

Name \_\_\_\_\_ Date \_\_\_\_\_ Period \_\_\_\_\_

**Worksheet 9.2—Parametric & Vector Accumulation**

Show all work. No calculator except unless specifically stated.

**Short Answer/Free Response**

1. If  $x = e^{2t}$  and  $y = \sin(3t)$ , find  $\frac{dy}{dx}$  in terms of  $t$ .

2. Write an integral expression to represent the length of the path described by the parametric equations  $x = \cos^3 t$  and  $y = \sin^2 t$  for  $0 \leq t \leq \frac{\pi}{2}$ .

3. For what value(s) of  $t$  does the curve given by the parametric equations  $x = t^3 - t^2 - 1$  and  $y = t^4 + 2t^2 - 8t$  have a vertical tangent?

4. Find the equation of the tangent line to the curve given by the parametric equations  $x(t) = 3t^2 - 4t + 2$  and  $y(t) = t^3 - 4t$  at the point on the curve where  $t = 1$ .
5. If  $x(t) = e^t + 1$  and  $y = 2e^{2t}$  are the equations of the path of a particle moving in the  $xy$ -plane, write an equation for the path of the particle in terms of  $x$  and  $y$ .
6. (Calculator) A particle moves in the  $xy$ -plane so that its position at any time  $t$  is given by  $x = \cos(5t)$  and  $y = t^3$ . What is the speed of the particle when  $t = 2$ ?

7. (Calculator) The position of a particle at time  $t \geq 0$  is given by the parametric equations

$$x(t) = \frac{(t-2)^3}{3} + 4 \text{ and } y(t) = t^2 - 4t + 4.$$

(a) Find the magnitude of the velocity vector at  $t = 1$ .

(b) Find the total distance traveled by the particle from  $t = 0$  to  $t = 1$ .

(c) When is the particle at rest? What is its position at that time?

8. (Calculator) An object moving along a curve in the  $xy$ -plane has position  $(x(t), y(t))$  at time  $t \geq 0$  with  $\frac{dx}{dt} = 1 + \tan(t^2)$  and  $\frac{dy}{dt} = 3e^{\sqrt{t}}$ . Find the acceleration vector and the speed of the object when  $t = 5$ .

9. (Calculator) A particle moves in the  $xy$ -plane so that the position of the particle is given by  $x(t) = t + \cos t$  and  $y(t) = 3t + 2 \sin t$ ,  $0 \leq t \leq \pi$ . Find the velocity vector when the particle's vertical position is  $y = 5$ .

10. (Calculator) An object moving along a curve in the  $xy$ -plane has position  $(x(t), y(t))$  at time  $t$  with

$$\frac{dx}{dt} = 2 \sin(t^3) \text{ and } \frac{dy}{dt} = \cos(t^2) \text{ for } 0 \leq t \leq 4. \text{ At time } t = 1, \text{ the object is at the position } (3, 4).$$

(a) Write an equation for the line tangent to the curve at  $(3, 4)$ .

(b) Find the speed of the object at time  $t = 2$ .

(c) Find the total distance traveled by the object over the time interval  $0 \leq t \leq 1$ .

(d) Find the position of the object at time  $t = 2$ .

**Multiple Choice:**

11. (Calculator) An object moving along a curve in the  $xy$ -plane has position  $(x(t), y(t))$  with  $\frac{dx}{dt} = \cos(t^2)$  and  $\frac{dy}{dt} = \sin(t^3)$ . At time  $t = 0$ , the object is at position  $(4, 7)$ . Where is the particle when  $t = 2$ ?
- (A)  $\langle -0.564, 0.989 \rangle$     (B)  $\langle 0.461, 0.452 \rangle$     (C)  $\langle 3.346, 7.989 \rangle$   
(D)  $\langle 4.461, 7.452 \rangle$     (E)  $\langle 5.962, 8.962 \rangle$
12. (Calculator) The path of a particle moving in the plane is defined parametrically as a function of time  $t$  by  $x = \sin 2t$  and  $y = \cos 5t$ . What is the speed of the particle at  $t = 2$ ?
- (A) 1.130    (B) 3.018    (C)  $\langle -1.307, 2.720 \rangle$     (D)  $\langle 0.757, 0.839 \rangle$     (E)  $\langle 1.307, 2.720 \rangle$

13. For what values of  $t$  does the curve given by the parametric equations  $x = t^3 - t^2 - 1$  and  $y = t^4 + 2t^2 - 8t$  have a vertical tangent?  
(A) 0 only    (B) 1 only    (C) 0 and 2/3 only    (D) 0, 2/3, and 1    (E) No value

14. The distance traveled by a particle from  $t = 0$  to  $t = 4$  whose position is given by the vector  $\mathbf{s}(t) = \langle t^2, t \rangle$  is given by

(A)  $\int_0^4 \sqrt{4t+1} dt$     (B)  $2 \int_0^4 \sqrt{t^2+1} dt$     (C)  $\int_0^4 \sqrt{2t^2+1} dt$     (D)  $\int_0^4 \sqrt{4t^2+1} dt$     (E)  $2\pi \int_0^4 \sqrt{4t^2+1} dt$