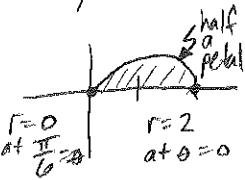


① one petal of $r = 2\cos(3\theta)$

polar zeros: $2\cos 3\theta = 0$
 $\cos 3\theta = 0$
 $3\theta = \frac{\pi}{2}, \frac{3\pi}{2}, \dots$



$\theta = \frac{\pi}{6}, \frac{\pi}{2}$

using symmetry:

$$\text{Area} = 2 \left[\frac{1}{2} \int_0^{\pi/2} (2\cos(3\theta))^2 d\theta \right]$$

$$= 4 \int_0^{\pi/2} \cos^2(3\theta) d\theta$$

*power-reducing ID

$$= 4 \left(\frac{1}{2} \right) \int_0^{\pi/2} (1 + \cos(6\theta)) d\theta$$

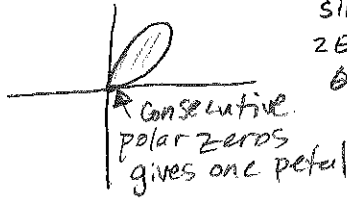
$$= 2 \left[\theta + \frac{1}{6} \sin(6\theta) \right]_0^{\pi/2}$$

$$= 2 \left[\left(\frac{\pi}{2} + 0 \right) - (0 + 0) \right]$$

$$= \boxed{\pi}$$

② one petal of $r = 4\sin(2\theta)$

polar zeros: $4\sin 2\theta = 0$
 $\sin 2\theta = 0$
 $2\theta = 0, \pi, \dots$
 $\theta = 0, \frac{\pi}{2}$



$$\text{Area} = \frac{1}{2} \int_0^{\pi/2} (4\sin(2\theta))^2 d\theta$$

$$= 8 \left(\frac{1}{2} \right) \int_0^{\pi/2} (1 - \cos(4\theta)) d\theta$$

*square & power-reducing ID.

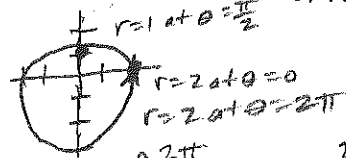
$$= 4 \left[\theta - \frac{1}{4} \sin(4\theta) \right]_0^{\pi/2}$$

$$= 4 \left[\left(\frac{\pi}{2} - 0 \right) - (0 - 0) \right]$$

$$= \boxed{2\pi}$$

④ interior of $r = 2 - \sin\theta$

polar zeros: $2 - \sin\theta = 0$
 $\sin\theta = 2 \rightarrow$ no zeros



$$\text{Area} = \frac{1}{2} \int_0^{2\pi} (2 - \sin\theta)^2 d\theta$$

$$= \frac{1}{2} \int_0^{2\pi} (4 - 4\sin\theta + \sin^2\theta) d\theta$$

$$= \frac{1}{2} \int_0^{2\pi} \left(4 - 4\sin\theta + \frac{1}{2}(1 - \cos(2\theta)) \right) d\theta$$

$$= \frac{1}{2} \int_0^{2\pi} \left(\frac{9}{2} - 4\sin\theta - \frac{1}{2}\cos(2\theta) \right) d\theta$$

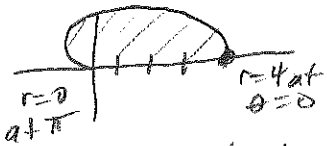
$$= \frac{1}{2} \left[\frac{9}{2}\theta + 4\cos\theta - \frac{1}{4}\sin(2\theta) \right]_0^{2\pi}$$

$$= \frac{1}{2} \left[(9\pi + 4 - 0) - (0 + 4 - 0) \right]$$

$$= \boxed{\frac{9\pi}{2}}$$

③ interior of $r = 2 + 2\cos\theta$

polar zeros: $2 + 2\cos\theta = 0$
 $\cos\theta = -1$
 $\theta = \pi$



using symmetry:

$$\text{Area} = 2 \left[\frac{1}{2} \int_0^{\pi} (2 + 2\cos\theta)^2 d\theta \right]$$

$$= \int_0^{\pi} (4 + 8\cos\theta + 4\cos^2\theta) d\theta$$

$$= \int_0^{\pi} (4 + 8\cos\theta + 2(1 + \cos(2\theta))) d\theta$$

$$= \int_0^{\pi} (6 + 8\cos\theta + 2\cos(2\theta)) d\theta$$

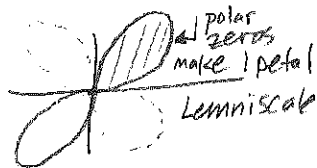
$$= 6\theta + 8\sin\theta + \sin(2\theta) \Big|_0^{\pi}$$

$$= [(6\pi + 0 + 0) - (0 + 0 + 0)]$$

$$= \boxed{6\pi}$$

⑤ interior of $r^2 = 4\sin(2\theta)$

polar zeros: $4\sin 2\theta = 0$
 $\sin 2\theta = 0$
 $2\theta = 0, \pi$
 $\theta = 0, \frac{\pi}{2}$



Using symmetry:

$$\text{Area} = 2 \left[\frac{1}{2} \int_0^{\pi/2} (4\sin(2\theta)) d\theta \right]$$

$$= 4 \int_0^{\pi/2} \sin 2\theta d\theta$$

$$= -2\cos 2\theta \Big|_0^{\pi/2}$$

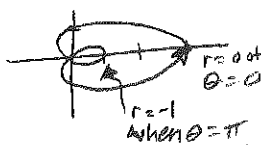
$$= -2[\cos \pi - \cos 0]$$

$$= -2[-1 - 1]$$

$$= \boxed{4}$$

⑥ inner loop $r = 1 + 2\cos\theta$

polar zeros: $1 + 2\cos\theta = 0$
 $\cos\theta = -\frac{1}{2}$
 $\theta = \frac{2\pi}{3}, \frac{4\pi}{3}$ (define inner loop)



$$\text{Area} = \frac{1}{2} \int_{\frac{2\pi}{3}}^{\frac{4\pi}{3}} (1 + 2\cos\theta)^2 d\theta$$

$$= \frac{1}{2} \int_{\frac{2\pi}{3}}^{\frac{4\pi}{3}} (1 + 4\cos\theta + 4\cos^2\theta) d\theta$$

$$= \frac{1}{2} \int_{\frac{2\pi}{3}}^{\frac{4\pi}{3}} (1 + 4\cos\theta + 4(\frac{1}{2})(1 + \cos 2\theta)) d\theta$$

$$= \frac{1}{2} \int_{\frac{2\pi}{3}}^{\frac{4\pi}{3}} (3 + 4\cos\theta + 2\cos 2\theta) d\theta$$

$$= \frac{1}{2} [3\theta + 4\sin\theta + \sin 2\theta] \Big|_{\frac{2\pi}{3}}^{\frac{4\pi}{3}}$$

$$= \frac{1}{2} \left[(4\pi + 4\sin \frac{4\pi}{3} + \sin \frac{8\pi}{3}) - (2\pi + 4\sin \frac{2\pi}{3} + \sin \frac{4\pi}{3}) \right]$$

$$= \frac{1}{2} \left[4\pi - 2\sqrt{3} + \frac{\sqrt{3}}{2} - 2\pi - 2\sqrt{3} + \frac{\sqrt{3}}{2} \right]$$

$$= \frac{1}{2} [2\pi - 4\sqrt{3} + \sqrt{3}] = \frac{1}{2} [2\pi - 3\sqrt{3}]$$

$$= \boxed{\pi - \frac{3\sqrt{3}}{2}}$$

⑦ between loops of $r = 1 + 2\cos\theta$

* same curve as in #6.

using symmetry:

$$\text{Area} = 2 \left[\frac{1}{2} \int_0^{\frac{2\pi}{3}} (1 + 2\cos\theta)^2 d\theta \right] - \left(\pi - \frac{3\sqrt{3}}{2} \right)$$



$$= \int_0^{\frac{2\pi}{3}} (1 + 4\cos\theta + 4\cos^2\theta) d\theta - \pi + \frac{3\sqrt{3}}{2}$$

$$= \int_0^{\frac{2\pi}{3}} (3 + 4\cos\theta + 2\cos 2\theta) d\theta - \pi + \frac{3\sqrt{3}}{2}$$

$$= [3\theta + 4\sin\theta + \sin 2\theta] \Big|_0^{\frac{2\pi}{3}} - \pi + \frac{3\sqrt{3}}{2}$$

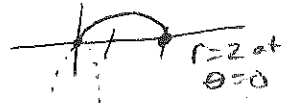
$$= \left[\left(2\pi + 4\sin \frac{2\pi}{3} + \sin \frac{4\pi}{3} \right) - (0 + 0 + 0) \right] - \pi + \frac{3\sqrt{3}}{2}$$

$$= 2\pi + 2\sqrt{3} - \frac{\sqrt{3}}{2} - \pi + \frac{3\sqrt{3}}{2}$$

$$= \boxed{\pi + 3\sqrt{3}}$$

⑧ one loop of $r^2 = 4\cos(2\theta)$

polar zeros: $\cos 2\theta = 0$
 $2\theta = \frac{\pi}{2}, \frac{3\pi}{2}$
 $\theta = \frac{\pi}{4}, \frac{3\pi}{4}$



using symmetry already squared

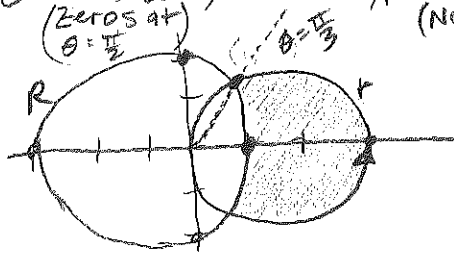
$$\text{Area} = 2 \left[\frac{1}{2} \int_0^{\frac{\pi}{4}} 4\cos(2\theta) d\theta \right]$$

$$= 2\sin(2\theta) \Big|_0^{\pi/4}$$

$$= 2\sin \frac{\pi}{2} - 2\sin 0$$

$$= \boxed{2}$$

9) inside $r = 3\cos\theta$, outside $R = 2 - \cos\theta$
 (zeros at $\theta = \frac{\pi}{3}$) (NO ZEROS)



point of intersection:

$$3\cos\theta = 2 - \cos\theta$$

$$4\cos\theta = 2$$

$$\cos\theta = \frac{1}{2}$$

$$\theta = \frac{\pi}{3}$$

using symmetry:

$$\text{Area} = 2 \left[\frac{1}{2} \int_0^{\pi/3} (3\cos\theta)^2 d\theta - \frac{1}{2} \int_0^{\pi/3} (2 - \cos\theta)^2 d\theta \right]$$

$$= \int_0^{\pi/3} [9\cos^2\theta - (4 - 4\cos\theta + \cos^2\theta)] d\theta$$

$$= \int_0^{\pi/3} [8\cos^2\theta + 4\cos\theta - 4] d\theta$$

$$= \int_0^{\pi/3} [4(1 + \cos 2\theta) + 4\cos\theta - 4] d\theta$$

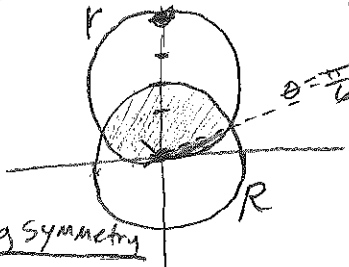
$$= \int_0^{\pi/3} [4\cos(2\theta) + 4\cos\theta] d\theta$$

$$= 2\sin(2\theta) + 4\sin\theta \Big|_0^{\pi/3}$$

$$= (2\sin(\frac{2\pi}{3}) + 4\sin(\frac{\pi}{3})) - (0)$$

$$= \sqrt{3} + 2\sqrt{3} = \boxed{3\sqrt{3}}$$

10) common interior of $r = 4\sin\theta$, $R = 2$



point of intersect

$$4\sin\theta = 2$$

$$\sin\theta = \frac{1}{2}$$

$$\theta = \frac{\pi}{6}$$

using symmetry

$$\text{Area} = 2 \left[\frac{1}{2} \int_0^{\pi/6} (4\sin\theta)^2 d\theta + \frac{1}{2} \int_{\pi/6}^{\pi/2} (2)^2 d\theta \right]$$

$$= \int_0^{\pi/6} 16\sin^2\theta d\theta + \int_{\pi/6}^{\pi/2} 4 d\theta$$

$$= 8 \int_0^{\pi/6} (1 - \cos 2\theta) d\theta + \int_{\pi/6}^{\pi/2} 4 d\theta$$

$$= 8 \left[\theta - \frac{1}{2}\sin 2\theta \right]_0^{\pi/6} + 4\theta \Big|_{\pi/6}^{\pi/2}$$

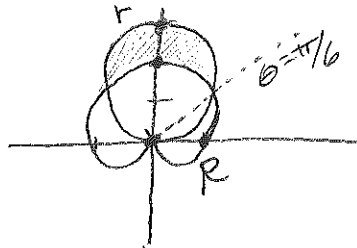
$$= 8 \left[\left(\frac{\pi}{6} - \frac{1}{2} \left(\frac{\sqrt{3}}{2} \right) \right) - (0 - 0) \right] + 4 \left[\frac{\pi}{2} - \frac{\pi}{6} \right]$$

$$= 8 \left[\frac{\pi}{6} - \frac{\sqrt{3}}{4} \right] + 4 \left[\frac{\pi}{3} \right]$$

$$= \frac{4\pi}{3} - 2\sqrt{3} + \frac{4\pi}{3}$$

$$= \boxed{\frac{8\pi}{3} - 2\sqrt{3}}$$

(11) inside $r = 3\sin\theta$, outside $R = 1 + \sin\theta$



zero at $\theta = \frac{3\pi}{2}$
 point of intersect
 $3\sin\theta = 1 + \sin\theta$
 $2\sin\theta = 1$
 $\sin\theta = \frac{1}{2}$
 $\theta = \frac{\pi}{6}$

using symmetry:

$$\text{Area} = 2 \left[\frac{1}{2} \int_{\pi/6}^{\pi/2} (3\sin\theta)^2 d\theta - \frac{1}{2} \int_{\pi/6}^{\pi/2} (1 + \sin\theta)^2 d\theta \right]$$

$$= 9 \int_{\pi/6}^{\pi/2} \sin^2\theta d\theta - \int_{\pi/6}^{\pi/2} (1 + 2\sin\theta + \sin^2\theta) d\theta$$

$$= \int_{\pi/6}^{\pi/2} \left[\frac{9}{2}(1 - \cos 2\theta) - 1 - 2\sin\theta - \frac{1}{2}(1 - \cos 2\theta) \right] d\theta$$

$$= \int_{\pi/6}^{\pi/2} [3 - 4\cos 2\theta - 2\sin\theta] d\theta$$

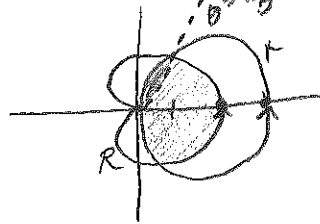
$$= 3\theta - 2\sin 2\theta + 2\cos\theta \Big|_{\pi/6}^{\pi/2}$$

$$= \left[\frac{3\pi}{2} - 0 + 0 \right] - \left[\frac{\pi}{2} - \sqrt{3} + \sqrt{3} \right]$$

$$= \frac{3\pi}{2} - \frac{\pi}{2}$$

$$= \boxed{\pi}$$

(12) common interior of $r = 3\cos\theta$, $R = 1 + \cos\theta$
 polar zero at $r = \frac{\pi}{2}$
 zero: $\cos\theta = -1$
 $\theta = \pi$



intersection:
 $3\cos\theta = 1 + \cos\theta$
 $2\cos\theta = 1$
 $\cos\theta = \frac{1}{2}$
 $\theta = \frac{\pi}{3}$

using symmetry

$$\text{Area} = 2 \left[\frac{1}{2} \int_0^{\pi/3} (1 + \cos\theta)^2 d\theta + \frac{1}{2} \int_{\pi/3}^{\pi/2} (3\cos\theta)^2 d\theta \right]$$

$$= \int_0^{\pi/3} (1 + 2\cos\theta + \frac{1}{2}(1 + \cos 2\theta)) d\theta + \int_{\pi/3}^{\pi/2} \frac{9}{2}(1 + \cos 2\theta) d\theta$$

$$= \int_0^{\pi/3} \left(\frac{3}{2} + 2\cos\theta + \frac{1}{4}\cos 2\theta \right) d\theta + \frac{9}{2} \int_{\pi/3}^{\pi/2} (1 + \cos 2\theta) d\theta$$

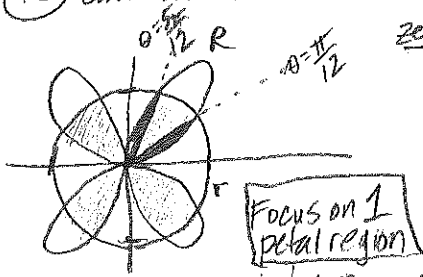
$$= \left[\frac{3}{2}\theta + 2\sin\theta + \frac{1}{4}\sin 2\theta \right] \Big|_0^{\pi/3} + \frac{9}{2} \left[\theta + \frac{1}{2}\sin 2\theta \right] \Big|_{\pi/3}^{\pi/2}$$

$$= \left[\frac{\pi}{2} + \sqrt{3} + \frac{\sqrt{3}}{8} \right] - 0 + \frac{9}{2} \left[\left(\frac{\pi}{2} + 0 \right) - \left(\frac{\pi}{3} + \frac{\sqrt{3}}{4} \right) \right]$$

$$= \frac{\pi}{2} + \sqrt{3} + \frac{\sqrt{3}}{8} + \frac{9\pi}{4} - \frac{3\pi}{2} - \frac{9\sqrt{3}}{8}$$

$$= \boxed{\frac{5\pi}{4}}$$

⑬ common interior of $r = 4\sin(2\theta)$ & $R = 2$



zeros:
 $\sin 2\theta = 0$
 $2\theta = 0, \pi$
 $\theta = 0, \frac{\pi}{2}$
 (one petal)

intersection

1: Area contained by 1 Rose Curve.
 (using symmetry)

$$\begin{aligned} \text{Area}_1 &= 2 \left[\frac{1}{2} \int_0^{\pi/12} (4\sin 2\theta)^2 d\theta \right] \\ &= 16 \int_0^{\pi/12} \sin^2 2\theta d\theta \\ &= 8 \int_0^{\pi/12} (1 - \cos 4\theta) d\theta \\ &= 8 \left[\theta - \frac{1}{4} \sin 4\theta \right]_0^{\pi/12} \\ &= 8 \left[\frac{\pi}{12} - \frac{\sqrt{3}}{8} \right] \\ &= \boxed{\frac{2\pi}{3} - \sqrt{3}} \end{aligned}$$

2: Area of 1 sector contained by circle
 (using symmetry)

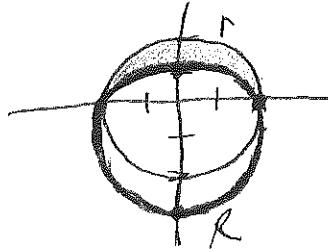
$$\begin{aligned} \text{Area}_2 &= \int_{\pi/12}^{\pi/2} (2)^2 d\theta \\ &= 4\theta \Big|_{\pi/12}^{\pi/2} \\ &= 4 \left[\frac{5\pi}{12} - \frac{\pi}{12} \right] = \boxed{\frac{4\pi}{3}} \end{aligned}$$

Total Common Area:

$$\begin{aligned} \text{Area} &= 4 \left[\left(\frac{2\pi}{3} - \sqrt{3} \right) + \left(\frac{4\pi}{3} \right) \right] \\ &= 4 \left[2\pi - \sqrt{3} \right] \\ &= \boxed{8\pi - 4\sqrt{3}} \end{aligned}$$

4 regions

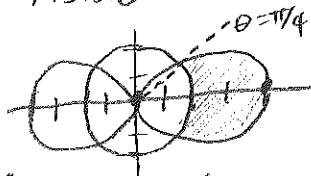
⑭ inside $r = 2$ & outside $R = 2 - \sin \theta$



(using symmetry)

$$\begin{aligned} \text{Area} &= 2 \left[\frac{1}{2} \int_0^{\pi/2} (2)^2 d\theta - \frac{1}{2} \int_0^{\pi/2} (2 - \sin \theta)^2 d\theta \right] \\ &= \int_0^{\pi/2} \left[4 - (4 - 4\sin \theta + \sin^2 \theta) \right] d\theta \\ &= \int_0^{\pi/2} \left[4\sin \theta - \frac{1}{2}(1 - \cos 2\theta) \right] d\theta \\ &= \int_0^{\pi/2} \left[4\sin \theta - \frac{1}{2} + \frac{1}{2}\cos 2\theta \right] d\theta \\ &= -4\cos \theta - \frac{1}{2}\theta + \frac{1}{4}\sin 2\theta \Big|_0^{\pi/2} \\ &= \left(0 - \frac{\pi}{4} + 0 \right) - (-4) \\ &= \boxed{4 - \frac{\pi}{4}} \end{aligned}$$

⑮ inside $r = 2 + 2\cos(2\theta)$ & outside $R = 2$



zeros of r:
 $\cos 2\theta = -1$
 $2\theta = \pi$
 $\theta = \frac{\pi}{2}$

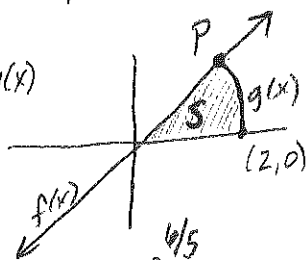
intersect:
 $2 + 2\cos 2\theta = 2$
 $\cos 2\theta = 0$
 $\theta = \frac{\pi}{4}$

(using symmetry)

$$\begin{aligned} \text{Area} &= 2 \left[\frac{1}{2} \int_0^{\pi/4} (2 + 2\cos 2\theta)^2 d\theta - \frac{1}{2} \int_0^{\pi/4} (2)^2 d\theta \right] \\ &= \int_0^{\pi/4} \left((2 + 2\cos 2\theta)^2 - 4 \right) d\theta \\ &= \boxed{5.5707} \text{ (from calculator)} \end{aligned}$$

(Dang, this is a LONG worksheet, but totally worth it!)

(16) $y = \frac{2}{3}x = f(x)$, $y = \sqrt{1 - \frac{x^2}{4}} = g(x)$



x-intercept of $g(x)$

$1 = \frac{x^2}{4}$, $x^2 = 4$, $x = 2$

$\sqrt{1 - \frac{x^2}{4}} = \sqrt{\frac{4 - x^2}{4}} = \frac{1}{2}\sqrt{4 - x^2}$

(a) point of intersection

$\frac{2}{3}x = \sqrt{1 - \frac{x^2}{4}}$

$\frac{4}{9}x^2 = 1 - \frac{1}{4}x^2$

$(\frac{4}{9} + \frac{1}{4})x^2 = 1$

$x^2 = \frac{36}{25}$

$x = -\frac{6}{5}$ or $\frac{6}{5}$

$y = \frac{2}{3}(\frac{6}{5}) = \frac{4}{5}$

So Point P is $(\frac{6}{5}, \frac{4}{5})$

(b) Area = $\int_0^{6/5} (\frac{2}{3}x - 0) dx + \int_{6/5}^2 (\sqrt{1 - \frac{x^2}{4}} - 0) dx$

= $\int_0^{6/5} \frac{2}{3}x dx + \int_{6/5}^2 \frac{1}{2}\sqrt{4 - x^2} dx$

need calculator to integrate

= 0.9272956436

(c) Polar eq. for curve C

$y = \sqrt{1 - \frac{1}{4}x^2}$, $y = r \sin \theta$, $x = r \cos \theta$, $x^2 + y^2 = r^2$

$y^2 = 1 - \frac{1}{4}x^2$ $\left\{ \begin{aligned} (r \cos \theta)^2 + 4(r \sin \theta)^2 &= 4 \\ r^2 \cos^2 \theta + 4r^2 \sin^2 \theta &= 4 \end{aligned} \right.$

$r^2 = \frac{4}{\cos^2 \theta + 4 \sin^2 \theta}$

or $r = \frac{2}{\sqrt{\cos^2 \theta + 4 \sin^2 \theta}}$

(d) $y = \frac{2}{3}x$

= $r \sin \theta = \frac{2}{3}(r \cos \theta)$

$\tan \theta = \frac{2}{3}$

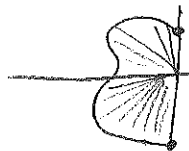
$\theta = 0.588 \text{ rads} = A$

Area = $\frac{1}{2} \int_0^A \left(\frac{4}{\cos^2 \theta + 4 \sin^2 \theta} \right) d\theta$

= 0.927295218

* note answers from (b) & (d) are equivalent to 5 decimals only because of the different ways the calculator numerically computes the integrals.

(17) $r = \theta + \cos(3\theta)$, $\theta \in \left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$



(a) Area = $\frac{1}{2} \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} (\theta + \cos 3\theta)^2 d\theta$
1.610

(b) $y = -1$

$y = r \sin \theta = (\theta + \cos 3\theta) \sin \theta = -1$
 * from calculator

$\theta = 3.484$ or $\theta = 3.485$

(c) $\frac{dr}{d\theta} = 1 - 3\sin(3\theta) = 0$

* from calculator

$\theta = 2.207 = A, 3.028 = B, 4.302 = C$

$\frac{dr}{d\theta} > 0$ for $\theta \in \left[\frac{\pi}{2}, 2.207\right) \cup (3.028, 4.302)$. On these intervals the radius is increasing with respect to θ , which means the curve is moving away from the pole/origin.

(d) the distance from the origin = r , so we want to maximize r on $\frac{\pi}{2} \leq \theta \leq \frac{3\pi}{2}$

$\frac{dr}{d\theta} = 1 - 3\sin(3\theta) = 0$

at $\theta = A, B, C$ (from part (c)). These are the critical values.

$\theta = \frac{\pi}{2}, \frac{3\pi}{2}$ are the endpoints.

* using the EVT and the closed interval argument.

from calculator

$r(A) = 3.150$
 $r(B) = 2.085$
 $r(C) = 5.244$
 $r\left(\frac{\pi}{2}\right) = 1.570$
 $r\left(\frac{3\pi}{2}\right) = 4.712$

Justification

so the curve is the greatest distance from the origin when $\theta = 4.302$ radians. At this time it is 5.244 units from the origin

(18) $r = \frac{4}{1 + \sin \theta}$, $0 \leq \theta \leq \pi$



(a) Area = $\frac{1}{2} \int_0^\pi \left(\frac{4}{1 + \sin \theta}\right)^2 d\theta$
 $= 10.667 = \frac{32}{3}$

(b) $r = \frac{4}{1 + \sin \theta}$
 $r = \frac{4}{1 + \frac{y}{r}} \left(\frac{r}{r}\right)$
 $r = \frac{4r}{r + y}$

$r + y = 4$, $\sqrt{x^2 + y^2} = 4 - y$
 $x^2 + y^2 = 16 - 8y + y^2$
 $8y = 16 - x^2$

(c) $y = \frac{1}{8}(16 - x^2)$

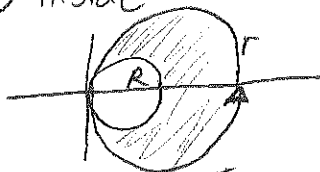
$y = 2 - \frac{1}{8}x^2$, x-ints at $x = -4, 4$

Area = $\int_{-4}^4 \left(2 - \frac{1}{8}x^2\right) dx$

or using symmetry

Area = $2 \int_0^4 \left(2 - \frac{1}{8}x^2\right) dx$

(19) inside $r = 2 \cos \theta$, outside $r = \cos \theta$



1 revolution for $\theta \in [0, \pi]$

Area = $\frac{1}{2} \int_0^\pi [2 \cos \theta]^2 d\theta - \frac{1}{2} \int_0^\pi [\cos \theta]^2 d\theta$

$= \frac{1}{2} \int_0^\pi [4 \cos^2 \theta - \cos^2 \theta] d\theta$

$= \frac{1}{2} \int_0^\pi 3 \cos^2 \theta d\theta$

$= \frac{3}{2} \int_0^\pi \cos^2 \theta d\theta$

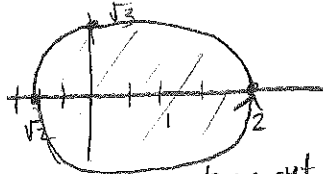
answer not there

So... using symmetry

$= 2 \left[\frac{3}{2} \int_0^{\pi/2} \cos^2 \theta d\theta \right]$

$= 3 \int_0^{\pi/2} \cos^2 \theta d\theta$ **A**

(20) $r = \sqrt{3 + \cos \theta}$



trace out once from 0 to 2π

Area = $\frac{1}{2} \int_0^{2\pi} (\sqrt{3 + \cos \theta})^2 d\theta$

$= \frac{1}{2} \int_0^{2\pi} (3 + \cos \theta) d\theta$

* using symmetry

$= 2 \left[\frac{1}{2} \int_0^\pi (3 + \cos \theta) d\theta \right]$

$= \int_0^\pi (3 + \cos \theta) d\theta$ **D**

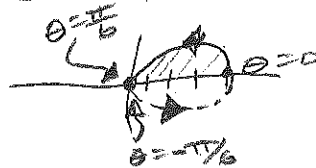
(21) one petal of $r = 4 \cos(3\theta)$

polar zeros:

$\cos 3\theta = 0$

$3\theta = \frac{\pi}{2}, \frac{3\pi}{2}$

$\theta = \frac{\pi}{6}, \frac{\pi}{2}$



From answer choices:

$A = \frac{1}{2} \left[\int_{-\pi/6}^{\pi/6} (4 \cos 3\theta)^2 d\theta \right]$

$= 8 \int_{-\pi/6}^{\pi/6} \cos^2 3\theta d\theta$ **E**

Finished already! 😊