Teaching Calculus

… a blog for high school calculus teachers and students

Why Radians?

Lin McMullin / October 12, 2012

Calculus is always done in radian measure. Degree (a right angle is 90 degrees) and gradian measure (a right angle is 100 grads) have their uses. Outside of the calculus they may be easier to use than radians. However, they are somewhat arbitrary. Why 90 or 100 for a right angles? Why not 10 or 217?

Radians make it possible to relate a linear measure and an angle measure. A unit circle is a circle whose radius is one unit. The one unit radius is the same as one unit along the circumference. Wrap a number line counterclockwise around a unit circle starting with zero at (1, 0). The length of the arc subtended by the central angle becomes the radian measure of the angle.

This keeps all the important numbers like the sine and cosine of the central angle, on the same scale. When you graph *y* = sin(*x*) one unit in the x-direction is the same as one unit in the *y*-direction. When graphing using degrees, the vertical scale must be stretched a lot to even see that the graph goes up and down. Try graphing on a calculator *y* = sin(*x*) in degree mode in a square window and you will see what I mean.

But the utility of radian measure is even more obvious in calculus. To develop the derivative of the sine function you first work with this inequality (At the request of a reader I have added an explanation of this inequality at the end of the post):

$$\frac{1}{2}\cos\left(\theta\right)\sin\left(\theta\right)\le\frac{1}{2}\theta\le\frac{1}{2}\tan\left(\theta\right)$$

From this inequality you determine that $\lim\limits_{\theta\to 0}\dfrac{\sin\left(\theta\right)}{\theta}=1$

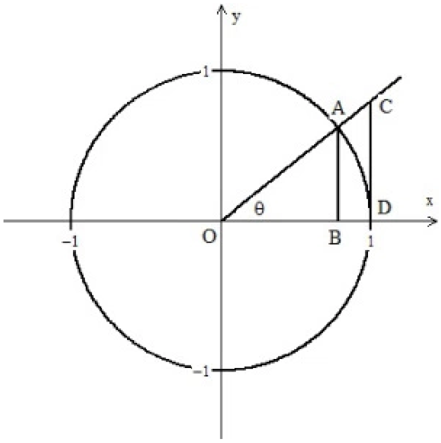
The middle term of the inequality is the area of a sector of a unit circle with central angles of θ radians. If you work in degrees, this sector's area is $\dfrac{\pi}{360}\theta$ and you will find that $\lim\limits_{\theta\to 0}\dfrac{\sin\left(\theta\right)}{\theta}=\dfrac{\pi}{180}$.

This limit is used to find the derivative of the sin(*x*). Thus, with *x* in degrees, $\dfrac{d}{dx}\sin\left(x\right)=\dfrac{\pi}{180}\cos\left(x\right)$. This means that with the derivative or antiderivative of any trigonometric function that $\dfrac{\pi}{180}$ is there getting in the way.

Who needs that?

Do your calculus in radians.

Revision December 7, 2014: The inequality above is derived this way. Consider the unit circle shown below.



1. The central angle is θ and the coordinates of *A* are $(\cos(\theta), \sin(\theta))$.

Then the area of triangle OAB is $\frac{1}{2}\cos(\theta)\sin(\theta)$

2. The area of sector $OAD = \frac{\theta}{2\pi}\pi(1)^2 = \frac{1}{2}\theta$. The sector's area is larger than the area of triangle OAB.

3. By similar triangles $\dfrac{AB}{OB} = \dfrac{\sin(\theta)}{\cos(\theta)} = \tan(\theta) = \dfrac{CD}{1} = CD$.

Then the area of $\Delta OCD = \frac{1}{2}CD \cdot OD = \frac{1}{2}\tan(\theta)$ This is larger than the area of the sector, which establishes the inequality above.

Multiply the inequality by $\dfrac{2}{\sin(\theta)}$ and take the reciprocal to obtain $\dfrac{1}{\cos(\theta)} \geq \dfrac{\sin(\theta)}{\theta} \geq \cos(\theta)$.

Finally, take the limit of these expression as $\theta \to 0$ and the limit $\lim\limits_{\theta \to 0} \dfrac{\sin(\theta)}{\theta} = 1$ is established by the squeeze theorem.

Share this:

Facebook Twitter Reddit Google Pinterest Tumblr LinkedIn More

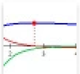
Like

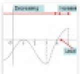
2 bloggers like this.

October 12, 2012 in Derivative Theory, Derivatives, Limits. Tags: definition of derivative, limits, Oct., radian

Related posts

Units


Good Question 1 2008 AB 6


Reading the Derivative's Graph

← Related Rate Problems II

Concepts Related to Graphs →

14 thoughts on "Why Radians?"


themannster October 14, 2017 at 11:42

Reblogged this on The Maths Mann.

★ Like

Reply

mrreja July 10, 2017 at 06:26

Reblogged this on mrreja and commented:
হুম

★ Like

Reply

sathasivam September 3, 2016 at 23:57

Lin,can you explain how d(sinx)/dx where x in degree is 180(cosx)/pi

★ Like

Reply

Lin McMullin September 4, 2016 at 12:13

Sathasivam
It is not 180(cosx)/pi, rather pi*cos(x)/180.

1. If x is in degrees, then to differentiate a trig function you must change the degrees to radians. So with x in degrees $\sin\left(x\right)=\sin\left(\frac{\pi}{180}x\right)$ with the argument now in radians. Then differentiate $\frac{d}{dx}\sin\left(\frac{\pi}{180}x\right)=\frac{\pi}{180}\cos\left(\frac{\pi}{180}x\right)$ or $\frac{\pi}{180}\cos\left(x\right)$ returning the argument to degrees.

2. If you wanted to work entirely in degrees from the start, then the middle term of the inequality in the post would be $\frac{2\pi\theta}{360}$ using the formula for arc length with θ in degrees. Then the $\frac{\pi}{180}$ will work its way through the inequalities resulting in $\lim_{\theta\to 0}\frac{\sin(\theta)}{\theta}=\frac{\pi}{180}$ and from there into the derivative formulas.

3. Try graphing $y=\sin(x)$ with x in degrees and your calculator set to degree mode. In a square window (that is, with equal units on both axes) the graph will appear to be very flat – almost linear. Thus, you would expect the slopes (derivatives) to be much smaller than when working in radians. The factor $\frac{\pi}{180}\approx 0.001745\ldots$ takes care of that "flattening."

★ Like

Reply

Pingback: What are Radians? Where Do They Come From? - Quirky Science

---

RJ Davis December 14, 2015 at 21:30

Could you do this same example with Cos (x) instead of Sin (x)?

★ Like

Reply

Lin McMullin December 15, 2015 at 09:36

Is this what you mean? With *x* in degrees you must change the argument to radians and then differentiate using the Chain Rule:
$\frac{dy}{dx}\cos\left(\frac{\pi}{180}x\right)=-\frac{\pi}{180}\sin\left(\frac{\pi}{180}x\right)$
This works the same way with any trig function.

★ Like

Reply

RJ Davis December 15, 2015 at 10:11

That is want I meant. I just wanted to make sure that the pi/180 would stay constant throughout and the chain rule would then be applicable. Thanks!

★ Like

Ranjay December 7, 2014 at 02:49

Hi Lin,
I like the way you have explained for easy understanding. However, please expalain (or give reference) of the inequality you have used for the explanation.

★ Like

Reply

Lin McMullin December 7, 2014 at 12:30

Hi Ranjay
I have added an explanation of the inequality at the end of the post. Thanks for writing; you were probably not the only one who was wondering about this.

★ Like

Reply

Jim October 1, 2014 at 10:15

Yes, if one were to use anything other than radian measure than one would get constants popping up upon differentiating trig functions and once they appeared these constants would mestasize. For the same reason one uses e as exponential base rather than the seemly more simple choice of say 10. Differentiating 10^x and the constant ln(10) appears. This must be nipped in the bud.

In elementary geometry where one is not using calculus the use of degrees or grads is perfectly OK.

★ Like

Reply

Bill Mahanna 6908 23 Ave W.,Bradenton, FL 34209 August 1, 2013 at 08:51

The radian is a linear measure of an angle, and it works best if you have your delta x and delta y in the same units. Degrees are a made up unit. Try plotting a sine wave using radians vs degrees for the x-axis. The y-axis will be the amplitude. A linear measurement. I'm sure more could be said but this might get your foot in the door.
Bill Retired math/physics teacher

★ Like

Reply

Steve Pulford October 22, 2012 at 18:59

Hi Lin,
Small typo – "a unit circles is a circle…"
Love your stuff!
Steve

★ Like

Reply

Lin McMullin October 22, 2012 at 19:01

Thanks. I fixed it. Glad you like the blog.

★ Like

Reply

Leave a Reply

Enter your comment here...