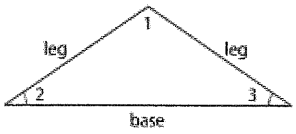
Geometry **Ch. 4.6 Isosceles and Equilateral Triangles**

Properties of Isosceles Triangles:

The two congruent sides are called the **legs of an isosceles triangle**, and the angle with sides that are the legs is called the **vertex angle**. The side of the triangle opposite the vertex angle is called the *base*. The two angles formed by the base and the congruent sides are called the **base angles**.

$\angle 1$ is the vertex angle.

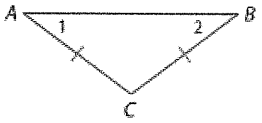
$\angle 2$ and $\angle 3$ are the base angles.



Theorems Isosceles Triangle

4.10 Isosceles Triangle Theorem
If two sides of a triangle are congruent, then the angles opposite those sides are congruent.

Example If $\overline{AC} \cong \overline{BC}$, then $\angle 2 \cong \angle 1$.



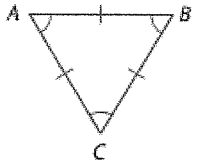
4.11 Converse of Isosceles Triangle Theorem
If two angles of a triangle are congruent, then the sides opposite those angles are congruent.

Example If $\angle 1 \cong \angle 2$, then $\overline{FE} \cong \overline{DE}$.

Corollaries Equilateral Triangle

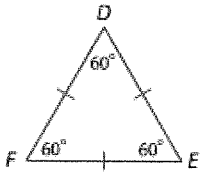
4.3 A triangle is equilateral if and only if it is equiangular.

Example If $\angle A \cong \angle B \cong \angle C$, then $\overline{AB} \cong \overline{BC} \cong \overline{CA}$.



4.4 Each angle of an equilateral triangle measures 60.

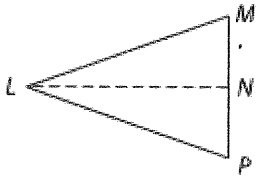
Example If $\overline{DE} \cong \overline{EF} \cong \overline{FE}$, then $m\angle A = m\angle B = m\angle C = 60$.



Proof Isosceles Triangle Theorem

Given: $\triangle LMP$; $\overline{LM} \cong \overline{LP}$

Prove: $\angle M \cong \angle P$

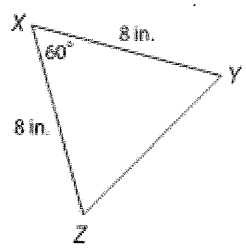


Proof:

Statements	Reasons

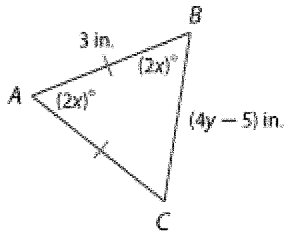
Find each measure.

a. $m\angle Y$



b. YZ

Find the value of each variable:



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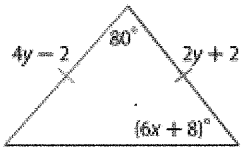
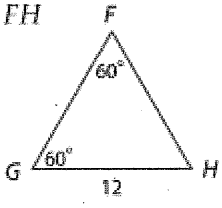
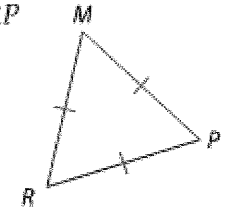


Find each measure.

3. FH

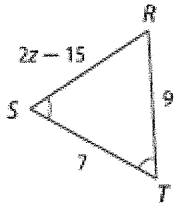


4. $m\angle MRP$



CCSS **SENSE-MAKING** **Find the value of each variable.**

5.



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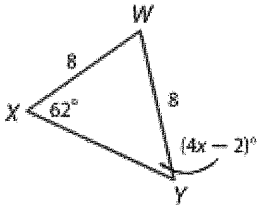
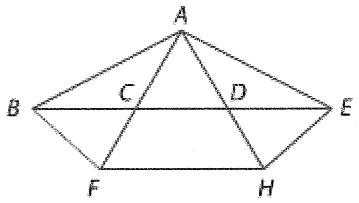


Refer to the figure at the right.



9 If $\overline{AB} \cong \overline{AE}$, name two congruent angles.

10. If $\angle ABF \cong \angle AFB$, name two congruent segments.

11. If $\overline{CA} \cong \overline{DA}$, name two congruent angles.

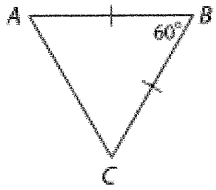
12. If $\angle DAE \cong \angle DEA$, name two congruent segments.

13. If $\angle BCF \cong \angle BFC$, name two congruent segments.

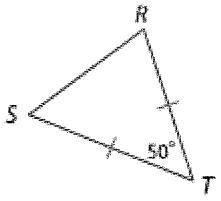
14. If $\overline{FA} \cong \overline{AH}$, name two congruent angles.

Find each measure.

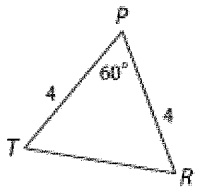
15. $m\angle BAC$



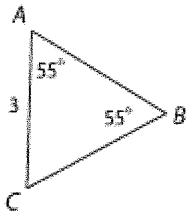
16. $m\angle SRT$



17. TR

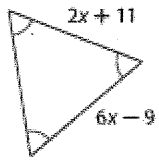


18. CB



CCSS **REGULARITY** **Find the value of each variable.**

19



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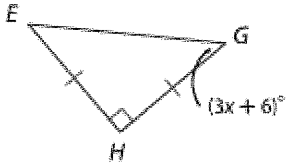

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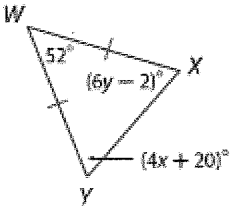

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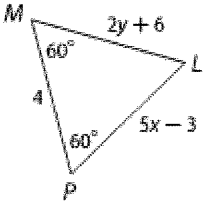
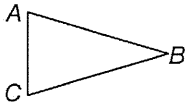


4-6 WorkSheet

Isosceles and Equilateral Triangles

Properties of Isosceles Triangles An **isosceles triangle** has two congruent sides called the ***legs***. The angle formed by the legs is called the **vertex angle**. The other two angles are called **base angles**. You can prove a theorem and its converse about isosceles triangles.

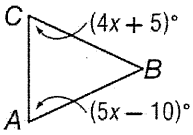
- If two sides of a triangle are congruent, then the angles opposite those sides are congruent. **(Isosceles Triangle Theorem)**
- If two angles of a triangle are congruent, then the sides opposite those angles are congruent. **(Converse of Isosceles Triangle Theorem)**



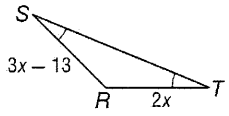
If $\overline{AB} \cong \overline{CB}$, then $\angle A \cong \angle C$.

If $\angle A \cong \angle C$, then $\overline{AB} \cong \overline{CB}$.

Example 1: Find x, given $\overline{BC} \cong \overline{BA}$.



Example 2: Find x.



Exercises

ALGEBRA Find the value of each variable.

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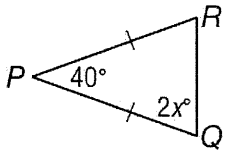

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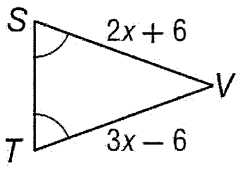

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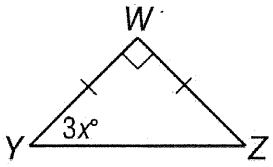

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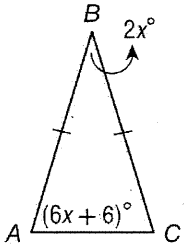

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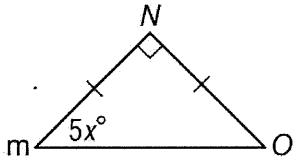

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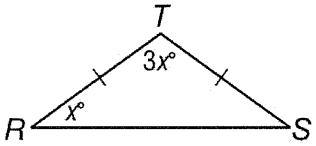
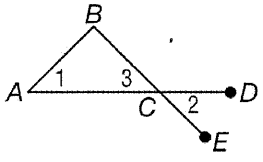


7. PROOF Write a two–column proof.

Given: $\angle 1 \cong \angle 2$

Prove: $\overline{AB} \cong \overline{CB}$

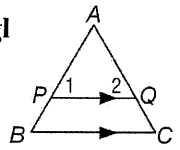


4-6 Worksheet *(continued)*
Isosceles and Equilateral Triangles

Properties of Equilateral Triangles An **equilateral triangle** has three congruent sides. The Isosceles Triangle Theorem leads to two corollaries about equilateral triangles.

1. A triangle is equilateral if and only if it is equiangular.
2. Each angle of an equilateral triangle measures 60°.

Example: Prove that if a line is parallel to one side of an equilateral triangl then it forms another equilateral triangle.



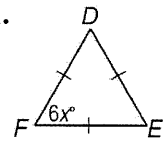
Proof:

Statements	Reasons
1. $\triangle ABC$ is equilateral; $\overline{PQ} \parallel \overline{BC}$.	**1.** Given
2. $m\angle A = m\angle B = m\angle C = 60$	**2.**
3. $\angle 1 \cong \angle B$, $\angle 2 \cong \angle C$	**3.**
4. $m\angle 1 = 60$, $m\angle 2 = 60$	**4.**
5. $\triangle APQ$ is equilateral.	**5.**

Exercises

ALGEBRA Find the value of each variable.

1.



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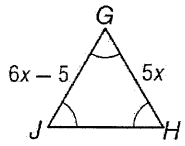

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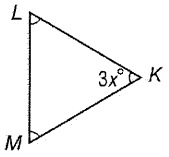

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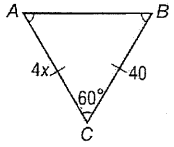

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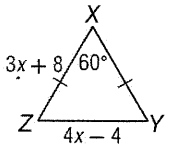

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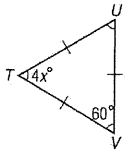
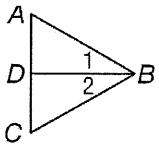


7. PROOF Write a two-column proof.
Given: $\triangle ABC$ is equilateral; $\angle 1 \cong \angle 2$.
Prove: $\angle ADB \cong \angle CDB$



4-6 WorkSheet

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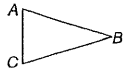

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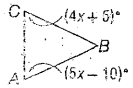
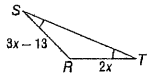
Example 1: Find x, given $\overline{BC} \cong \overline{BA}$.



Example 2: Find x.



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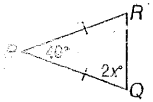
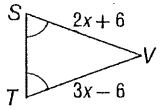
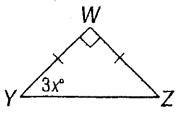
1. 

2. 

3. 

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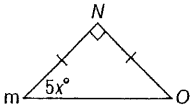
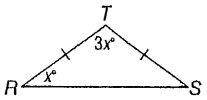
5. 

6. 

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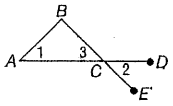
Prove: $\overline{AB} \cong \overline{CB}$



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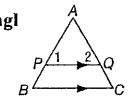
2. Each angle of an equilateral triangle measures 60°.

Example: Prove that if a line is parallel to one side of an equilateral triangl then it forms another equilateral triangle.

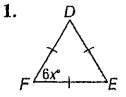


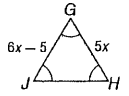
Proof:

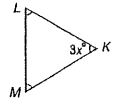
Statements	Reasons
1. $\triangle ABC$ is equilateral; $\overline{PQ} \parallel \overline{BC}$.	1. Given
2. $m\angle A = m\angle B = m\angle C = 60$	2.
3. $\angle 1 \cong \angle B$, $\angle 2 \cong \angle C$	3.
4. $m\angle 1 = 60$, $m\angle 2 = 60$	4.
5. $\triangle APQ$ is equilateral.	5.

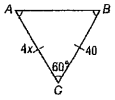
Exercises

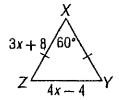
ALGEBRA Find the value of each variable.

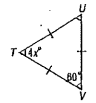
1. 

2. 

3. 

4. 

5. 

6. 

7. **PROOF** Write a two-column proof.
Given: $\triangle ABC$ is equilateral; $\angle 1 \cong \angle 2$.
Prove: $\angle ADB \cong \angle CDB$