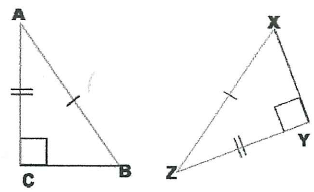
**Geometry** **Ch. 4.5 Notes - Triangle Congruence Day 2 (HL, ASA, AAS)** Nov 30, 2015 Key

**Hypotenuse Leg (HL):** a special case of (Side-Side-Angle) SSA

In a right triangle, you can find the measure of the third side if you know the measures of the other 2 sides by using the pythagorean theorem. Thus, creating a special case of SSA known as Hypotenuse-Leg Theorem. If the hypotenuse and a leg of one right triangle are congruent to the hypotenuse and a leg of another right triangle, then the two triangles are congruent.

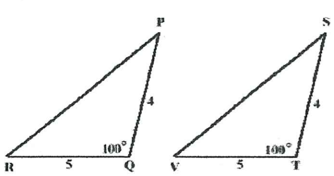



- Since the hypotenuse and leg of the right triangles are congruent, then $\Delta ABC \cong \Delta ZXY$ by the **Hypotenuse Leg (HL) Theorem.**
- Since $\Delta ABC \cong \Delta ZXY$ are congruent by Hypotenuse-Leg (HL) Theorem, then $BC = XY$, $\angle A = \angle Z$, and $\angle B = \angle X$ because CPCTC (corresponding parts of congruent triangles are congruent)

| RECAP | | | |
|---|---|---|---|
| **Side–Side–Side (SSS)** | Triangle congruence | **Angle–Angle–Angle (AAA)** | not enough info |
| **Side–Angle–Side (SAS)** | Triangle congruence | **Side– Side– Angle (SSA or ASS)** | not enough info |
| **Hypotenuse–Leg (HL)** | Triangle congruence | only works if right triangle | |

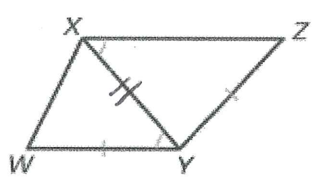
Examples: Determine if the triangles are congruent or not. If so, list the reason and a congruence statement. Also, list the congruent corresponding parts.

1.



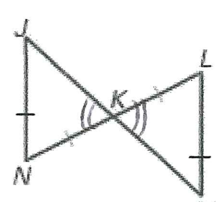

SAS $\Delta RQP \cong \Delta VTS$

2.




not enough info

3.



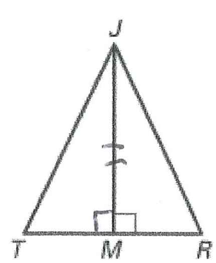

SSA - not enough info

4.


HL

$\Delta MON \cong \Delta RNO$

5.

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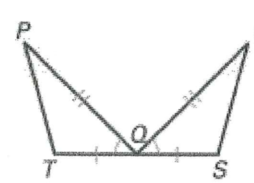
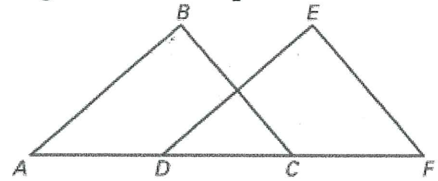
6.




SAS

State the third congruence that must be given to prove that $\Delta ABC \cong \Delta FED$ using the indicated postulate or theorem.




7. Given: $\overline{BC} \cong \overline{ED}$, $\overline{AC} \cong \overline{FD}$, ∠C ≅ ∠F using SAS.

8. Given: $\overline{AB} \cong \overline{FE}$, $\overline{AC} \cong \overline{FD}$, BC ≅ ED using SSS.

9. Given: $\overline{BC} \cong \overline{ED}$, ∠B is a right angle and ∠B ≅ ∠E, AC ≅ DF using HL.

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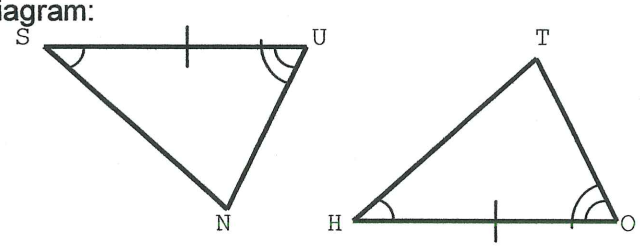
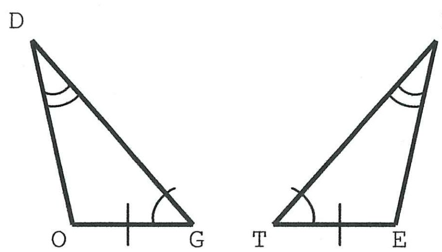
**Angle-Side-Angle Congruence Postulate (ASA ≅ Postulate):** If two angles and the included side of one triangle are congruent with two angles and the included side of another triangle, then the triangles are congruent.

Diagram:




**Angle-Angle-Side Congruence Theorem (AAS ≅ Theorem):** If two angles and a non-included side of one triangle are congruent with two angles and the non-included side of a second triangle, then the two triangles are congruent.

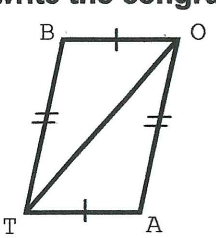
Diagram:




Remember, SSA ≅ and AAA ≅ are **NOT** a valid ways to prove triangles congruent.

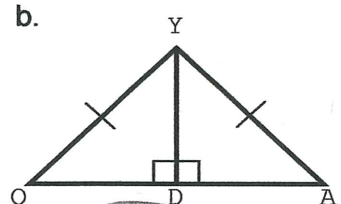
**Example 10:** Determine if there is enough information given in the figures to have two triangles congruent. If so, **write the congruence statement** and **name the postulate or theorem used**.

a.



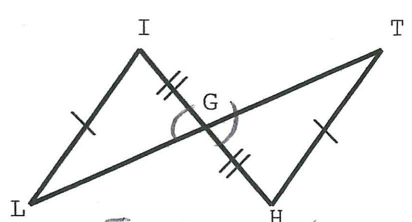

SSS ΔOTB ≅ ΔTOA

b.




HL ΔOYD ≅ ΔAYD

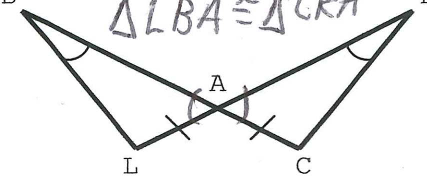
c.




SSA – not enough info

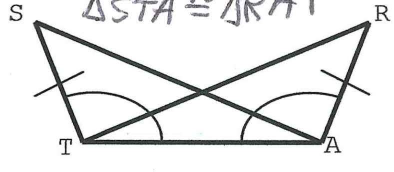
d. AAS

ΔLBA ≅ ΔCKA

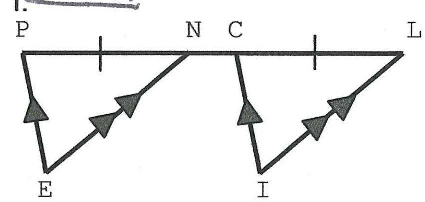



e. SAS

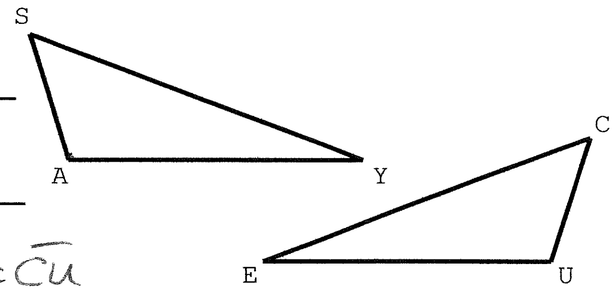
ΔSTA ≅ ΔRAT




f. ASA ΔPEN ≅ ΔCIL



**Example 11:** State the third congruence that is needed to prove that □SAY ≅ □CUE using the indicated postulate or theorem.

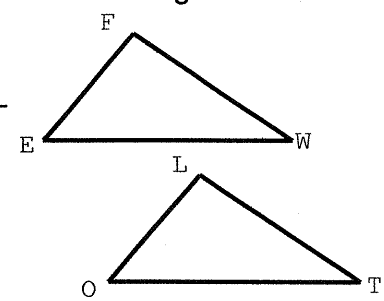



a. GIVEN: $\overline{SA} \cong \overline{CU}$, $\overline{SY} \cong \overline{CE}$, and AY ≅ EU
Use the SSS ≅ Postulate.

b. GIVEN: $\angle A \cong \angle U$, $\angle Y \cong \angle E$, and SY ≅ CE
Use the AAS ≅ Theorem.

or $\overline{SA} \cong \overline{CU}$

**Example 12:** State the third congruence that is needed to prove that □FEW ≅ □LOT using the indicated postulate or theorem.



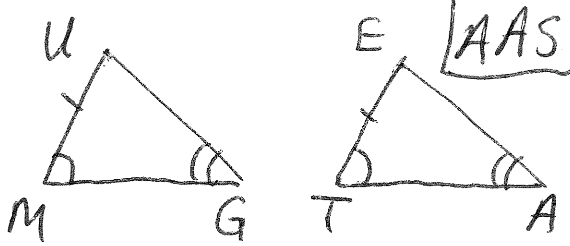

a. GIVEN: $\angle F$ is a right angle, $\angle F \cong \angle L$, $\overline{FE} \cong \overline{LO}$, and WE ≅ OT
Use the HL ≅ Theorem.

b. GIVEN: $\overline{EW} \cong \overline{OT}$, $\angle E \cong \angle O$, and $\angle W \cong \angle T$
Use the ASA ≅ Postulate.

**Example 13:** Tell whether you can use the given information to determine whether □MUG andΔ□TEA are congruent. *Explain* your reasoning. It might help to draw the given information.

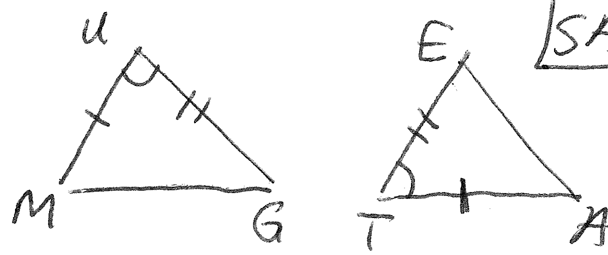
a. $\overline{MU} \cong \overline{TE}$, $\angle M \cong \angle T$, $\angle G \cong \angle A$

$\boxed{AAS}$

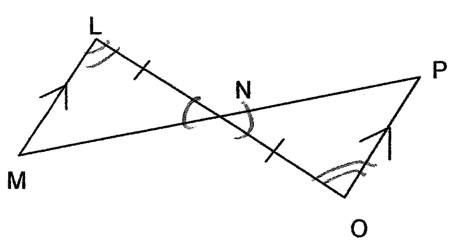



b. $\overline{MU} \cong \overline{TA}$, $\overline{UG} \cong \overline{TE}$, $\angle U \cong \angle T$

$\boxed{SAS}$




**Example 14: Proving Triangles are Congruent**




Given: $\overline{LM} \parallel \overline{OP}$, $\overline{LN} \cong \overline{NO}$
Prove: △MNL ≅ △PNO

| Statements | Reasons |
|---|---|
| 1. $LM \parallel OP$ | 1. Given |
| 2. $LN = NO$ | 2. Given |
| 3. $\angle LNM \cong \angle PNO$ | 3. vertical angles ≅ |
| 4. $\angle L \cong \angle O$ | 4. alt. interior angles ≅ |
| 5. $\triangle MNL \cong \triangle PNO$ | 5. ASA |

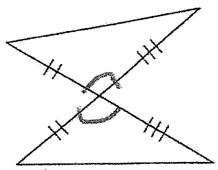
Analytic Geometry

Name____________________

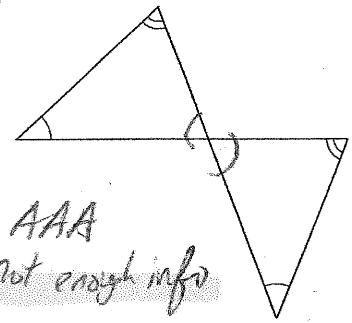
# Triangle congruence

Date__________ Period____

**State if the two triangles are congruent. If they are, state how you know.**

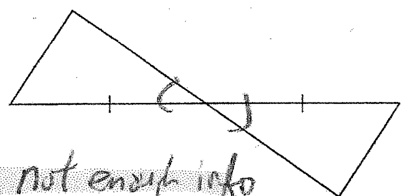
1) 

SAS

2) 

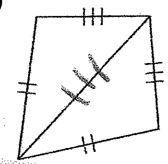
AAA
not enough info

Not congruent

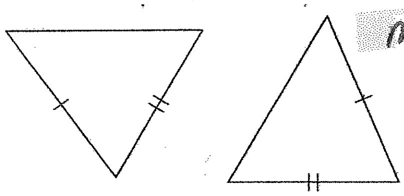
3) 

not enough info

Not congruent

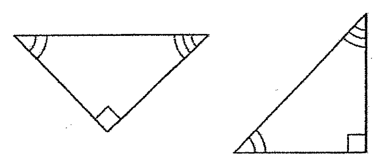
4) 

SSS

5) 

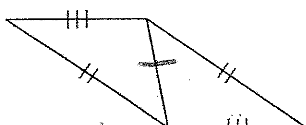
not enough info

Not congruent

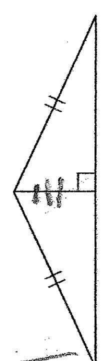
6) 

AAA
not enough info

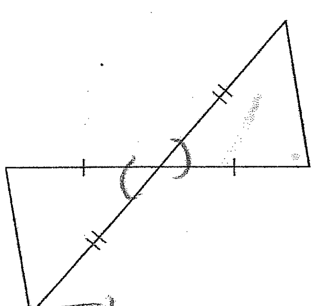
Not congruent

7) 

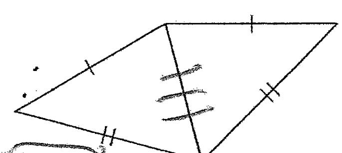
SSS

8) 

HL

9) 

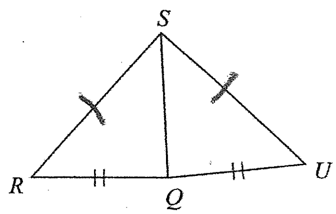
SAS

10) 

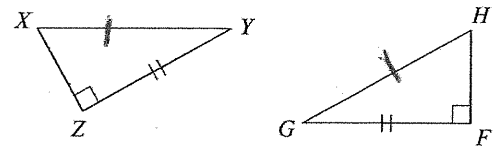
SSS

**State what additional information is required in order to know that the triangles are congruent for the reason given.**

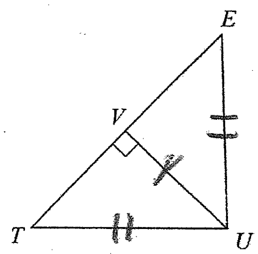
11) SSS $\overline{RS} \cong \overline{US}$



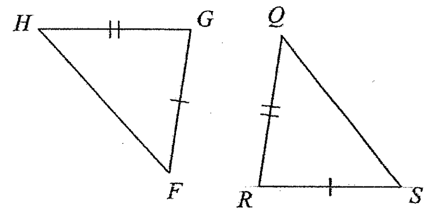

12) HL $\overline{YX} \cong \overline{GH}$



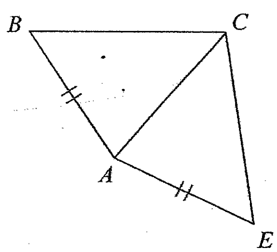

13) HL $\overline{UT} \cong \overline{UE}$



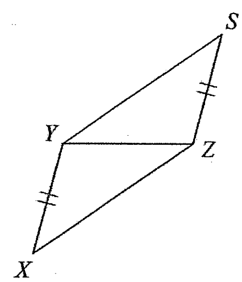

14) SAS $\angle G \cong \angle R$



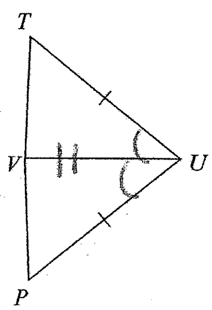

15) SSS $\overline{BC} \cong \overline{EC}$



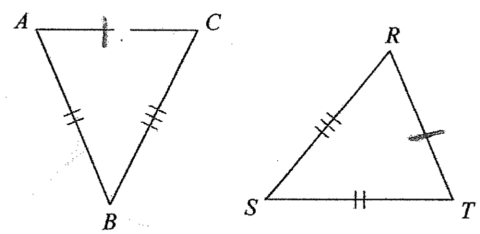

16) SAS $\angle XYZ \cong \angle SZY$



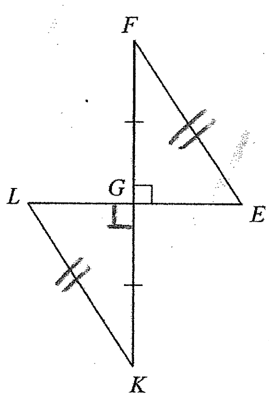

17) SAS $\angle TUV \cong \angle PUV$



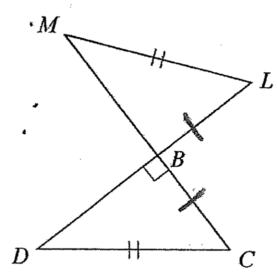

18) SSS $\overline{CA} \cong \overline{RT}$



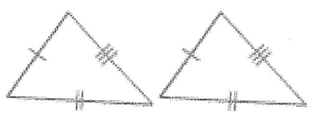
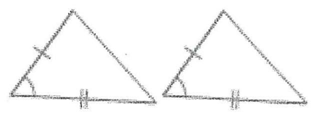
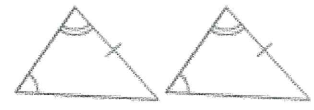

19) HL $\overline{FE} \cong \overline{KL}$




20) HL $\overline{BC} \cong \overline{BL}$ or $\overline{DB} \cong \overline{MB}$



**Geometry** **Ch. 4.5 Proving Triangle Congruence (ASA, AAS) Homework** Key

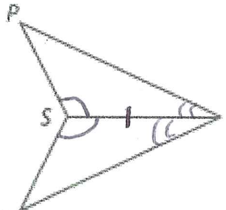
| SSS | SAS | ASA | AAS |
|---|---|---|---|
|  |  |  |  |
| Three pairs of corresponding sides are congruent. | Two pairs of corresponding sides and their included angles are congruent. | Two pairs of corresponding angles and their included sides are congruent. | Two pairs of corresponding angles and the corresponding nonincluded sides are congruent. |

**Example 1** Use ASA to Prove Triangles Congruent

**Write a two-column proof.**

**Given:** $\overline{QS}$ bisects $\angle PQR$; $\angle PSQ \cong \angle RSQ$.

**Prove:** $\triangle PQS \cong \triangle RQS$




**Proof:**

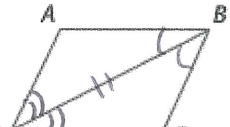
| Statements | Reasons |
|---|---|
| 1) $\overline{QS}$ bisects $\angle PQR$, $\angle PSQ = \angle RSQ$ | 1) Given |
| 2) $\angle PQS \cong \angle SQR$ | 2) Def. of Angle Bisector |
| 3) $\overline{SQ} = \overline{SQ}$ | 3) Reflexive property of congruence |
| 4) $\triangle PQS \cong \triangle RQS$ | 4) ASA |

**PROOF** **Write the specified type of proof.**

**1.** two-column proof

**Given:** $\overline{CB}$ bisects $\angle ABD$ and $\angle ACD$.

**Prove:** $\triangle ABC \cong \triangle DBC$

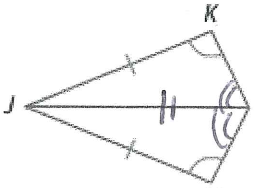



| Statement | Reason |
|---|---|
| 1) $\overline{CB}$ bisects $\angle ABD$ and $\angle ACD$ | 1) Given |
| 2) $\angle ABC = \angle CBD$ | 2) Def of Angle Bisector |
| 3) $BC = BC$ | 3) Reflexive property |
| 4) $\angle ACB = \angle BCD$ | 4) Def. of Angle Bisector |
| 5) $\triangle ABC = \triangle DBC$ | 5) ASA |

**3.** paragraph proof

**Given:** $\angle K \cong \angle M$, $\overline{JK} \cong \overline{JM}$, $\overline{JL}$ bisects $\angle KLM$.

**Prove:** $\triangle JKL \cong \triangle JML$

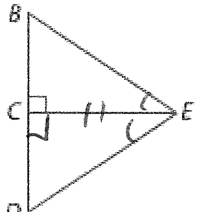



| Statement | Reason |
|---|---|
| 1) $\angle K = \angle M$, $JK = JM$, $JL$ bisects $\angle KLM$ | 1) Given |
| 2) $\angle KLJ = \angle JLM$ | 2) Def of Angle Bisector |
| 3) $JL = JL$ | 3) Reflexive property |
| 4) $\triangle JKL = \triangle JML$ | 4) AAS |

**PROOF** **Write a paragraph proof.**

**6. Given:** $\overline{CE}$ bisects $\angle BED$; $\angle BCE$ and $\angle ECD$ are right angles.

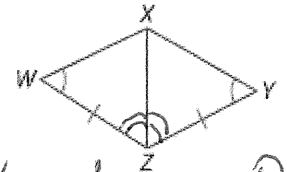
**Prove:** $\triangle ECB \cong \triangle ECD$




| Statement | Reason |
|---|---|
| 1) $\overline{CE}$ bisects $\angle BED$, $\angle BCE$ and $\angle ECD$ are right angles | 1) Given |
| 2) $\angle BEC \cong \angle DEC$ | 2) Def. of Angle bisector |
| 3) $\angle BCE = \angle DCE$ | 3) Right angles congruent |
| 4) $CE = CE$ | 4) Reflexive property |
| ) $\Delta ECB \cong \Delta ECD$ | 5) ASA |

**7. Given:** $\angle W \cong \angle Y$, $\overline{WZ} \cong \overline{YZ}$, $\overline{XZ}$ bisects $\angle WZY$.

**Prove:** $\triangle XWZ \cong \triangle XYZ$

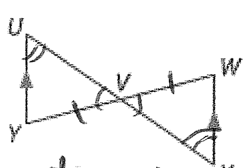



| Statement | Reason |
|---|---|
| 1) $\angle W \cong \angle Y$, $WZ = YZ$, $\overline{XZ}$ bisects $\angle WZY$ | 1) Given |
| 2) $\angle WZX = \angle YZX$ | 2) Def. of Angle Bisector |
| 3) $\Delta XWZ = \Delta XYZ$ | 3) ASA |

**PROOF** **Write a two-column proof.**

**9** **Given:** $V$ is the midpoint of $\overline{YW}$; $\overline{UY} \parallel \overline{XW}$.

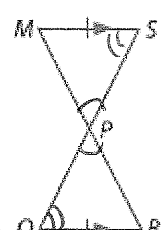
**Prove:** $\triangle UVY \cong \triangle XVW$




| Statement | Reason |
|---|---|
| 1) $V$ is midpt of $\overline{YW}$, $\overline{UY} \parallel XW$ | 1) Given |
| 2) $YV = VW$ | 2) Def. of midpt. |
| 3) $\angle UVY = \angle WVX$ | 3) Vertical angles $\cong$ |
| .) $\angle U \cong \angle X$ | 4) Alt. Interior angles $\cong$ |
| 5) $\Delta UVY = \Delta XVW$ | 5) AAS |

**10. Given:** $\overline{MS} \cong \overline{RQ}$, $\overline{MS} \parallel \overline{RQ}$

**Prove:** $\triangle MSP \cong \triangle RQP$




| Statement | Reason |
|---|---|
| 1) $MS = RQ$, $MS \parallel RQ$ | 1) Given |
| 2) $\angle MPS \cong \angle RPQ$ | 2) Vertical angles $\cong$ |
| 3) $\angle S \cong \angle Q$ | 3) Alt. Interior angles $\cong$ |
| 4) $\Delta MSP \cong \Delta RQP$ | 4) AAS |