Key

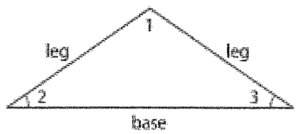
Geometry **Ch. 4.6 Isosceles and Equilateral Triangles**

Properties of Isosceles Triangles:

The two congruent sides are called the **legs of an isosceles triangle**, and the angle with sides that are the legs is called the **vertex angle**. The side of the triangle opposite the vertex angle is called the *base*. The two angles formed by the base and the congruent sides are called the **base angles**.

$\angle 1$ is the vertex angle.

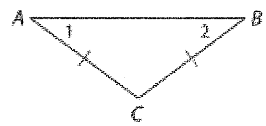
$\angle 2$ and $\angle 3$ are the base angles.



Theorems Isosceles Triangle

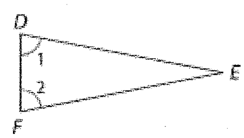
4.10 Isosceles Triangle Theorem
If two sides of a triangle are congruent, then the angles opposite those sides are congruent.

Example If $\overline{AC} \cong \overline{BC}$, then $\angle 2 \cong \angle 1$.



4.11 Converse of Isosceles Triangle Theorem
If two angles of a triangle are congruent, then the sides opposite those angles are congruent.

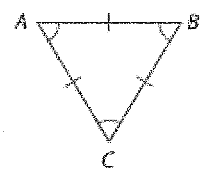
Example If $\angle 1 \cong \angle 2$, then $\overline{FE} \cong \overline{DE}$.



Corollaries Equilateral Triangle

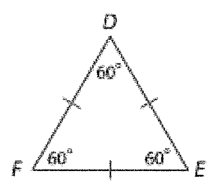
4.3 A triangle is equilateral if and only if it is equiangular.

Example If $\angle A \cong \angle B \cong \angle C$, then $\overline{AB} \cong \overline{BC} \cong \overline{CA}$.



4.4 Each angle of an equilateral triangle measures 60.

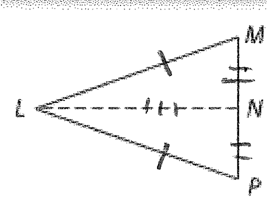
Example If $\overline{DE} \cong \overline{EF} \cong \overline{FE}$, then $m\angle A = m\angle B = m\angle C = 60$.



Proof Isosceles Triangle Theorem

Given: $\triangle LMP$; $\overline{LM} \cong \overline{LP}$, N is midpt. of MP

Prove: $\angle M \cong \angle P$

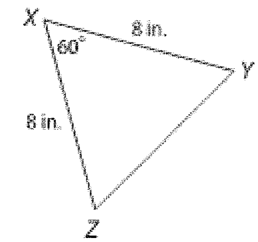


Proof:

Statements	Reasons
1) $\triangle LMP$ $\overline{LM} = \overline{LP}$, N is midpt of MP	1) Given
2) $MN = NP$	2) Def. of midpt.
3) $\overline{LN} = \overline{LN}$	3) Reflexive property
4) $\triangle LMN = \triangle LPN$	4) SSS
5) $\angle M = \angle P$	5) CPCTC

Geometry Ch. 4.6 Isosceles and Equilateral Triangles Homework

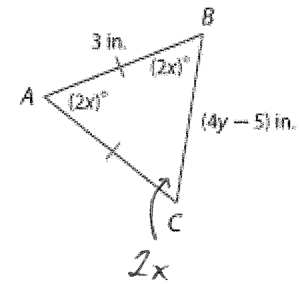
Find each measure.



a. $m\angle Y$ $= \boxed{60°}$

b. YZ $\approx \boxed{8 \text{ in.}}$

Find the value of each variable:



$2x + 2x + 2x = 180$

$6x = 180$

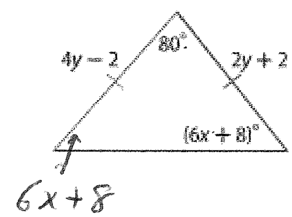
$\boxed{x = 30}$

$3 = 4y - 5$

$8 = 4y$

$\boxed{2 = y}$

Find the value of each variable:



$6x + 8 + 6x + 8 + 80 = 180$

$12x + 96 = 180$

$12x = 84$

$\boxed{x = 7}$

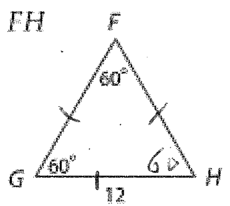
$4y - 2 = 2y + 2$

$2y = 4$

$\boxed{y = 2}$

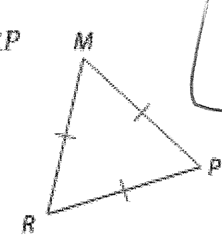
Find each measure.

3. FH



$\boxed{FH = 12}$

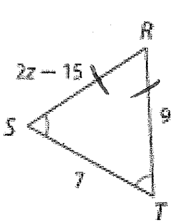
4. $m\angle MRP$



$\boxed{m\angle MRP = 60°}$

CCSS SENSE-MAKING Find the value of each variable.

5.

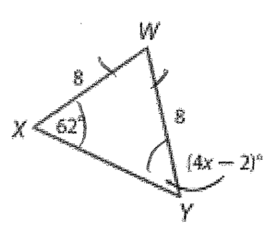


$2z - 15 = 9$

$2z = 24$

$\boxed{z = 12}$

6.

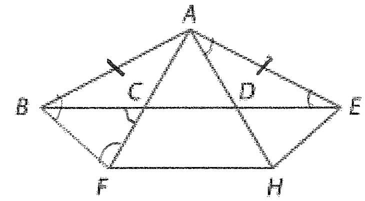


$4x - 2 = 62$

$4x = 64$

$\boxed{x = 16}$

Refer to the figure at the right.



9 If $\overline{AB} \cong \overline{AE}$, name two congruent angles. $\angle ABE \cong \angle AEB$

10. If $\angle ABF \cong \angle AFB$, name two congruent segments. $AB = AF$

11. If $\overline{CA} \cong \overline{DA}$, name two congruent angles. $\angle ACE = \angle ADB$

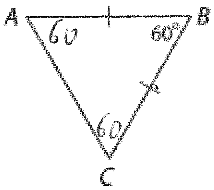
12. If $\angle DAE \cong \angle DEA$, name two congruent segments. $AD = DE$

13. If $\angle BCF \cong \angle BFC$, name two congruent segments. $BC = BF$

14. If $\overline{FA} \cong \overline{AH}$, name two congruent angles. $\angle AFH = \angle AHF$

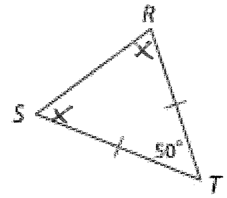
Find each measure.

15. $m\angle BAC$



$\boxed{60°}$

16. $m\angle SRT$

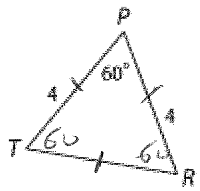


$50 + 2x = 180$

$2x = 130$

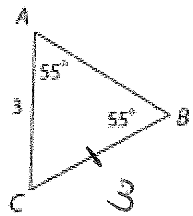
$\boxed{x = 65°}$

17. TR



$\boxed{TR = 4}$

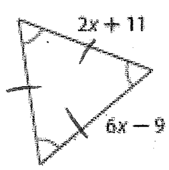
18. CB



$\boxed{CB = 3}$

CCSS REGULARITY Find the value of each variable.

19

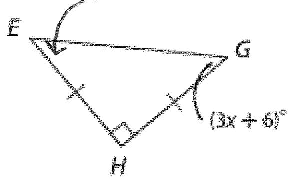


$6x - 9 = 2x + 11$

$4x = 20$

$\boxed{x = 5}$

20.



$3x + 6 + 3x + 6 + 90 = 180$

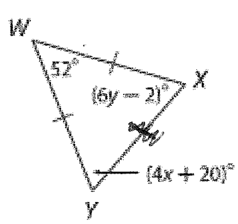
$6x + 12 + 90 = 180$

$6x + 102 = 180$

$6x = 78$

$\boxed{x = 13}$

21.



$4x + 20 = 6y - 2$

$4x - 6y = -22$

$4x + 20 + 6y - 2 + 52 = 180$

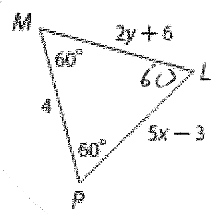
$4x + 6y = 110$

$4x + 6y = 110$

$4x - 6y = -22$

$8x = 88$ $\boxed{x = 11}$ $\boxed{y = 11}$

22.



$2y + 6 = 4$

$2y = -2$

$\boxed{y = -1}$

$5x - 3 = 4$

$5x = 7$

$\boxed{x = 7/5}$

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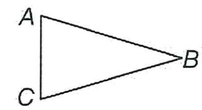
4-6 Study Guide and Intervention

Isosceles and Equilateral Triangles

Properties of Isosceles Triangles An **isosceles triangle** has two congruent sides called the ***legs***. The angle formed by the legs is called the **vertex angle**. The other two angles are called **base angles**. You can prove a theorem and its converse about isosceles triangles.

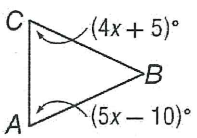
- If two sides of a triangle are congruent, then the angles opposite those sides are congruent. **(Isosceles Triangle Theorem)**
- If two angles of a triangle are congruent, then the sides opposite those angles are congruent. **(Converse of Isosceles Triangle Theorem)**



If $\overline{AB} \cong \overline{CB}$, then $\angle A \cong \angle C$.

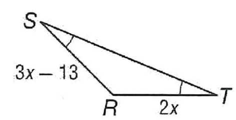
If $\angle A \cong \angle C$, then $\overline{AB} \cong \overline{CB}$.

Example 1: Find x, given $\overline{BC} \cong \overline{BA}$.



$BC = BA$, so	
$m\angle A = m\angle C$	Isos. Triangle Theorem
$5x - 10 = 4x + 5$	Substitution
$x - 10 = 5$	subtract $4x$ from each side.
$x = 15$	Add 10 to each side.

Example 2: Find x.

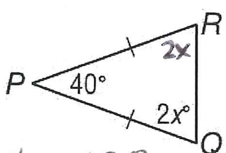


$m\angle S = m\angle T$, so	
$SR = TR$	Converse of Isos. ΔThm.
$3x - 13 = 2x$	Substitution
$3x = 2x + 13$	Add 13 to each side.
$x = 13$	Subtract $2x$ from each side.

Exercises

ALGEBRA Find the value of each variable.

1.

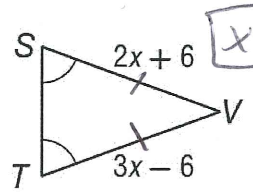


$4x+40=180$

$4x=140$

$\boxed{x=35}$

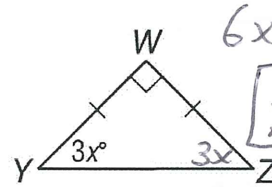
2.



$3x-6=2x+6$

$\boxed{x=12}$

3.

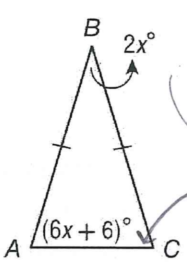


$6x+90=180$

$6x=90$

$\boxed{x=15}$

4.



$14x+12=180$

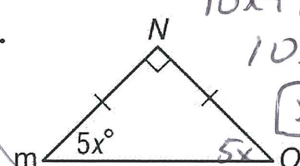
$14x=168$

$\boxed{x=12}$

6x+6

$2x+6x+6+6x+6=180$

5.

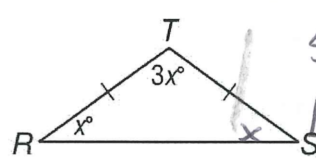


$10x+90=180$

$10x=90$

$\boxed{x=9}$

6.



$3x+2x=180$

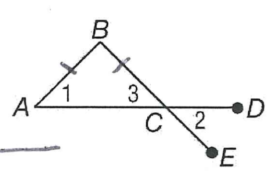
$5x=180$

$\boxed{x=36}$

7. PROOF Write a two–column proof.

Given: $\angle 1 \cong \angle 2$

Prove: $\overline{AB} \cong \overline{CB}$



Statement	Reason
1) $\angle 1 \cong \angle 2$	1) Given
2) $\angle 3 = \angle 2$	2) Vertical angles $\cong$
3) $\angle 1 = \angle 3$	3) Transitive property (substitution)

Statement	Reason
4) $\overline{AB} = \overline{CB}$	5) Def. of Isosceles Triangle Theorem

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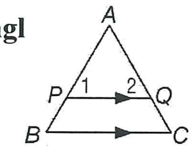
4-6 Study Guide and Intervention *(continued)*

Isosceles and Equilateral Triangles

Properties of Equilateral Triangles An **equilateral triangle** has three congruent sides. The Isosceles Triangle Theorem leads to two corollaries about equilateral triangles.

1. A triangle is equilateral if and only if it is equiangular. **2.** Each angle of an equilateral triangle measures 60°.

Example: Prove that if a line is parallel to one side of an equilateral triangl then it forms another equilateral triangle.

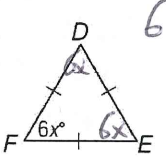


Proof:

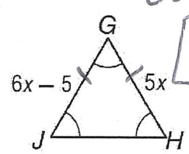
Statements	**Reasons**
1. $\triangle ABC$ is equilateral; $\overline{PQ} \parallel \overline{BC}$.	**1.** Given
2. $m\angle A = m\angle B = m\angle C = 60$	**2.** Each $\angle$ of an equilateral $\triangle$ measures 60°.
3. $\angle 1 \cong \angle B$, $\angle 2 \cong \angle C$	**3.** If $\parallel$ lines, then corres. $\angle$s are $\cong$.
4. $m\angle 1 = 60$, $m\angle 2 = 60$	**4.** Substitution
5. $\triangle APQ$ is equilateral.	**5.** If a $\triangle$ is equiangular, then it is equilateral.

Exercises

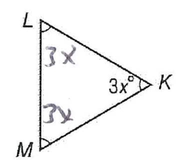
ALGEBRA Find the value of each variable.

1. 

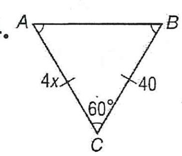
6x +6x+6x =180
18x =180
$\boxed{x=10}$

2. 

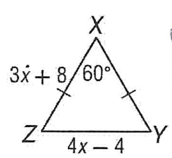
6x−5=5x
$\boxed{x=5}$

3. 

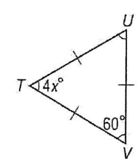
9x=180
$\boxed{x=20}$

4. 

4x=40
$\boxed{x=10}$

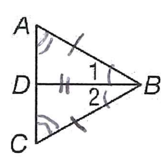
5. 

4x−4=3x+8
$\boxed{x=12}$

6. 

4x=60
$\boxed{x=15}$

7. PROOF Write a two-column proof.
Given: $\triangle ABC$ is equilateral; $\angle 1 \cong \angle 2$.
Prove: $\angle ADB \cong \angle CDB$



Statement	Reason
1) $\triangle ABC$ is equilateral $\angle 1 = \angle 2$	1) Given
2) $\angle A = \angle C$	2) property of equilateral/equiangular triangle
3) BD = BD	3) Reflexive Property
4) $\triangle ADB = \triangle CDB$	4) ASA
5) $\angle ADB = \angle CDB$	5) CPCTC