Calculus **Chapter 4.5b** **U-Substitution Method for Definite Integrals**

U-Substitution with definite integrals: Be sure the bounds matches the variable of the problem

Ex. 1: $\int_{1}^{2} 2x(x^2-2)^3\,dx$

Ex. 2: $\int_{1}^{5} \frac{x}{\sqrt{2x-1}}dx$

Integrals of Odd and Even Functions

Review: Suppose $\int_{10}^{3} f(x)dx = 9$ and $\int_{-1}^{3} f(x)dx = 5$, find $\int_{-1}^{10} f(x)dx$

Even/Odd Rules:

Even: $\int_{-a}^{a} f(x)dx = 2\int_{0}^{a} f(x)dx$

Odd: $\int_{-a}^{a} f(x)dx = 0$

Ex. 3: Suppose g(x) is an even function where $\int_{0}^{3} g(x)dx = 2$ and $\int_{-4}^{-3} g(x)dx = 4$. Find $\int_{-4}^{3} g(x)dx$.

(Sketch a possible graph using the above given information)

Ex. 4: Same as Example 3, but g(x) is an odd function: $\int_{0}^{3} g(x)dx = 2$ and $\int_{-4}^{-3} g(x)dx = 4$. Find $\int_{-4}^{3} g(x)dx$.

Ex. 5: If f(x) is even and $\int_{3}^{6} f(x)dx = 7$ and $\int_{-6}^{3} f(x)dx = 12$, find $\int_{0}^{6} f(x)dx$

Key

U-Substitution with definite integrals: Be sure the bounds matches the variable of the problem

Ex. 1: $\int_1^2 2x(x^2-2)^3\,dx$

$u = x^2 - 2$

$\frac{du}{dx} = 2x$

$dx = \frac{du}{2x}$

$\int \cancel{2x} \cdot u^3 \cdot \frac{du}{\cancel{2x}}$

$\int u^3\,du$

Convert bounds:

if $x=1$, $u = 1^2 - 2 = -1$

if $x=2$, $u = 2^2 - 2 = 2$

$\int_{-1}^{2} u^3\,du$

$= \frac{u^4}{4}\Big]_{-1}^{2} = \frac{2^4}{4} - \left(\frac{(-1)^4}{4}\right) = \frac{16}{4} - \frac{1}{4}$

$= \boxed{\frac{15}{4}}$

OR:

$\int u^3\,du = \frac{u^4}{4} = \frac{(x^2-2)^4}{4}\Big]_{-1}^{2}$

$= \frac{16}{4} - \frac{1}{4} = \boxed{\frac{15}{4}}$

Ex. 2: $\int_1^5 \frac{x}{\sqrt{2x-1}}dx$

$u = 2x - 1$

$\frac{du}{dx} = 2$

$dx = \frac{du}{2}$

$\int \frac{x}{u^{1/2}} \cdot \frac{du}{2}$

* Need to use change of variable method:

$u = 2x - 1$

$\frac{u+1}{2} = x$

$\int \frac{\frac{u+1}{2}}{u^{1/2}} \cdot \frac{du}{2}$

$\frac{1}{4}\int (u+1)u^{-1/2}\,du$

$\frac{1}{4}\int u^{1/2} + u^{-1/2}\,du$

If $x=1$, $u = 2(1)-1 = 1$

if $x=5$, $u = 2(5)-1 = 9$

$\frac{1}{4}\frac{u^{3/2}}{3/2} + \frac{1}{4}\frac{u^{1/2}}{1/2}$

$\frac{1}{6}u^{3/2} + \frac{1}{2}u^{1/2}\Big]_1^9 = \frac{1}{6}(9)^{3/2} + \frac{1}{2}(9)^{1/2} - \left(\frac{1}{6} + \frac{1}{2}\right)$

$= \frac{1}{6}(27) + \frac{1}{2}(3) - \frac{1}{6} - \frac{1}{2}$

$= \boxed{\frac{16}{3}}$

OR $\frac{1}{6}u^{3/2} + \frac{1}{2}u^{1/2}$

$= \frac{1}{6}(2x-1)^{3/2} + \frac{1}{2}(2x-1)^{1/2}\Big]_1^5$

$= \frac{1}{6}(9)^{3/2} + \frac{1}{2}(9)^{1/2} - \left(\frac{1}{6} + \frac{1}{2}\right) = \boxed{\frac{16}{3}}$

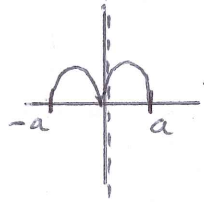
Integrals of Odd and Even Functions

Review: Suppose $\int_{10}^{3} f(x)dx = 9$ and $\int_{-1}^{3} f(x)dx = 5$, find $\int_{-1}^{10} f(x)dx$

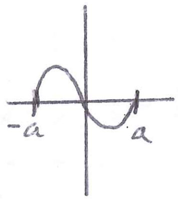
$$\int_{-1}^{10} f(x)dx = \int_{-1}^{3} f(x)dx + \int_{3}^{10} f(x)dx = 5 + (-9) = \boxed{-4}$$

Even/Odd Rules:

Even: $\int_{-a}^{a} f(x)dx = 2\int_{0}^{a} f(x)dx$

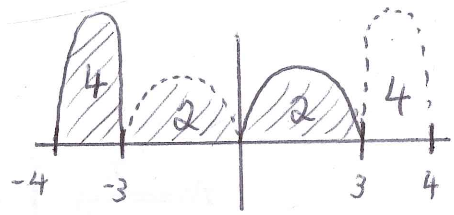


Odd: $\int_{-a}^{a} f(x)dx = 0$



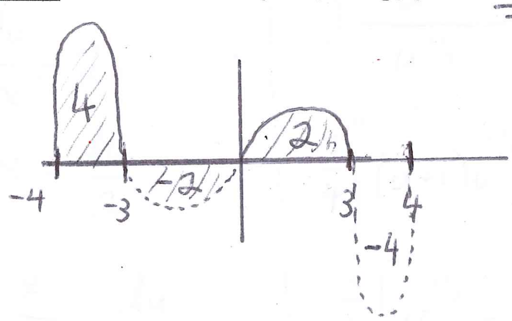
Ex. 3: Suppose g(x) is an even function where $\int_{0}^{3} g(x)dx = 2$ and $\int_{-4}^{-3} g(x)dx = 4$. Find $\int_{-4}^{3} g(x)dx$.

(Sketch a possible graph using the above given information)



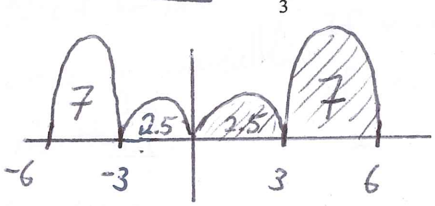
$$\int_{-4}^{3} g(x)dx = 4 + 2 + 2 = \boxed{8}$$

Ex. 4: Same as Example 3, but g(x) is an odd function: $\int_{0}^{3} g(x)dx = 2$ and $\int_{-4}^{-3} g(x)dx = 4$. Find $\int_{-4}^{3} g(x)dx$.



$$\int_{-4}^{3} g(x)dx = \boxed{4}$$

Ex. 5: If f(x) is even and $\int_{3}^{6} f(x)dx = 7$ and $\int_{-6}^{3} f(x)dx = 12$, find $\int_{0}^{6} f(x)dx$



$$\int_{0}^{6} f(x)dx = 2.5 + 7 = \boxed{9.5}$$