Name____ Date_______________________ Period______

Worksheet 10.2—Polar Area
Show all work. No calculator except unless specifically stated.

Short Answer: Sketch a graph, shade the region, and find the area.

1. one petal of $r = 2\cos(3\theta)$

2. one petal of $r = 4\sin(2\theta)$

3. interior of $r = 2 + 2\cos\theta$

4. interior of $r = 2 - \sin\theta$

5. interior of $r^2 = 4\sin(2\theta)$

6. inner loop of $r = 1 + 2\cos\theta$

7. between the loops of $r = 1 + 2\cos\theta$

8. one loop of $r^2 = 4\cos(2\theta)$

9. inside $r = 3\cos\theta$ and outside $r = 2 - \cos\theta$

10. common interior of $r = 4\sin\theta$ and $r = 2$

11. inside $r = 3\sin\theta$ and outside $r = 1 + \sin\theta$

12. common interior of $r = 3\cos\theta$ and $r = 1 + \cos\theta$

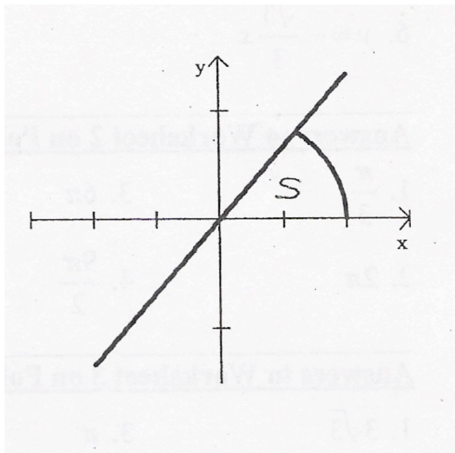
13. common interior of $r = 4\sin(2\theta)$ and $r = 2$

14. inside $r = 2$ and outside $r = 2 - \sin\theta$

15. (Calculator Permitted) inside $r = 2 + 2\cos(2\theta)$ and outside $r = 2$

Free Response

16. (Calculator Permitted) The figure shows the graphs of the line $y = \frac{2}{3}x$ and the curve C given by $y = \sqrt{1 - \frac{x^2}{4}}$. Let S be the region in the first quadrant bounded by the two graphs and the x-axis. The line and the curve intersect at point P.



(a) Find the coordinates of P.

(b) Set up and evaluate an integral expression with respect to x that gives the area of S.

(b) Find a polar equation to represent curve C.

(d) Use the polar equation found in (c) to set up and evaluate an integral expression with respect to the polar angle θ that gives the area of S.

17. (Calculator Permitted) A curve is drawn in the xy-plane and is described by the equation in polar coordinates $r = \theta + \cos(3\theta)$ for $\frac{\pi}{2} \le \theta \le \frac{3\pi}{2}$, where r is measured in meters and θ is measured in radians.

(a) Find the area bounded by the curve and the y-axis.

(b) Find the angle θ that corresponds to the point on the curve with y-coordinate -1.

(c) For what values of θ, $\pi \le \theta \le \frac{3\pi}{2}$ is $\frac{dr}{d\theta}$ positive? What does this say about r?

(d) Find the value of θ on the interval $\pi \le \theta \le \frac{3\pi}{2}$ that corresponds to the point on the curve with the greatest distance from the origin. What is this greatest distance? Justify your answer.

18. (Calculator Permitted) A region R in the xy-plane is bounded below by the x-axis and above by the polar curve defined by $r = \dfrac{4}{1+\sin\theta}$ for $0 \le \theta \le \pi$.

(a) Find the area of R by evaluating an integral in polar coordinates.

(b) The curve resembles an arch of the parabola $8y = 16 - x^2$. Convert the polar equation to rectangular coordinates, and prove that the curves are the same.

(c) Set up an integral in rectangular coordinates that gives the area of R.

Multiple Choice

19. Which of the following is equal to the area of the region inside the polar curve $r = 2\cos\theta$ and outside the polar curve $r = \cos\theta$?

(A) $3\int_0^{\frac{\pi}{2}} \cos^2\theta d\theta$ (B) $3\int_0^{\pi} \cos^2\theta d\theta$ (C) $\frac{3}{2}\int_0^{\frac{\pi}{2}} \cos^2\theta d\theta$ (D) $3\int_0^{\frac{\pi}{2}} \cos\theta d\theta$ (E) $3\int_0^{\pi} \cos\theta d\theta$

20. (Calculator permitted) The area of the region enclosed by the polar graph of $r = \sqrt{3+\cos\theta}$ is given by which integral?

(A) $\int_0^{2\pi} \sqrt{3+\cos\theta} d\theta$ (B) $\int_0^{\pi} \sqrt{3+\cos\theta} d\theta$ (C) $2\int_0^{\pi/2} (3+\cos\theta) d\theta$

(D) $\int_0^{\pi} (3+\cos\theta) d\theta$ (E) $\int_0^{\pi/2} \sqrt{3+\cos\theta} d\theta$

21. The area enclosed by one petal of the 3-petaled rose curve $r = 4\cos(3\theta)$ is given by which integral?

(A) $16\int_{-\pi/3}^{\pi/3} \cos(3\theta) d\theta$ (B) $8\int_{-\pi/6}^{\pi/6} \cos(3\theta) d\theta$ (C) $8\int_{-\pi/3}^{\pi/3} \cos^2(3\theta) d\theta$

(D) $16\int_{-\pi/6}^{\pi/6} \cos(3\theta) d\theta$ (E) $8\int_{-\pi/6}^{\pi/6} \cos^2(3\theta) d\theta$