

**AB Calculus Ch. 3.3 Select HW Problems**

Key

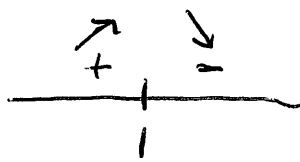
**Applying the First Derivative Test** In Exercises 17-40,  
 (a) find the critical numbers of  $f$  (if any), (b) find the open interval(s) on which the function is increasing or decreasing,  
 (c) apply the First Derivative Test to identify all relative extrema,  
 and (d) use a graphing utility to confirm your results.

19.  $f(x) = -2x^2 + 4x + 3$

$f'(x) = -4x + 4$

$0 = -4(x-1)$

$x = 1$



Increasing on  $(-\infty, 1)$   
 b/c  $f'(x) > 0$   
 Decreasing on  $(1, \infty)$   
 b/c  $f'(x) < 0$   
 Relative max at  $(1, 5)$   
 b/c  $f'(x)$  changes from + to -

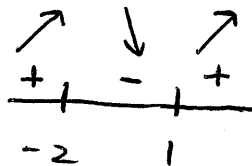
21.  $f(x) = 2x^3 + 3x^2 - 12x$

$f'(x) = 6x^2 + 6x - 12$

$0 = 6(x^2 + x - 2)$

$0 = 6(x+2)(x-1)$

$x = -2, -1$



Increasing on  $(-\infty, -2)$   
 $\cup (1, \infty)$  b/c  $f'(x) > 0$   
 Decreasing on  $(-2, 1)$   
 b/c  $f'(x) < 0$   
 Rel. max  $(-2, 20)$  b/c  $f'(x)$  changes from + to -  
 Rel. min  $(1, -7)$  b/c  $f'(x)$  changes from - to +

25.  $f(x) = \frac{x^5 - 5x}{5} = \frac{1}{5}(x^5 - 5x)$

$f(x) = \frac{1}{5}x^5 - \frac{5}{5}x = \frac{1}{5}x^5 - x$

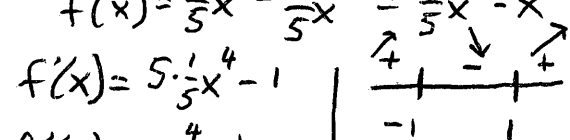
$f'(x) = 5 \cdot \frac{1}{5}x^4 - 1$

$f'(x) = x^4 - 1$

$0 = (x^2 + 1)(x^2 - 1)$

$0 = (x^2 + 1)(x-1)(x+1)$

$x = -1$



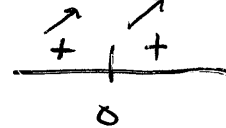
Inc:  $(-\infty, -1) \cup (1, \infty)$   
 Dec:  $(-1, 1)$   
 Rel. max:  $(-1, 4/5)$   
 Rel. min:  $(1, -4/5)$

27.  $f(x) = x^{1/3} + 1$

$f'(x) = \frac{1}{3}x^{-2/3} + 0$

$0 = \frac{1}{3x^{2/3}}$

$x = 0$



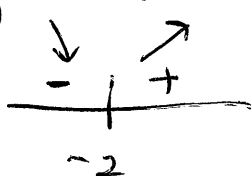
Increasing:  $(-\infty, 0] \cup (0, \infty)$   
 b/c  $f'(x) > 0$   
 No relative extrema

29.  $f(x) = (x+2)^{2/3}$  \* Apply chain rule

$f'(x) = \frac{2}{3}(x+2)^{-1/3} (1)$

$0 = \frac{2}{3(x+2)^{1/3}}$

$x = -2$



Dec:  $(-\infty, -2)$   
 b/c  $f'(x) < 0$   
 Inc:  $(-2, \infty)$   
 b/c  $f'(x) > 0$   
 Rel. min  $(-2, 0)$   
 b/c  $f'(x)$  changes from - to +

33.  $f(x) = 2x + \frac{1}{x}$

$f(x) = 2x + x^{-1}$

$f'(x) = 2 - 1x^{-2}$

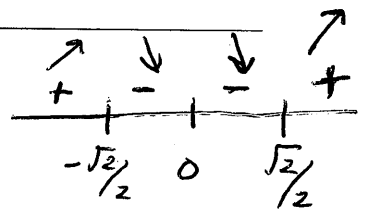
$f'(x) = 2 - \frac{1}{x^2}$

$f'(x) = \frac{2x^2 - 1}{x^2}$

$2x^2 - 1 = 0$  |  $x^2 = 0$

$x^2 = 1/2$

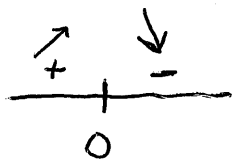
$x = \pm\sqrt{1/2} = \frac{\sqrt{2}}{2}$



Inc:  $(-\infty, -\frac{\sqrt{2}}{2}) \cup (\frac{\sqrt{2}}{2}, \infty)$   
 b/c  $f'(x) > 0$   
 Dec:  $(-\frac{\sqrt{2}}{2}, 0) \cup (0, \frac{\sqrt{2}}{2})$   
 b/c  $f'(x) < 0$   
 Rel. max:  $(-\frac{\sqrt{2}}{2}, -2\sqrt{2})$   
 Rel. min:  $(\frac{\sqrt{2}}{2}, 2\sqrt{2})$

$$37. f(x) = \begin{cases} 4 - x^2, & x \leq 0 \\ -2x, & x > 0 \end{cases}$$

$$f'(x) = \begin{cases} -2x & x \leq 0 \\ -2 & x > 0 \end{cases}$$



Inc:  $(-\infty, 0)$

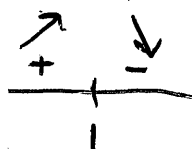
Dec:  $(0, \infty)$

Rel. max:  $(0, 4)$

$$39. f(x) = \begin{cases} 3x + 1, & x \leq 1 \\ 5 - x^2, & x > 1 \end{cases}$$

$$f'(x) = \begin{cases} 3, & x \leq 1 \\ -2x, & x > 1 \end{cases}$$

$-2x = 0$   
critical pts  $x = 0, 1$



Inc:  $(1, \infty)$

Dec:  $(-\infty, 1)$

Rel. max:  $(1, 4)$

### Finding and Analyzing Derivatives Using Technology

In Exercises 49–54, (a) use a computer algebra system to differentiate the function, (b) sketch the graphs of  $f$  and  $f'$  on the same set of coordinate axes over the given interval, (c) find the critical numbers of  $f$  in the open interval, and (d) find the interval(s) on which  $f'$  is positive and the interval(s) on which it is negative. Compare the behavior of  $f$  and the sign of  $f'$ .

49.  $f(x) = 2x\sqrt{9-x^2}$ ,  $[-3, 3]$  \* Apply product, chain rule to find  $f'(x)$ .

$$f(x) = 2x(9-x^2)^{1/2}$$

$$f'(x) = 2(9-x^2)^{1/2} + 2x \cdot \frac{1}{2}(9-x^2)^{-1/2}(-2x)$$

$$= \frac{2\sqrt{9-x^2}}{1} - \frac{2x^2}{\sqrt{9-x^2}}$$

$$f'(x) = \frac{2(9-x^2) - 2x^2}{\sqrt{9-x^2}} = \frac{18-2x^2-2x^2}{\sqrt{9-x^2}} = \frac{18-4x^2}{\sqrt{9-x^2}}$$

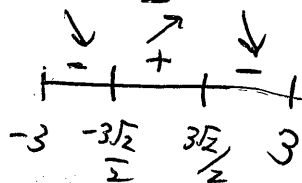
$$18-4x^2 = 0 \quad \sqrt{9-x^2} = 0$$

$$4x^2 = 18$$

$$x^2 = \frac{18}{4} = \frac{9}{2}$$

$$x = \pm \frac{3\sqrt{2}}{2}$$

$$x = \pm 3$$



Inc:  $(-\frac{3\sqrt{2}}{2}, \frac{3\sqrt{2}}{2})$

Dec:  $(-3, -\frac{3\sqrt{2}}{2}) \cup$

$(\frac{3\sqrt{2}}{2}, 3)$

### Motion Along a Line

In Exercises 81–84, the function  $s(t)$  describes the motion of a particle along a line. For each function, (a) find the velocity function of the particle at any time  $t \geq 0$ , (b) identify the time interval(s) in which the particle is moving in a positive direction, (c) identify the time interval(s) in which the particle is moving in a negative direction, and (d) identify the time(s) at which the particle changes direction.

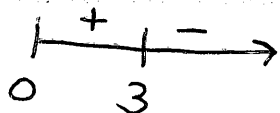
81.  $s(t) = 6t - t^2$

a)  $v(t) = s'(t) = 6 - 2t$

b) set  $v(t) = 0$

$$0 = 6 - 2t$$

$$2t = 6 \quad \underline{\underline{t = 3}}$$



particle moving right  $0 \leq t < 3$

c) particle moving left  $t > 3$

d) particle changes direction at  $t = 3$ .

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$$37. f(x) = \begin{cases} 4 - x^2, & x \leq 0 \\ -2x, & x > 0 \end{cases}$$

$$39. f(x) = \begin{cases} 3x + 1, & x \leq 1 \\ 5 - x^2, & x > 1 \end{cases}$$

### Finding and Analyzing Derivatives Using Technology

In Exercises 49–54, (a) use a computer algebra system to differentiate the function, (b) sketch the graphs of  $f$  and  $f'$  on the same set of coordinate axes over the given interval, (c) find the critical numbers of  $f$  in the open interval, and (d) find the interval(s) on which  $f'$  is positive and the interval(s) on which it is negative. Compare the behavior of  $f$  and the sign of  $f'$ .

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$$81. s(t) = 6t - t^2$$