2017

3.2 MVT and Rolle's Theorem

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#5-15 odd, 27, 32, 33-41 odd

Rolle's Theorem

7) $f(x) = x\sqrt{x+4}$ x-ints: $(0,0)$ and $(-4,0)$

$f(x) = x(x+4)^{1/2}$ $f'(x) = 1(x+4)^{1/2} + x \cdot \frac{1}{2}(x+4)^{-1/2}(1)$

$f'(x) = \sqrt{x+4} + \frac{x}{2\sqrt{x+4}} = \frac{2(x+4)+x}{2\sqrt{x+4}} = \frac{2x+8+x}{2\sqrt{x+4}} = \frac{3x+8}{2\sqrt{x+4}}$

$f'(x) = \frac{3x+8}{2\sqrt{x+4}}$ set $f'(x)=0$ $3x+8=0$ $\boxed{x = -8/3}$

11) $f(x) = (x-1)(x-2)(x-3)$ $[1,3]$ $f(1)=0$, $f(3)=0$ ✓

$f(x) = x^3 - 6x^2 + 11x - 6$

$f'(x) = 3x^2 - 12x + 11$

$0 = 3x^2 - 12x + 11$

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$x = \frac{12 \pm \sqrt{144 - 4(3)(11)}}{2(3)} = \frac{12 \pm \sqrt{12}}{6} = \frac{12 \pm 2\sqrt{3}}{6}$

$\boxed{x = 2 \pm \frac{\sqrt{3}}{3}}$

13) $f(x) = x^{2/3} - 1$ $[-8, 8]$ $f(-8) = (-8)^{2/3} - 1 = 3$
$f(8) = 8^{2/3} - 1 = 3$

$f'(x) = \frac{2}{3}x^{-1/3} - 0$

$f'(x) = \frac{2}{3x^{1/3}}$ critical pt: $x = 0$

$f(x)$ is not differentiable at $x=0$; does not fulfill differentiable condition of Rolle's theorem. Rolle's theorem does not apply.

15) $f(x) = \dfrac{x^2-2x-3}{x+2}$ $[-1,3]$ $\left.\begin{array}{l} f(-1)=0 \\ f(3)=0 \end{array}\right]$

$$f'(x) = \frac{(2x-2)(x+2)-(x^2-2x-3)(1)}{(x+2)^2} = \frac{2x^2+4x-2x-4-x^2+2x+3}{(x+2)^2}$$

$$f'(x) = \frac{x^2+4x-1}{(x+2)^2} \qquad x = \frac{-4\pm\sqrt{4^2-4(1)(-1)}}{2(1)} = \frac{-4\pm\sqrt{20}}{2} = \frac{-4\pm2\sqrt{5}}{2}$$

$x = -2\pm\sqrt{5}$ $\boxed{\text{only } x=-2+\sqrt{5} \text{ is in interval } [-1,3]}$

27) $f(t) = -16t^2+48t+6$ a) $\left.\begin{array}{l} f(1)=38 \\ f(2)=38 \end{array}\right]$ ✓

b) set $f'(t)=0$ by Rolle's Theorem

$f'(t) = -32t+48$ | $32t=48$

$0=-32t+48$ | $\boxed{t = \frac{48}{32} = \frac{3}{2} \text{ sec}}$

35) MVT: $f(x) = -x^2+5$

a) equation of secant line through $(-1,4)$ and $(2,1)$ $m = \frac{1-4}{2--1} = \frac{-3}{3} = -1$

$y-1 = -1(x-2) \rightarrow y-1=-x+2 \rightarrow \boxed{y=-x+3} \rightarrow$ slope $m=-1$

b) set $f'(x)=-1$

$f'(x) = -2x$

$-2x=-1$

$x = 1/2$ $\boxed{c=1/2}$ $f(1/2) = -\left(\frac{1}{2}\right)^2+5 = -\frac{1}{4}+\frac{20}{4} = \frac{19}{4}$

c) Tangent line: point $(1/2, 19/4)$ slope $m=-1$

$\boxed{y-\frac{19}{4} = -1(x-1/2)} \rightarrow 4x+4y-21=0$

2017

3.2 Determine if MVT applies

32) f is not differentiable at $x=2$ (sharp point), not smooth curve at $x=2$.

37) $f(x)=x^2$ $[-2,1]$ $f(-2)=4$ $f(1)=1$ slope $=\frac{4-1}{-2-1}=\frac{3}{-3}=-1$

f(x) continuous on $[-2,1]$, differentiable on $(-2,1)$

set $f'(x)=$ slope secant line

$2x=-1$ $x=-1/2$ $c=-1/2$ on interval $[-2,1]$

39) $f(x)=x^3+2x$ on $[-1,1]$ $f(-1)=-1-2=-3$ $f(1)=1+2=3$ $m_{Avg}=\frac{3-(-3)}{1-(-1)}=\frac{6}{2}$

f(x) continuous on $[-1,1]$, differentiable on $(-1,1)$ $m_{Avg}=3$

set $f'(x)=m_{Avg.}$

$f'(x)=3x^2+2$

$3x^2+2=3$

$3x^2=1$

$x^2=\frac{1}{3}$

$x=\pm\sqrt{1/3}$ or $\pm\frac{\sqrt{3}}{3}$

$\boxed{c=\pm\frac{\sqrt{3}}{3} \text{ on interval } [-1,1]}$

41) $f(x)=x^{2/3}$ $[0,1]$ $f(0)=0$ $f(1)=1$ $M_{Avg}=\frac{1-0}{1-0}=\frac{1}{1}=1$

$f'(x)=\frac{2}{3}x^{-1/3}=\frac{2}{3x^{1/3}}$ $x=0$, f(x) not differentiable at $x=0$.

However, MVT still applies since f(x) is continuous on $[0,1]$ and differentiable on the open interval $(0,1)$ * f(x) Does not need to be differentiable at $x=0$.

set $f'(x)=M_{Avg.}$

$\frac{2}{3x^{1/3}}=1$

$2=3x^{1/3}$

$\frac{2}{3}=x^{1/3}$

$x^{1/3}=\frac{2}{3}$

$(x^{1/3})^3=\left(\frac{2}{3}\right)^3=\frac{8}{27}$

$\boxed{c=8/27}$