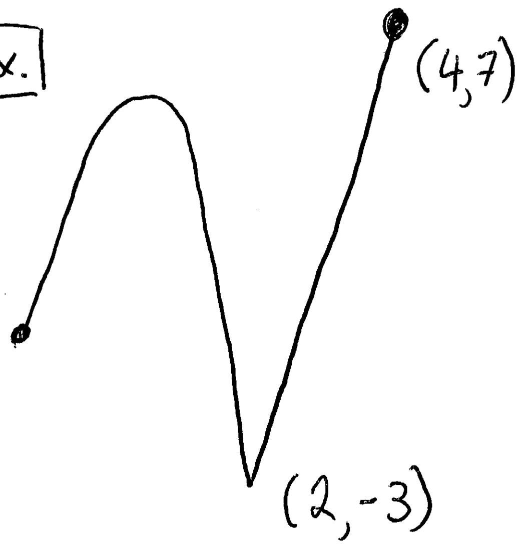
Ch. 3.1

Extreme Value Theorem (EVT) Ch. 3.1

Ex.

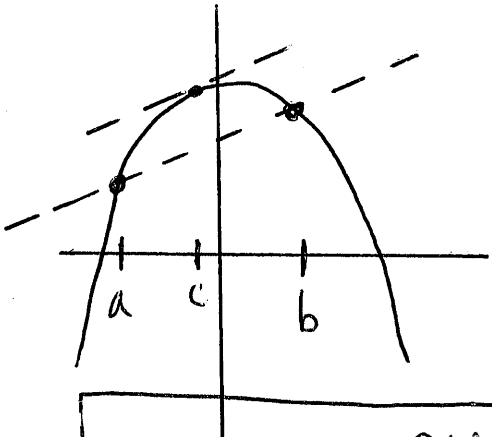


Purpose: Find Abs max/min on closed interval

* $f(x)$ continuous $[a, b]$
* find critical points
  - a) set $f'(x) = 0$
  - b) set denominator of $f'(x) = 0$
* test critical points and endpoints into $f(x)$ to find absolute max/min
* Abs max is 7 at $x = 4$

  Abs min is -3 at $x = 2$

3.2a Mean Value Theorem (MVT)



purpose: find the location on the curve where the guaranteed slope occurs.

Conditions:

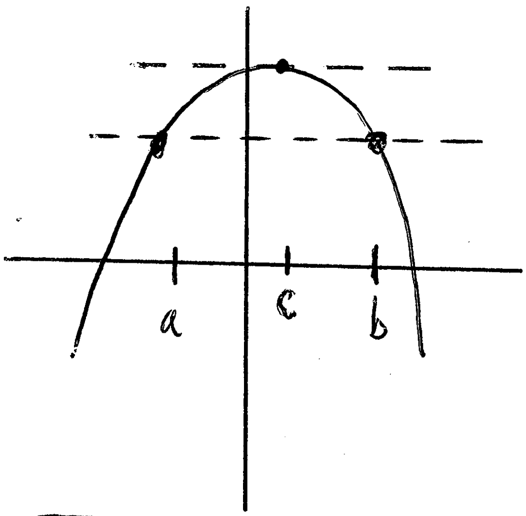
* $f(x)$ continuous on $[a, b]$
  (no VA, no holes on interval)
* $f(x)$ differentiable on (a, b)
  (no sharp turns, no slope undefined on (a, b))

$$\boxed{\text{MVT: } f'(c) = \frac{f(b) - f(a)}{b - a}}$$

Steps:

1) find slope between endpoints $\left[\frac{f(b) - f(a)}{b - a}\right]$
2) find $f'(x)$
3) set $f'(x) =$ slope value, solve for x (c-value)
4) Keep the c-values in interval (a, b)

3.2b Rolle's Theorem



Purpose: Find the location on the curve where the guaranteed slope of 0 occurs.

Conditions:

* $f(x)$ continuous $[a,b]$ (no breaks, no vertical asymptote, no holes)
* $f(x)$ differentiable (a,b) (no sharp turns, no location with undefined slope)
* $f(a)=f(b)$ (endpoints with same y-value)

$$\boxed{\text{Rolle's Theorem: } f'(c)=0}$$

Steps:

1) confirm endpts have same y-values
2) find $f'(x)$
3) set numerator of $f'(x)=0$, solve for x (c-value)
4) Keep the c-values in interval (a,b)

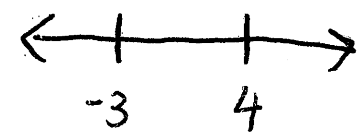
Ch. 3.3 1st Derivative Test

Purpose: Use $f'(x)$ to determine slope behavior of graph and find relative max, relative mins of graph

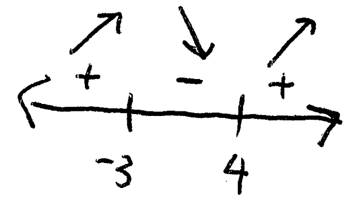
1) Find critical points
   a) find $f'(x)$
   b) set numerator of $f'(x)=0$
   c) set denominator of $f'(x)=0$

$\boxed{\text{Ex:}}$

2) Place critical values on $f'(x)$ sign line



3) Test intervals, plug in x-values into $f'(x)$



   a) Rel. max at $(-3, \_)$ b/c $f'(x)$ changes from + to -

   b) Rel. min at $(4, \_)$ b/c $f'(x)$ changes from - to +

   c) $f(x)$ increasing $(-\infty, 3), (4, \infty)$ b/c $f'(x) > 0$

   d) $f(x)$ decreasing $(-3, 4)$ b/c $f'(x) < 0$

Ch. 3.4 Concavity Test:

Purpose: Use $f''(x)$ to determine concavity behavior of graph and find Points of Inflection (POI)

1) Find critical points
   a) find $f''(x)$
   b) set numerator, denominator of $f''(x) = 0$

2) Place critical points on $f''(x)$ sign line

   Example

   $f''(x)$ $\longleftarrow\!\!+\!\!-\!\!-\!\!+\!\!\longrightarrow$
   $-2 \quad 3$

3) Test intervals, plug x-values into $f''(x)$

   $f''(x)$ $\quad \cap \quad \cup \quad \cap$
   $\quad - \quad + \quad -$
   $-2 \quad 3$

   a) POI at $(-2, \_)$ and $(3, \_)$ b/c $f''(x)$ change signs.

   b) $f(x)$ concave up $(-2, 3)$ b/c $f''(x) > 0$

   c) $f(x)$ concave down $(-\infty, -2), (3, \infty)$ b/c $f''(x) < 0$

Ch. 3.4 2^{nd} derivative test

Purpose: Use $f''(x)$ to determine relative max/mins of graph

<u>Steps:</u>

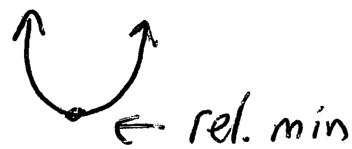
$\boxed{\text{example}}$

1) find $f'(x)$ and critical points (set $f'(x)=0$) $x=a, b$

2) find $f''(x)$

3) plug in critical points from $f'(x)$ <u>into</u> $f''(x)$

4) If $f''(a) > 0$, concave up, Rel. min at $x=a$



5) If $f''(b) < 0$, concave down, Rel. max at $x=b$