Geometry 4.3a Congruent Triangles **Congruence and Corresponding Parts**

Fri (11/13)

Key

Congruent	Not Congruent
	
While positioned differently, Figures 1, 2, and 3 are exactly the same shape and size.	Figures 4 and 5 are exactly the same shape but not the same size. Figures 5 and 6 are the same size but not exactly the same shape.

In two congruent polygons, all parts of one polygon are congruent to the Corresponding parts or matching parts of another polygon. These *corresponding parts* include *corresponding angles* and *corresponding sides.*

KeyConcept Definition of Congruent Polygons

Words Two polygons are congruent if and only if their corresponding parts are congruent.

Example

Corresponding Angles

$\angle A \cong \angle H$ $\angle B \cong \angle J$ $\angle C \cong \angle K$

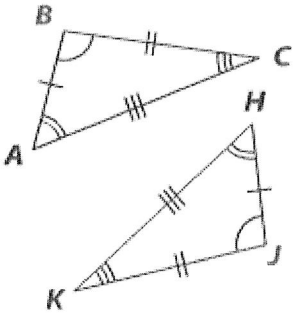
Corresponding Sides

$\overline{AB} \cong \overline{HJ}$ $\overline{BC} \cong \overline{JK}$ $\overline{AC} \cong \overline{HK}$

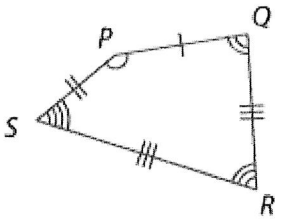
Congruence Statement

$\triangle ABC \cong \triangle HJK$

Model

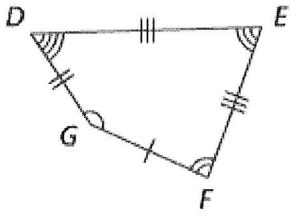


Show that the polygons are congruent by identifying all the congruent corresponding parts. Then write a congruence statement.



Angles: $\angle P \cong \angle G$, $\angle Q \cong \angle F$, $\angle R \cong \angle E$, $\angle S \cong \angle D$

Sides: $\overline{PQ} \cong \overline{GF}$, $\overline{QR} \cong \overline{FE}$, $\overline{RS} \cong \overline{ED}$, $\overline{SP} \cong \overline{DG}$

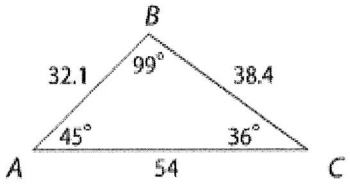


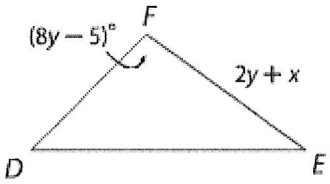
All corresponding parts of the two polygons are congruent. Therefore, polygon $PQRS \cong$ polygon $GFED$.

The phrase "if and only if" in the congruent polygon definition means that both the conditional and its converse are true. So, if two polygons are congruent, then their corresponding parts are congruent. For triangles, we say *Corresponding parts of congruent triangles are congruent*, or CPCTC.

In the diagram, $\triangle ABC \cong \triangle DFE$. Find the values of x and y.

Use Corresponding Parts of Congruent Triangles





$\angle F \cong \angle B$	CPCTC
$m\angle F = m\angle B$	Definition of congruence
$8y - 5 = 99$	Substitution
$8y = 104$	Add 5
$y = 13$	Divide by 8

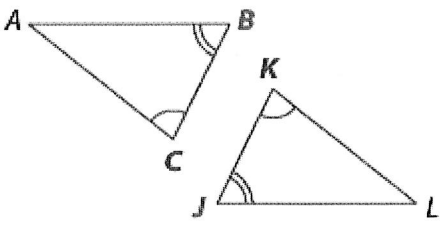
$\overline{FE} \cong \overline{BC}$	CPCTC
$FE = BC$	Definition of congruence
$2y + x = 38.4$	Substitution
$2(13) + x = 38.4$	Substitution
$26 + x = 38.4$	Simplify
$x = 12.4$	Subtract

Third Angles Theorem :

If two angles of one triangle are congruent to two angles of a second triangle, then the third angles of the triangles are congruent

If $\angle C \cong \angle K$ and $\angle B \cong \angle J$,

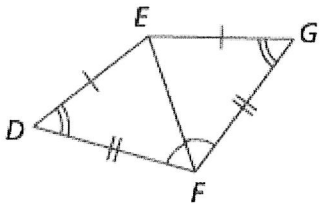
then $\angle A \cong \angle L$



Write a two-column proof.

Given: $\overline{DE} \cong \overline{GE}, \overline{DF} \cong \overline{GF}, \angle D \cong \angle G, \angle DFE \cong \angle GFE$

Prove: $\triangle DEF \cong \triangle GEF$

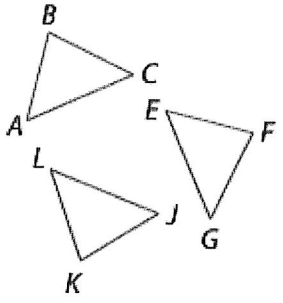


Proof:

Statements	Reasons
1) $\overline{DE} \cong \overline{GE}$, $\overline{DF} \cong \overline{GF}$	1) Given
2) $\overline{EF} \cong \overline{EF}$	2) Reflexive property of congruence
3) $\angle D \cong \angle G$, $\angle DFE \cong \angle GFE$	3) Given
4) $\angle DEF \cong \angle GEF$	4) Third Angles Theorem
5) $\triangle DEF \cong \triangle GEF$	5) Definition of Congruent Polygons

Properties of Triangle Congruence:

Reflexive Property of Congruence – $\triangle ABC \cong \triangle ABC$



Symmetric Property of Congruence – ABC ↔ EFG

If $\triangle ABC \cong \triangle EFG$,
then $\triangle EFG \cong \triangle ABC$

Transitive Property of Congruence –

If $A = B$ and $B = C$, then $A = C$

If $\triangle ABC \cong \triangle EFG$ and $\triangle EFG \cong \triangle JKL$,
then $\triangle ABC \cong \triangle JKL$

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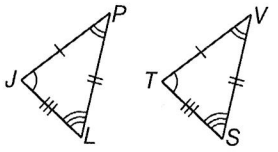
4-3 Skills Practice

Congruent Triangles

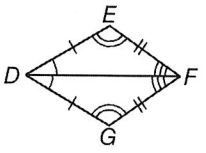
Show that polygons are congruent by identifying all congruent corresponding parts. Then write a congruence statement.

1.



$\angle J \cong \angle T$; $\angle P \cong \angle V$;
$\angle L \cong \angle S$; $\overline{JP} \cong \overline{TV}$
$\overline{PL} \cong \overline{VS}$; $\overline{JL} \cong \overline{TS}$
$\triangle JPL \cong \triangle TVS$

2.

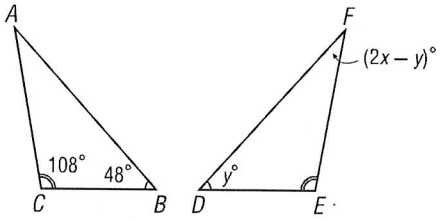


$\angle EDF \cong \angle GDF$; $\angle E \cong \angle G$;
$\angle EFD \cong \angle GFD$; $\overline{DE} \cong \overline{DG}$;
$\overline{GF} \cong \overline{EF}$; $\triangle DEF \cong \triangle DGF$

In the figure, $\triangle ABC \cong \triangle FDE$.

3. Find the value of x. **36**

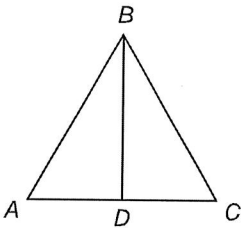
4. Find the value of y. **48**



5. PROOF Write a two-column proof.

Given: $\overline{AB} \cong \overline{CB}, \overline{AD} \cong \overline{CD}$, $\angle ABD \cong \angle CBD$, $\angle ADB \cong \angle CDB$

Prove: $\triangle ABD \cong \triangle CBD$



Proof:

Statements	Reasons
1. $\overline{AB} \cong \overline{CB}$, $\overline{AD} \cong \overline{CD}$	1. Given
2. $\angle ABD \cong \angle CBD$, $\angle ADB \cong \angle CDB$	2. Given
3. $\angle A \cong \angle C$	3. Third Angle Theorem
4. $\triangle ABD \cong \triangle CBD$	4. CPCTC

Lesson 4-3