① $x = t^2 - 1,\ y = e^{t^3}$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{3t^2 e^{t^3}}{2t} = \boxed{\frac{3}{2} t e^{t^3}}$$

② $\vec{s}(t) = \langle \ln(t^2+5t), 3t^2 \rangle,\ t > 0$

$$\vec{s}\,'(t) = \vec{v}(t) = \left\langle \frac{2t+5}{t^2+5t}, 6t \right\rangle$$

$$\vec{v}(2) = \left\langle \frac{2(2)+5}{2^2+5(2)}, 6(2) \right\rangle = \boxed{\left\langle \frac{9}{14}, 12 \right\rangle}$$

③ $x = t^5 - 1,\ y = 3t^4 - 2t^3$

$$\vec{s}(t) = \langle t^5 - 1, 3t^4 - 2t^3 \rangle$$

$$\vec{s}\,'(t) = \vec{v}(t) = \langle 5t^4, 12t^3 - 6t^2 \rangle$$

$$\vec{s}\,''(t) = \vec{v}\,'(t) = \vec{a}(t) = \langle 20t^3, 36t^2 - 12t \rangle$$

$$\boxed{\vec{a}(1) = \langle 20, 24 \rangle}$$

④ $\vec{s}(t) = \left\langle \sin\left(3t - \frac{\pi}{2}\right), 3t^2 \right\rangle$

$$\vec{s}\,'(t) = \vec{v}(t) = \left\langle 3\cos\left(3t - \frac{\pi}{2}\right), 6t \right\rangle$$

$$\vec{v}\left(\frac{\pi}{2}\right) = \left\langle 3\cos(\pi), \frac{6\pi}{2} \right\rangle = \boxed{\langle -3, 3\pi \rangle}$$

⑤ $x'(t) = t + 1,\ t \geq 0,\ y = \ln x$

$$x(t) = \frac{1}{2}t^2 + t + C$$

when $t = 0;\ x = 1$:

$$1 = 0 + 0 + C,\ C = 1$$

so $x(t) = \frac{1}{2}t^2 + t + 1$

$$x(1) = \frac{1}{2} + 1 + 1 = \frac{5}{2}$$

when $x = \frac{5}{2},\ y = \ln\frac{5}{2}$

so at $t = 2$, the particle is at $\boxed{(x, y) = \left(\frac{5}{2}, \ln\frac{5}{2}\right)}$

⑥ $\vec{v}(t) = \langle 1 + t, t^3 \rangle,\ \vec{s}(0) = \langle 5, 0 \rangle = \langle x(0), y(0) \rangle$

$$x(2) = 5 + \int_0^2 (1+t)\,dt = 5 + \left[t + \frac{1}{2}t^2\right]_0^2$$

$$= 5 + [(2+2) - 0] = \boxed{9}$$

$$y(2) = 0 + \int_0^2 t^3\,dt = \frac{1}{4}t^4 \Big|_0^2 = \frac{16}{4} - 0 = \boxed{4}$$

so $\boxed{\vec{s}(2) = \langle 9, 4 \rangle}$ or particle is at $(9, 4)$ at $t = 2$.

⑦ $xy = 10,\ x = 2,\ \frac{dy}{dt} = 3,\ \frac{dx}{dt} = ?$

$$y = 10x^{-1}$$

$$\frac{dy}{dx} = \frac{-10}{x^2}$$

when $x = 2$:

$$\frac{dy}{dx} = \frac{-10}{4} = \frac{dy/dt}{dx/dt}$$

So $-\frac{5}{2} = \frac{3}{dx/dt}$

$$\frac{dx}{dt} = 3\left(-\frac{2}{5}\right) = \boxed{\frac{-6}{5}}$$

⑧ $x(t) = t^3 - \frac{3}{2}t^2 - 18t + 5,\ y(t) = t^3 - 6t^2 + 9t + 4$

particle is at rest when $\vec{v}(t) = \langle x'(t), y'(t) \rangle = \langle 0, 0 \rangle$

$$x'(t) = 3t^2 - 3t - 18 = 0$$
$$3(t^2 - t - 6) = 0$$
$$3(t-3)(t+2) = 0$$
$$t = 3,\ t = -2$$

$$y'(t) = 3t^2 - 12t + 9 = 0$$
$$3(t^2 - 4t + 3) = 0$$
$$3(t-3)(t-1) = 0$$
$$t = 3,\ t = 1$$

when $\boxed{t = 3}$, $\vec{v}(3) = \langle 0, 0 \rangle$

So the particle is not moving in either the horizontal or vertical direction and is, thus, at rest.

(9) $x(t)=t^3,\ y(t)=t^2-5t+2$

tangent line at $(x,y)=(8,-4)$

$x'(t)=\frac{dx}{dt}=3t^2,\ y'(t)=\frac{dy}{dt}=2t-5$

Find t @ $(8,-4)$:

$8=t^3$ so $\boxed{t=2}$

$-4=t^2-5t+2$

$t^2-5t+6=0$

$(t-3)(t-2)=0$

$t=3,\ \boxed{t=2}$

so $\left.\frac{dy}{dx}\right|_{t=2}=\frac{dy/dt|_{t=2}}{dx/dt|_{t=2}}$

$=\frac{y'(2)}{x'(2)}=\frac{-1}{12}=m$

so tangent line eq is

$\boxed{y=-4-\frac{1}{12}(x-8)}$

(10) $x(t)=5t+3\sin t,\ y(t)=(8-t)(1-\cos t)$

Find $\vec{v}(t)=\langle x'(t),y'(t)\rangle$ when $x=25$.

$25=5t+3\sin t$

$t=5.445...=A$ (find and store on calculator)

so $\vec{v}(A)=\langle x'(A),y'(A)\rangle$

$\boxed{\vec{v}(A)=\langle 7.008,-2.228\rangle}$

MATH #8 on calculator at $t=$ A

(11) $t\geq 0,\ x(t)=t^2-3,\ y(t)=\frac{2}{3}t^3$

(a) $\|\vec{v}(5)\|=?$, $x'(t)=2t$, $y'(t)=2t^2$

$x'(5)=10$, $y'(5)=50$

$\|\vec{v}(5)\|=\sqrt{10^2+50^2}=\sqrt{2600}=\boxed{10\sqrt{26}}$

(b) Total Distance = Arc Length $=\int_a^b |v(t)|\,dt$

$=\int_0^5\sqrt{(x'(t))^2+(y'(t))^2}\,dt$

$=\int_0^5\sqrt{4t^2+4t^4}\,dt=\int_0^5\sqrt{4t^2(1+t^2)}\,dt$

$=\int_0^5 2t(1+t^2)^{1/2}dt=\left.\frac{2}{3}(1+t^2)^{3/2}\right|_0^5$

$=\boxed{\frac{2}{3}\left[26^{3/2}-1^{3/2}\right]}$

(c) $y=\frac{2}{3}t^3\rightarrow t=\left(\frac{3}{2}y\right)^{1/3}$ *eliminate the parameter

so $x=\left(\left(\frac{3}{2}y\right)^{1/3}\right)^2-3=\left(\frac{3}{2}y\right)^{2/3}-3$

*Now solve for y

$x=\left(\frac{3}{2}\right)^{2/3}y^{2/3}-3$

$\left(\frac{3}{2}\right)^{-2/3}(x+3)=y^{2/3}$

$y=\left(\left(\frac{3}{2}\right)^{-2/3}(x+3)\right)^{3/2}$

$y=\frac{2}{3}(x+3)^{3/2}$

$\boxed{\frac{dy}{dx}=(x+3)^{1/2}}$

(12) $\frac{dx}{dt}=\frac{1}{t+1},\ \frac{dy}{dt}=2t,\ t\geq 0$

(a) $t=1,\ x=\ln 2,\ y=0$

$\int\frac{1}{t+1}dt=\ln|t+1|+C=x(t)$

$x(1)=\ln 2+C=\ln 2$, so $C=0$

$\int 2t\,dt=t^2+C=y(t)$

$y(1)=1+C=0,\ C=-1$

So $P(x,y)=(x(t),y(t))=\boxed{(\ln|t+1|,t^2-1)}$

(b) eliminate parameter: $x=\ln|t+1|,\ t\geq 0$

$e^x=t+1,\ t=e^x-1$

$y=t^2-1\rightarrow\boxed{y=(e^x-1)^2-1}$

(c) Avg R-o-C $=\frac{y(4)-y(0)}{x(4)-x(0)}=\frac{15-(-1)}{\ln 5-\ln 1}=\boxed{\frac{16}{\ln 5}}$

(d) $\frac{dy}{dx}=2(e^x-1)^1\cdot e^x$

$\left.\frac{dy}{dx}\right|_{\substack{t=1\\x=\ln 2}}=2e^{\ln 2}(e^{\ln 2}-1)$

$=2(2)(2-1)$

$=\boxed{4}$

(13) $x = 2 - 3\cos t,\ y = 3 + 2\sin t,\ t \in [-\frac{\pi}{2}, \frac{\pi}{2}]$

(a) $\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \boxed{\frac{2\cos t}{3\sin t}}$ or $= \boxed{\frac{2}{3}\cot t}$

(b) $\left.\frac{dy}{dx}\right|_{t=\pi/4} = \frac{2\cos(\frac{\pi}{4})}{3\sin(\frac{\pi}{4})} = \frac{2(\frac{\sqrt{2}}{2})}{3(\frac{\sqrt{2}}{2})} = \boxed{\frac{2}{3}} = m$

$x(\frac{\pi}{4}) = 2 - 3\cos\frac{\pi}{4} = 2 - \frac{3\sqrt{2}}{2}$

$y(\frac{\pi}{4}) = 3 + 2\sin(\frac{\pi}{4}) = 3 + \sqrt{2}$

tangent line eq: $\boxed{y = (3+\sqrt{2}) + \frac{2}{3}\left(x - \left(2 - \frac{3\sqrt{2}}{2}\right)\right)}$

(c) On y-axis, $x = 0$

$2 - 3\cos t = 0$

From calculator on $[-\frac{\pi}{2}, \frac{\pi}{2}]$

$t = -0.841 = A$

$t = 0.841 = B$

Arclength

$= \int_A^B \sqrt{(2-3\cos t)^2 + (3+2\sin t)^2}\,dt$

$= \boxed{5.196}$ (from calculator)

$y1 = 2 - 3\cos t$

$y2 = 3 + 2\sin t$

(14) $x = \sin t,\ y = \csc t,\ 0 < t < \frac{\pi}{2}$

$x'(t) = \cos t = \frac{dx}{dt}$

$x''(t) = -\sin t$

$y'(t) = -\csc t\cot t = -\frac{\cos t}{\sin^2 t}$

$\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left[\frac{dy}{dx}\right]}{\frac{dx}{dt}}$

$\frac{dy}{dx} = \frac{y'(t)}{x'(t)} = \frac{-\csc t\cot t}{\cos t}$

$= -\frac{1}{\sin^2 t} < 0$

for $0 < t < \frac{\pi}{2}$

So Decreasing

$\frac{dy}{dx} = -(\csc t)^2,\quad \frac{d^2y}{dx^2} = \frac{-2\csc t\cdot(-\csc t\cot t)}{\cos t}$

$= \frac{2\csc^2 t\cot t}{\cos t} = \frac{2}{\sin^3 t} > 0$

for $0 < t < \frac{\pi}{2}$, so

Concave up $\boxed{C}$

(15) $x = \ln t,\ y = t,\ t > 0$

*eliminate the parameter

$x = \ln y,\ \boxed{y = e^x},\ y > 0$ on $e^x\ \forall x$

$y > 0$

$\boxed{C}$

(16) $x(t) = t^2 + 1,\ y = \ln(2t+3),\ t \geq 0$

$x'(t) = 2t,\ y'(t) = \frac{2}{2t+3} = 2(2t+3)^{-1}$

$x''(t) = 2,\ y''(t) = -2(2t+3)^{-2}(2) = \frac{-4}{(2t+3)^2}$

So $\vec{a}(t) = \left\langle 2, -\frac{4}{(2t+3)^2} \right\rangle$

or $\left(2, -\frac{4}{(2t+3)^2}\right)$ $\boxed{E}$

*vectors aren't always written in chevrons < >.