① $y = 4$

* $y = r\sin\theta$

so $4 = r\sin\theta$

$r = \frac{4}{\sin\theta}$

or $\boxed{r = 4\csc\theta}$

② $3x - 5y + 2 = 0$

* $x = r\cos\theta$, $y = r\sin\theta$

so $3(r\cos\theta) - 5(r\sin\theta) + 2 = 0$

$r(3\cos\theta - 5\sin\theta) = -2$

$\boxed{r = \frac{-2}{3\cos\theta - 5\sin\theta}}$

③ $x^2 + y^2 = 25$

* $x^2 + y^2 = r^2$

So $r^2 = 25$

$\boxed{r = 5 \text{ or } r = -5}$

↙ traces out differently, but same graph

④ $r = 3\sec\theta$

* $\sec\theta = \frac{r}{x}$

$r = 3\left(\frac{r}{x}\right)$

$1 = \frac{3}{x}$

$\boxed{x = 3}$

⑤ $r = 2\sin\theta$

* $\sin\theta = \frac{y}{r}$

$r = 2\left(\frac{y}{r}\right)$

$r^2 = 2y$

$\boxed{x^2 + y^2 = 2y}$

⑥ $\theta = \frac{5\pi}{6}$

$\tan\theta = \tan\frac{5\pi}{6}$ * take tangent of both sides

$\frac{y}{x} = -\frac{\sqrt{3}}{3}$

$\boxed{y = -\frac{\sqrt{3}}{3}x}$

⑦ $r = 2 + 3\sin\theta$, $\theta = \frac{3\pi}{2}$

* $x = r\cos\theta$, $y = r\sin\theta$

$x = (2 + 3\sin\theta)\cos\theta$, $y = (2 + 3\sin\theta)\sin\theta$

$x'(\theta) = (3\cos\theta)(\cos\theta) + (2 + 3\sin\theta)(-\sin\theta)$

$y'(\theta) = (3\cos\theta)(\sin\theta) + (2 + 3\sin\theta)(\cos\theta)$

$x'\left(\frac{3\pi}{2}\right) = 3\left(\cos\frac{3\pi}{2}\right)^2 - \left(\sin\frac{3\pi}{2}\right)\left(2 + 3\sin\frac{3\pi}{2}\right)$

$= 0 - (-1)(2 - 3) = -1$

$y'\left(\frac{3\pi}{2}\right) = \left(3\cos\frac{3\pi}{2}\right)\left(\sin\frac{3\pi}{2}\right) + \left(2 + 3\sin\frac{3\pi}{2}\right)\left(\cos\frac{3\pi}{2}\right)$

$= 0 + (2 - 3)(0) = 0$

$\left.\frac{dy}{dx}\right|_{\theta = \frac{3\pi}{2}} = \frac{y'\left(\frac{3\pi}{2}\right)}{x'\left(\frac{3\pi}{2}\right)} = \frac{0}{-1} = \boxed{0}$

⑧ $r = 3(1 - \cos\theta)$, $\theta = \frac{\pi}{2}$

$x = 3(1 - \cos\theta)\cos\theta = 3\cos\theta - 3\cos^2\theta$

$x' = -3\sin\theta - 6\cos\theta(-\sin\theta)$

$x'\left(\frac{\pi}{2}\right) = -3\sin\frac{\pi}{2} + 6\cos\frac{\pi}{2}\sin\frac{\pi}{2}$

$= \boxed{-3}$

$y = 3(1 - \cos\theta)\sin\theta$

$y' = 3(\sin\theta)\sin\theta + 3(1 - \cos\theta)\cos\theta$

$y'\left(\frac{\pi}{2}\right) = 3\left(\sin\frac{\pi}{2}\right)^2 + 3\left(1 - \cos\frac{\pi}{2}\right)\cos\frac{\pi}{2}$

$= 3 + 0 = \boxed{3}$

$\left.\frac{dy}{dx}\right|_{\theta = \frac{\pi}{2}} = \frac{y'\left(\frac{\pi}{2}\right)}{x'\left(\frac{\pi}{2}\right)} = \frac{3}{-3} = \boxed{-1}$

⑨ $r = 4\sin\theta$, $\theta = \frac{\pi}{3}$

$x = 4\sin\theta\cos\theta = 2\sin 2\theta$, $x' = 4\cos 2\theta$

$x'\left(\frac{\pi}{3}\right) = 4\left(-\frac{1}{2}\right) = \underline{-2}$

$y = 4\sin^2\theta$, $y' = 8\sin\theta\cos\theta = 4\sin 2\theta$

$y'\left(\frac{\pi}{3}\right) = 4\sin\frac{2\pi}{3} = 4\left(\frac{\sqrt{3}}{2}\right) = \underline{2\sqrt{3}}$

$\left.\frac{dy}{dx}\right|_{\theta = \frac{\pi}{3}} = \frac{2\sqrt{3}}{-2} = \boxed{-\sqrt{3}}$

⑩ $r = 2\sin(3\theta)$, $\theta = \frac{\pi}{4}$

$x = 2\sin(3\theta)\cos\theta$, $x' = 6\cos(3\theta)\cos\theta - 2\sin 3\theta\sin\theta$

$x'\left(\frac{\pi}{4}\right) = 6\cos\left(\frac{3\pi}{4}\right)\cos\left(\frac{\pi}{4}\right) - 2\sin\left(\frac{3\pi}{4}\right)\sin\left(\frac{\pi}{4}\right) =$

$= 6\left(-\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) - 2\left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) = -3 - 1 = \underline{-4}$

$y = 2\sin(3\theta)\sin\theta$, $y' = 6\cos(3\theta)\sin\theta + 2\sin(3\theta)\cos\theta$

$y'\left(\frac{\pi}{4}\right) = 6\left(-\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + 2\left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) = -3 + 1 = \underline{-2}$

$\left.\frac{dy}{dx}\right|_{\theta = \pi/4} = \frac{-2}{-4} = \boxed{\frac{1}{2}}$

(11) $r = 1+\sin\theta$

$x = (1+\sin\theta)\cos\theta$, $x' = \cos^2\theta - (1+\sin\theta)\sin\theta = \cos^2\theta - \sin\theta - \sin^2\theta = \cos 2\theta - \sin\theta$

$y = (1+\sin\theta)\sin\theta = \sin\theta + \sin^2\theta$, $y' = \cos\theta + 2\sin\theta\cos\theta = \cos\theta + \sin 2\theta$ ← easier to use below

Horizontal tangents when $y'=0$, $x'\neq 0$:

$\cos\theta + 2\sin\theta\cos\theta = 0$

$\cos\theta(1+2\sin\theta) = 0$

$\cos\theta = 0$ or $\sin\theta = -\frac{1}{2}$

$\theta = \frac{\pi}{2}, \frac{3\pi}{2}$ or $\theta = \frac{7\pi}{6}, \frac{11\pi}{6}$

$(r,\theta) = (1+\sin\theta, \theta)$

pts: $\left(2, \frac{\pi}{2}\right), \left(0, \cancel{\frac{3\pi}{2}}\right), \left(\frac{1}{2}, \frac{7\pi}{6}\right), \left(\frac{1}{2}, \frac{11\pi}{6}\right)$

throw out → $\frac{0}{0}$

Vert tangents when $x'=0$, $y'\neq 0$

$\cos 2\theta - \sin\theta = 0$

$(\cos^2\theta) - \sin^2\theta - \sin\theta = 0$

$(1-\sin^2\theta) - \sin^2\theta - \sin\theta = 0$

$-2\sin^2\theta - \sin\theta + 1 = 0$

$2\sin^2\theta + \sin\theta - 1 = 0$

$(2\sin\theta - 1)(\sin\theta + 1) = 0$

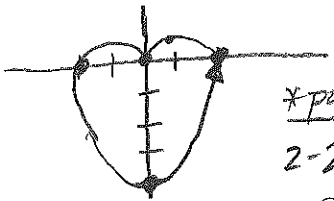
$\sin\theta = \frac{1}{2}$ or $\sin\theta = -1$

$\theta = \frac{\pi}{6}, \frac{5\pi}{6}$ or $\theta = \cancel{\frac{3\pi}{2}}$ ← throw out → $\frac{0}{0}$

pts: $\left(\frac{3}{2}, \frac{\pi}{6}\right), \left(\frac{3}{2}, \frac{5\pi}{6}\right)$,

(12) $r = 2 - 2\sin\theta$

θ	0	$\pi/6$	$\pi/2$	π	$3\pi/2$	2π
r	2	1	0	2	4	0



CARDIOID

*polar zeros:

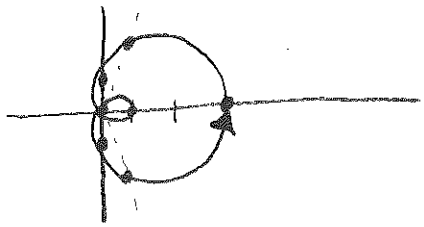
$2 - 2\sin\theta = 0$

$\sin\theta = 1$

$\theta = \frac{\pi}{2}$

(13) $r = 1 + 2\cos\theta$

θ	0	$\pi/3$	$\pi/2$	$2\pi/3$	π	$4\pi/3$	$3\pi/2$	2π
r	3	2	1	0	−1	0	1	3



LIMAÇON

*polar zeros:

$1 + 2\cos\theta = 0$

$\cos\theta = -\frac{1}{2}$

$\theta = \frac{2\pi}{3}, \frac{4\pi}{3}$

(14) $r = 4\cos(2\theta)$

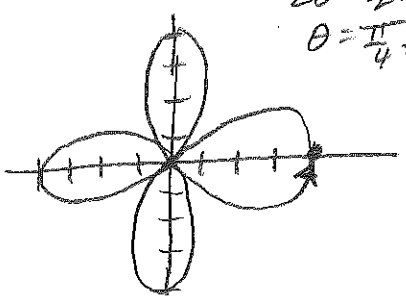
θ	0	$\pi/4$	$\frac{\pi}{2}$	...
r	4	0	−4	

*polar zeros: $4\cos 2\theta = 0$

$\cos 2\theta = 0$

$2\theta = \frac{\pi}{2} + \pi n$

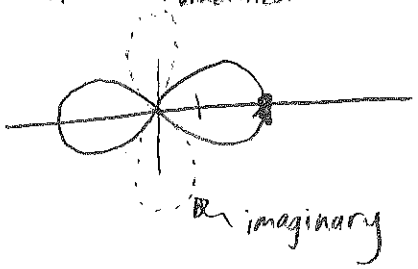
$\theta = \frac{\pi}{4} + \frac{\pi}{2} n$



ROSE CURVE

(15) $r^2 = 4\sin(2\theta)$, $r = \pm\sqrt{4\sin(2\theta)}$

θ	0	$\pi/2$	$3\pi/4$
r	2	0	−4 undefined



imaginary

LEMNISCATE

polar zeros:

$4\sin 2\theta = 0$

$\sin 2\theta = 0$

$2\theta = 0 + \pi n$

$\theta = 0 + \frac{\pi}{2} n$

(16) $a \neq 0, \theta \neq 0$

(A) (a, θ) (B) $(-a, -\theta)$ (C) $(-a, \theta - \pi)$ (D) $(-a, \theta + \pi)$ (E) $(a, \theta - 2\pi)$

doesn't fit other 4 **B**

(17) $r = f(\theta)$
$y = r\sin\theta$
$x = r\cos\theta$
, Slope $= \frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta}$ **D**

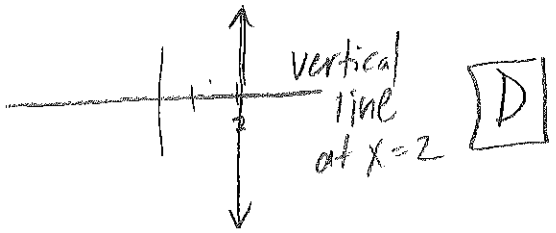
(18) $r = 2\sec\theta$

$r = 2\left(\frac{r}{x}\right)$

$(r) = \frac{2}{x}(r)$

$1 = \frac{2}{x}$

$x = 2$



vertical line at $x = 2$ **D**