2017

# AB Calculus Ch. 3.3 Select HW Problems

Key

**Applying the First Derivative Test** In Exercises 17–40, (a) find the critical numbers of $f$ (if any), (b) find the open interval(s) on which the function is increasing or decreasing, (c) apply the First Derivative Test to identify all relative extrema, and (d) use a graphing utility to confirm your results.

**19.** $f(x) = -2x^2 + 4x + 3$

$f'(x) = -4x + 4$

$0 = -4(x-1)$

$x = 1$

$\nearrow$ $+$ | $\searrow$ $-$

$1$

Increasing on $(-\infty, 1)$ b/c $f'(x) > 0$

Decreasing on $(1, \infty)$ b/c $f'(x) < 0$

Relative max at $(1, \underline{5})$ b/c $f'(x)$ changes from + to −

**21.** $f(x) = 2x^3 + 3x^2 - 12x$

$f'(x) = 6x^2 + 6x - 12$

$0 = 6(x^2 + x - 2)$

$0 = 6(x+2)(x-1)$

$x = 1, -2$

$\nearrow$ $+$ | $\searrow$ $-$ | $\nearrow$ $+$

$-2$ $\quad$ $1$

Increasing on $(-\infty, -2) \cup (1, \infty)$ b/c $f'(x) > 0$

Decreasing on $(-2, 1)$ b/c $f'(x) < 0$

Rel. max $(-2, \underline{20})$ b/c $f'(x)$ changes from + to −

Rel. min $(1, \underline{-7})$ b/c $f'(x)$ changes from − to +

**25.** $f(x) = \dfrac{x^5 - 5x}{5} = \frac{1}{5}(x^5 - 5x)$

$f(x) = \frac{1}{5}x^5 - \frac{5}{5}x = \frac{1}{5}x^5 - x$

$f'(x) = 5 \cdot \frac{1}{5}x^4 - 1$

$f'(x) = x^4 - 1$

$0 = (x^2+1)(x^2-1)$

$0 = (x^2+1)(x-1)(x+1)$

$x = 1, -1$

$\nearrow$ $+$ | $\searrow$ $-$ | $\nearrow$ $+$

$-1$ $\quad$ $1$

Inc: $(-\infty, -1) \cup (1, \infty)$

Dec: $(-1, 1)$

Rel. max: $(-1, 4/5)$

Rel. min: $(1, -4/5)$

**27.** $f(x) = x^{1/3} + 1$

$f'(x) = \frac{1}{3}x^{-2/3} + 0$

$0 = \dfrac{1}{3x^{2/3}}$

$x = 0$

$\nearrow$ $+$ | $\nearrow$ $+$

$0$

Increasing: $(-\infty, 0) \cup (0, \infty)$ b/c $f'(x) > 0$

No relative extrema

**29.** $f(x) = (x+2)^{2/3}$ $\quad$ * Apply chain rule

$f'(x) = \frac{2}{3}(x+2)^{-1/3}(1)$

$0 = \dfrac{2}{3(x+2)^{1/3}}$

$x = -2$

$\searrow$ $-$ | $\nearrow$ $+$

$-2$

Dec: $(-\infty, -2)$ b/c $f'(x) < 0$

Inc: $(-2, \infty)$ b/c $f'(x) > 0$

Rel. min $(-2, \underline{0})$ b/c $f'(x)$ changes from − to +

**33.** $f(x) = 2x + \dfrac{1}{x}$

$f(x) = 2x + x^{-1}$

$f'(x) = 2 - 1x^{-2}$

$f'(x) = 2 - \dfrac{1}{x^2}$

$f'(x) = \dfrac{2x^2 - 1}{x^2}$

$2x^2 - 1 = 0$ $\quad$ | $\quad$ $x^2 = 0$

$x^2 = 1/2$

$x = \pm\sqrt{1/2} = \frac{\sqrt{2}}{2}$

$\nearrow$ $+$ | $\searrow$ $-$ | $\searrow$ $-$ | $\nearrow$ $+$

$-\sqrt{2}/2$ $\quad$ $0$ $\quad$ $\sqrt{2}/2$

Inc: $(-\infty, -\sqrt{2}/2) \cup (\sqrt{2}/2, \infty)$ b/c $f'(x) > 0$

Dec: $(-\sqrt{2}/2, 0) \cup (0, \sqrt{2}/2)$ b/c $f'(x) < 0$

Rel. max: $(-\sqrt{2}/2, -2\sqrt{2})$

Rel. min: $(\sqrt{2}/2, 2\sqrt{2})$

**37.** $f(x) = \begin{cases} 4 - x^2, & x \le 0 \\ -2x, & x > 0 \end{cases}$

$f'(x) = \begin{cases} -2x & x \le 0 \\ -2 & x > 0 \end{cases}$

$+$ ↗ | ↘ $-$ at $0$

Inc: $(-\infty, 0)$

Dec: $(0, \infty)$

Rel. max: $(0, 4)$

**39.** $f(x) = \begin{cases} 3x + 1, & x \le 1 \\ 5 - x^2, & x > 1 \end{cases}$

$f'(x) = \begin{cases} 3, & x \le 1 \\ -2x, & x > 1 \end{cases}$

$-2x = 0$

critical pts $x = \cancel{0}, 1$

$+$ ↗ | ↘ $-$ at $1$

Inc: $(1, \infty)$

Dec: $(-\infty, 1)$

Rel. max: $(1, 4)$

**Finding and Analyzing Derivatives Using Technology**
**In Exercises 49–54, (a) use a computer algebra system to differentiate the function, (b) sketch the graphs of $f$ and $f'$ on the same set of coordinate axes over the given interval, (c) find the critical numbers of $f$ in the open interval, and (d) find the interval(s) on which $f'$ is positive and the interval(s) on which it is negative. Compare the behavior of $f$ and the sign of $f'$.**

**49.** $f(x) = 2x\sqrt{9 - x^2}$, $[-3, 3]$ * Apply product, chain rule to find $f'(x)$.

$f(x) = 2x(9-x^2)^{1/2}$

$f'(x) = 2(9-x^2)^{1/2} + 2x \cdot \frac{1}{2}(9-x^2)^{-1/2}(-2x)$

$= \frac{2\sqrt{9-x^2}}{1} - \frac{2x^2}{\sqrt{9-x^2}}$

$f'(x) = \frac{2(9-x^2) - 2x^2}{\sqrt{9-x^2}} = \frac{18-2x^2-2x^2}{\sqrt{9-x^2}} = \boxed{\frac{18-4x^2}{\sqrt{9-x^2}}}$

$18 - 4x^2 = 0$

$4x^2 = 18$

$x^2 = \frac{18}{4} = \frac{9}{2}$

$x = \pm \frac{3\sqrt{2}}{2}$

$\sqrt{9-x^2} = 0$

$x = \pm 3$

$-3$ | ↘ $-$ | $-\frac{3\sqrt{2}}{2}$ | ↗ $+$ | $\frac{3\sqrt{2}}{2}$ | ↘ $-$ | $3$

Inc: $\left(-\frac{3\sqrt{2}}{2}, \frac{3\sqrt{2}}{2}\right)$

Dec: $\left(-3, -\frac{3\sqrt{2}}{2}\right) \cup \left(\frac{3\sqrt{2}}{2}, 3\right)$

**Motion Along a Line** **In Exercises 81–84, the function $s(t)$ describes the motion of a particle along a line. For each function, (a) find the velocity function of the particle at any time $t \ge 0$, (b) identify the time interval(s) in which the particle is moving in a positive direction, (c) identify the time interval(s) in which the particle is moving in a negative direction, and (d) identify the time(s) at which the particle changes direction.**

**81.** $s(t) = 6t - t^2$

a) $v(t) = s'(t) = 6 - 2t$

b) set $v(t) = 0$

$0 = 6 - 2t$

$2t = 6$ $\quad \underline{t = 3}$

$0$ | $+$ | $3$ | $-$ →

particle moving right $0 \le t < 3$

c) particle moving left $t > 3$

d) particle changes direction at $t = 3$.

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**33.** $f(x) = 2x + \dfrac{1}{x}$

37. $f(x) = \begin{cases} 4 - x^2, & x \le 0 \\ -2x, & x > 0 \end{cases}$

39. $f(x) = \begin{cases} 3x + 1, & x \le 1 \\ 5 - x^2, & x > 1 \end{cases}$

**Finding and Analyzing Derivatives Using Technology** In Exercises 49–54, (a) use a computer algebra system to differentiate the function, (b) sketch the graphs of $f$ and $f'$ on the same set of coordinate axes over the given interval, (c) find the critical numbers of $f$ in the open interval, and (d) find the interval(s) on which $f'$ is positive and the interval(s) on which it is negative. Compare the behavior of $f$ and the sign of $f'$.

49. $f(x) = 2x\sqrt{9 - x^2}$, $[-3, 3]$

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