3x - 3$

*Curve with slope $2x+3$ means $\frac{dy}{dx} = 2x + 3$

*solve differential equation

$\frac{dy}{dx} = \frac{2x+3}{1}$

$\int dy = \int 2x + 3\,dx$

$\int 1\,dy = \int 2x + 3\,dx$

$y = \frac{2x^2}{2} + 3x + C$

$y = x^2 + 3x + C$

plug in (1,2) to find C

$2 = 1^2 + 3(1) + C$

$2 = 4 + C$

$-2 = C$

$y = x^2 + 3x - 2$

24) Of the following, which are solutions to the differential equation $y'' - 10y' + 9y = 0$?

I. $y = 2\sin(3x)$
II. $y = 5e^x$
III. $y = Ce^{9x}$, where C is a constant.

(A) I only
(B) II only
(C) III only
(D) II and III only

*Find y' and y''. Then test them against the differential equation

✓ II. $y = 5e^x$

$y' = 5e^x(1)$

$y'' = 5e^x(1)$

$y'' - 10y' + 9y = 0$

$5e^x - 10(5e^x) + 9(5e^x) = 0$

$5e^x - 50e^x + 45e^x = 0$ ✓

✓ III. $y = Ce^{9x}$

$y' = Ce^{9x} \cdot 9 \rightarrow 9Ce^{9x}$

$y'' = 9Ce^{9x} \cdot 9 \rightarrow 81Ce^{9x}$

$y'' - 10y' + 9y = 0$

$81Ce^{9x} - 10(9Ce^{9x}) + 9(Ce^{9x}) = 0$

$81Ce^{9x} - 90e^{9x} + 9Ce^{9x} = 0$ ✓

X. $y = 2\sin(3x)$

$y' = 2\cos(3x) \cdot 3 \rightarrow 6\cos(3x)$

$y'' = -6\sin(3x) \cdot 3 \rightarrow -18\sin(3x)$

$y'' - 10y' + 9y = 0$

$-18\sin(3x) - 10(6\cos 3x) + 9(2\sin 3x) \neq 0$

25)

For what value of k, if any, is $y = e^{2x} + ke^{-3x}$ a solution to the differential equation $4y - y'' = 10e^{-3x}$?

(A) -2

(B) $\frac{10}{3}$

(C) 10

(D) There is no such value of k.

*Next, make substitutions into differential equation.

$4y - y'' = 10e^{-3x}$

$4(e^{2x} + ke^{-3x}) - (4e^{2x} + 9ke^{-3x}) = 10e^{-3x}$

$4e^{2x} + 4ke^{-3x} - 4e^{2x} - 9ke^{-3x} = 10e^{-3x}$

$-5ke^{-3x} = 10e^{-3x}$

$k = \frac{10e^{-3x}}{-5e^{-3x}}$

$k = -2$

*First, find y' and y''

$y = e^{2x} + ke^{-3x}$

$y' = e^{2x}(2) + ke^{-3x}(-3)$

$y' = 2e^{2x} - 3ke^{-3x}$

$y'' = 2e^{2x}(2) - 3ke^{-3x}(-3)$

$y'' = 4e^{2x} + 9ke^{-3x}$

26)

Of the following, which are solutions to the differential equation $y'' - 6y' + 8y = 0$?

I. $y = 2\sin(4x)$

II. $y = 3e^{2x}$

III. $y = Ce^{4x}$, where C is a constant.

(A) I only

(B) II only

(C) III only

(D) II and III only

✗ I. $y = 2\sin(4x)$

$y' = 2\cos(4x)\cdot 4 \to 8\cos(4x)$

$y'' = -8\sin(4x)\cdot 4 \to -32\sin 4x$

$y'' - 6y' + 8y = 0$

$-32\sin 4x - 6(8\cos 4x) + 8(2\sin(4x)) \neq 0$

*not a solution ✗

✓ II. $y = 3e^{2x}$

$y' = 6e^{2x}$

$y'' = 12e^{2x}$

$y'' - 6y' + 8y = 0$

$12e^{2x} - 6(6e^{2x}) + 8(3e^{2x}) = 0$

$12e^{2x} - 36e^{2x} + 24e^{2x} = 0$ ✓

✓ III. $y = Ce^{4x}$ $y' = Ce^{4x}\cdot 4$

$y'' = 4Ce^{4x}\cdot 4 \to 16Ce^{4}$

$y'' - 6y' + 8y = 0$

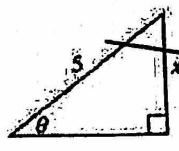
$16Ce^{4x} - 6(4Ce^{4x}) + 8(Ce$

$= 0$

$16Ce^{4x} - 24Ce^{4x} + 8Ce^{4x} =$

✓

27) ~~Review Topic: Trig Related Rates~~



In the triangle shown above, if θ increases at a constant rate of 3 radians per minute, at what rate is x increasing in units per minute when x equals 3 units?

(A) 3

(B) $\frac{15}{4}$

(C) 4

(D) 9

(E) 12

$\frac{d\theta}{dt} = 3$ rad/min

Find $\frac{dx}{dt} = ?$

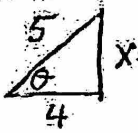
$x = 3$

$\sin\theta = \frac{x}{5}$

$\sin\theta = \frac{1}{5}x$

$\cos\theta\left(\frac{d\theta}{dt}\right) = \frac{1}{5}\left(\frac{dx}{dt}\right)$

*replace $\cos\theta$ with ratio from triangle



$\to \cos\theta = \frac{4}{5}$

$\left(\frac{4}{5}\right)(3) = \frac{1}{5}\left(\frac{dx}{dt}\right)$

$\frac{12}{5} = \frac{1}{5}\left(\frac{dx}{dt}\right)$

$\frac{dx}{dt} = 12$ units/m

$\frac{12}{5}\cdot\frac{5}{1} = \frac{dx}{dt}$