Key

Calculus **Ch. 5.2 Natural Log Integrals**

What would happen if we attempted to do power rule for this problem?

$$\int \frac{1}{x}dx \rightarrow \int x^{-1}dx = \cancel{\frac{x^0}{0}} + C$$

← undefined term! Power Rule would fail.

Recall Derivative Rule:

$$\frac{d}{dx}\ln u = \frac{u'}{u}$$

Natural Log Integral Rule

$$\int \frac{1}{u}du = \ln|u| + C$$

* Reminder: If "u" is raised to any other power, we will bring variable to the top and apply power rule.

***Example* 1:** $\int \frac{2x}{x^2+1}dx$

$u = x^2+1$ | $du = 2x\,dx$

$\frac{du}{dx} = 2x$ | $\frac{du}{2x} = dx$

$$\int \frac{2x}{x^2+1}dx$$

$$\int \frac{\cancel{2x}}{u}\cdot\frac{du}{\cancel{2x}}$$

$\rightarrow \int \frac{1}{u}du$

$\rightarrow \ln|u| + C$

$\rightarrow \boxed{\ln|x^2+1| + C}$

***Example* 2:** $\int \frac{1}{x\ln x}dx$

$u = \ln x$ | $x\,du = dx$

$\frac{du}{dx} = \frac{1}{x}$

$$\int \frac{1}{x\ln x}dx$$

$$\int \frac{1}{\cancel{x}\cdot u}\cdot \cancel{x}du$$

$$\int \frac{1}{u}du = \ln|u| + C$$

$$= \boxed{\ln|\ln x| + C}$$

***Example* 3:** $\int \frac{x^2+x+1}{x^2+1}dx$

$$\begin{array}{r} 1 \quad + \frac{x}{x^2+1} \\ x^2+1 \overline{)\, x^2+1+x} \\ \ominus x^2 \ominus 1 \\ \hline x \end{array}$$

← remainder

$$\int 1\,dx + \int \frac{x}{x^2+1}dx$$

power Rule

u-sub

$u = x^2+1$

$\frac{du}{dx} = 2x$

$dx\,2x = du$

$dx = \frac{du}{2x}$

$$\int \frac{x}{u}\cdot\frac{du}{2x}$$

$$\frac{1}{2}\int \frac{1}{u}du = \frac{1}{2}\ln|u| + C$$

$$\boxed{x + \frac{1}{2}\ln|x^2+1| + C}$$

Ch. 5.2 More Trig Integral Rules:

1) $\int \tan u \, du = -\ln|\cos u| + C$

2) $\int \cot u \, du = \ln|\sin u| + C$

3) $\int \sec u \, du = \ln|\sec u + \tan u| + C$

4) $\int \csc u \, du = -\ln|\csc u + \cot u| + C$

Example 4:

a) $\int \tan x \, dx$

$\int \frac{\sin x}{\cos x} dx$

$u = \cos x$

$\frac{du}{dx} = -\sin x$

$-dx \sin x = du$

$dx = \frac{du}{-\sin x}$

$\int \frac{\cancel{\sin x}}{u} \cdot \frac{du}{-\cancel{\sin x}}$

$-\int \frac{1}{u} du = -\ln|u| + C$

$\rightarrow \boxed{-\ln|\cos x| + C}$

b) $\int \cot x \, dx$

$\int \frac{\cos x}{\sin x} dx$

$u = \sin x$

$\frac{du}{dx} = \cos x$

$dx \cos x = du$

$dx = \frac{du}{\cos x}$

$\int \frac{\cancel{\cos x}}{u} \cdot \frac{du}{\cancel{\cos x}}$

$\int \frac{1}{u} du = \ln|u| + C$

$\boxed{= \ln|\sin x| + C}$

Example 5: (method 1)- long division

$\int \frac{x^3 - 6x - 20}{x+5} dx$

$$\begin{array}{r} x^2 - 5x + 19 - \frac{115}{x+5} \\ (x+5)\overline{)x^3 - 6x - 20} \\ -x^3 + 5x^2 \\ \hline -5x^2 - 6x - 20 \\ +5x^2 + 25x \\ \hline 19x - 20 \\ -19x - 95 \\ \hline -115 \end{array}$$

Remainder

$\int x^2 - 5x + 19 - \frac{115}{x+5} dx$

$\boxed{\frac{x^3}{3} - \frac{5x^2}{2} + 19x - 115\ln|x+5| + C}$

$-\int \frac{115}{x+5} dx$

$u = x+5$

$\frac{du}{dx} = 1$

$dx = du$

$-115 \int \frac{1}{u} du$

$\rightarrow -115 \ln|u|$

$\rightarrow -115 \ln|x+5|$

(Example 5: Method 2) – synthetic division

$\int \frac{x^3 - 6x - 20}{x+5} dx$

$$\begin{array}{r|rrrr} -5 & 1 & 0 & -6 & -20 \\ & \downarrow & -5 & 25 & -95 \\ \hline & 1 & -5 & 19 & -115 \end{array}$$

$\int x^2 - 5x + 19 - \frac{115}{x+5} dx$

$\boxed{\frac{x^3}{3} - \frac{5}{2}x^2 + 19x - 115\ln|x+5| + C}$

5.2 Homework p. 338-339 #1-39 odd, 47-53 odd, 61-73 odd #4

9) $\int \frac{x^2-4}{x}dx = \int \frac{x^2}{x} - \frac{4}{x}dx = \int x\,dx - 4\int \frac{1}{x}dx = \boxed{\frac{x^2}{2} - 4\ln|x| + C}$

11) $\int \frac{x^2+2x+3}{x^3+3x^2+9x}dx$

$u = x^3+3x^2+9x$

$\frac{du}{dx} = 3x^2+6x+9$

$dx = \frac{du}{3x^2+6x+9}$

$\int \frac{x^2+2x+3}{u} \cdot \frac{du}{3x^2+6x+9}$

$= \int \frac{\cancel{x^2+2x+3}}{u} \cdot \frac{du}{3(\cancel{x^2+2x+3})}$

$= \frac{1}{3}\int \frac{1}{u}du = \frac{1}{3}\ln|u| + C = \boxed{\frac{1}{3}\ln|x^3+3x^2+9x| + C}$

15) $\int \frac{x^3-3x^2+5}{x-3}dx$

$$\begin{array}{r} x^2 \qquad + \frac{5}{x-3} \\ x-3 \overline{\big) x^3 - 3x^2 + 5} \\ \ominus x^3 \oplus 3x^2 \\ \hline \end{array}$$

remainder $\longrightarrow$ 5

$\int x^2 + \frac{5}{x-3}dx$

$u = x-3$

$\frac{du}{dx} = 1$

$dx = du$

$5\int \frac{1}{u}du = 5\ln|u| + C$

$= 5\ln|x-3| + C$

$= \boxed{\frac{x^3}{3} + 5\ln|x-3| + C}$

17) $\int \frac{x^4+x-4}{x^2+2}dx$

$$\begin{array}{r} x^2 - 2 \qquad + \frac{x}{x^2+2} \\ x^2+2 \overline{\big) x^4 + x - 4} \\ \ominus x^4 \ominus 2x^2 \\ \hline -2x^2 + x - 4 \\ \oplus 2x^2 \oplus 4 \\ \hline \end{array}$$

x $\longleftarrow$ remainder

$\int x^2 - 2 + \frac{x}{x^2+2}dx \longrightarrow \int \frac{x}{x^2+2}dx$

$u = x^2+2$

$\frac{du}{dx} = 2x$

$dx = \frac{du}{2x}$

$\int \frac{x}{u} \cdot \frac{du}{2x} = \frac{1}{2}\int \frac{1}{u}du$

$= \frac{1}{2}\ln|u| + C$

$= \frac{1}{2}\ln|x^2+2| + C$

$\downarrow$

$\boxed{\frac{x^3}{3} - 2x + \frac{1}{2}\ln|x^2+2| + C}$

19) $\int \frac{(\ln x)^2}{x}\,dx$ $\quad u=\ln x \quad \frac{du}{dx}=\frac{1}{x} \quad dx = x\,du$

$\int \frac{u^2}{\cancel{x}}\cdot \cancel{x}\,du = \frac{u^3}{3}+C$

$= \boxed{\frac{1}{3}(\ln x)^3 + C}$

23) $\int \frac{2x}{(x-1)^2}\,dx$ $\quad u=x-1 \longrightarrow x=u+1$

$\frac{du}{dx}=1$ $\quad$ *use change of variable method

$dx=du$

$\int \frac{2x}{u^2}\cdot du = \int \frac{2(u+1)}{u^2}\,du = \int \frac{2u}{u^2}+\frac{2}{u^2}\,du = 2\int \frac{1}{u}\,du + 2\int u^{-2}\,du$

$= 2\ln|u| + \frac{2u^{-1}}{-1} + C = \boxed{2\ln|x-1| - \frac{2}{(x-1)} + C}$

25) $\int \frac{1}{1+\sqrt{2x}}\,dx$ $\quad u=1+\sqrt{2x}$

$u=1+(2x)^{1/2}$

$\frac{du}{dx}=\frac{1}{2}(2x)^{-1/2}(2)$

$\frac{du}{dx}=\frac{1}{\sqrt{2x}}$ $\quad$ *use change of variable

$dx=\sqrt{2x}\,du$ $\quad u=1+\sqrt{2x}$

$\sqrt{2x}=u-1$

$\int \frac{1}{u}\cdot\sqrt{2x}\,du \longrightarrow \int \frac{1}{u}(u-1)\,du = \int \frac{u-1}{u}\,du = \int \frac{u}{u}-\frac{1}{u}\,du$

$= \int 1-\frac{1}{u}\,du = u - \ln|u| + c = \underbrace{1}_{\text{constant}}+\sqrt{2x} - \ln|1+\sqrt{2x}| + c$

$= \boxed{\sqrt{2x} - \ln|1+\sqrt{2x}| + c}$

27) $\int \frac{\sqrt{x}}{\sqrt{x}-3}\,dx$

$u=\sqrt{x}-3$

$u=(x)^{1/2}-3$

$\frac{du}{dx}=\frac{1}{2}(x)^{-1/2}(1)=\frac{1}{2\sqrt{x}}$

$dx=2\sqrt{x}\,du$

$\int \frac{\sqrt{x}}{u}\cdot 2\sqrt{x}\,du$

$= \int \frac{2x}{u}\,du$

*use change of variable

$u+3=\sqrt{x}$

$(u+3)^2=x$

$= \int \frac{2(u+3)^2}{u}\,du$

$= \int \frac{2(u^2+6u+9)}{u}\,du$

$= \int \frac{2u^2}{u}+\frac{12u}{u}+\frac{18}{u}\,du$

$= \int 2u+12+\frac{18}{u}\,du$

$= \frac{2u^2}{2}+12u+18\ln|u|+c$

$= (\sqrt{x}-3)^2+12(\sqrt{x}-3)+18\ln|\sqrt{x}-3|+c$

or $x+6\sqrt{x}+18\ln|\sqrt{x}-3|+c$

31) $\int \csc 2x\,dx$

$u = 2x$

$\frac{du}{dx} = 2$

$dx = \frac{du}{2}$

*Recall: $\int \csc u\,du = -\ln|\csc u + \cot u| + C$

$\int \csc u \cdot \frac{du}{2} = \frac{1}{2}\int \csc u\,du$

$= -\frac{1}{2}\ln|\csc u + \cot u| + C$

$= \boxed{-\frac{1}{2}\ln|\csc 2x + \cot 2x| + C}$

35) $\int \frac{\sec x \tan x}{\sec x - 1}\,dx$

$u = \sec x - 1$

$\frac{du}{dx} = \sec x \tan x$

$dx = \frac{du}{\sec x \tan x}$

$\int \frac{\cancel{\sec x \tan x}}{u} \cdot \frac{du}{\cancel{\sec x \tan x}} = \int \frac{1}{u}\,du$

$= \ln|u| + C$

$= \boxed{\ln|\sec x - 1| + C}$

37) Solve differential equation: $\frac{dy}{dx} = \frac{3}{2-x}$ at $(1,0)$

$dy = \frac{3}{2-x}dx$

$\int dy = \int \frac{3}{2-x}dx$

$y = \int \frac{3}{2-x}dx$

$u = 2-x \quad dx = -du$

$\frac{du}{dx} = -1$

$y = 3\int \frac{1}{u}(-du)$

$y = -3\int \frac{1}{u}du$

$y = -3\ln|u| + C$

$y = -3\ln|2-x| + C$ *solve for C

$0 = -3\ln|2-1| + C$

$0 = -3\ln|1| + C$

$\boxed{0 = C}$

$\boxed{y = -3\ln|2-x|}$

39) $\frac{ds}{d\theta} = \tan 2\theta$ at $(0,2)$

$ds = \tan 2\theta\,d\theta$

$\int ds = \int \tan 2\theta\,d\theta$

$s = \int \tan 2\theta\,d\theta$

$u = 2\theta$

$\frac{du}{d\theta} = 2$

$d\theta = \frac{du}{2}$

$s = \int \tan u \cdot \frac{du}{2}$

$s = \frac{1}{2}\int \tan u\,du$

$s = \frac{1}{2}(-\ln|\cos u|) + C$

$s = -\frac{1}{2}\ln|\cos 2\theta| + C$

*solve for c

$2 = -\frac{1}{2}\ln|\cos(0)| + C$

$2 = -\frac{1}{2}\ln(1) + C$

$2 = -\frac{1}{2}(0) + C \quad \boxed{C = 2}$

$\boxed{s = -\frac{1}{2}\ln|\cos 2\theta| + 2}$

51) $\int_0^2 \frac{x^2-2}{x+1}\,dx$ $\quad \int x-1-\frac{1}{x+1}\,dx$ $\quad u=x+1$, $\frac{du}{dx}=1$

$$\begin{array}{r|l} & x-1 \quad -\frac{1}{x+1} \\ x+1 & x^2-2 \\ & \ominus x^2 \oplus x \\ \hline & -x-2 \\ & \oplus -x \oplus 1 \\ \hline & -1 \end{array}$$

-1 ← remainder

$\frac{x^2}{2}-x-\ln|x+1|\Big]_0^2$

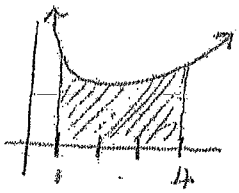
$\frac{4}{2}-2-\ln|3|-(0-0-0) = \boxed{-\ln 3}$

63) $F(x)=\int_1^{3x}\frac{1}{t}\,dt$ Find $F'(x)$ → $\boxed{\text{Use SFTC}}$

$\frac{d}{dx}\int_1^{3x}\frac{1}{t}\,dt = \frac{1}{3x}\cdot 3 = \boxed{\frac{1}{x}}$

69) $\int_0^{\pi/4}\tan x\,dx = -\ln|\cos x|\Big]_0^{\pi/4} = -\ln|\cos(\pi/4)| - (-\ln|\cos 0|) = -\ln\left(\frac{\sqrt{2}}{2}\right) \approx \boxed{0.3466}$ ($\ln|\cos 0| = 0$)

71) $y=\frac{x^2+4}{x}$, $x=1$, $x=4$, $y=0$

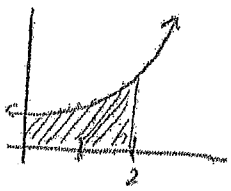


Find $\int_1^4 \frac{x^2+4}{x}\,dx$

$= \int \frac{x^2}{x}+\frac{4}{x}\,dx = \int x+\frac{4}{x}\,dx = \frac{x^2}{2}+4\ln|x|\Big]_1^4 = \frac{4^2}{2}+4\ln 4-\left(\frac{1}{2}+4\ln(1)\right)$

$= 8+4\ln 4-\frac{1}{2} \approx \boxed{13.045 \text{ square units}}$

73) $y=2\sec\left(\frac{\pi x}{6}\right)$, $x=0$, $x=2$, $y=0$



Find $\int_0^2 2\sec\left(\frac{\pi x}{6}\right)dx$

$u=\frac{\pi x}{6}$

$\frac{du}{dx}=\pi/6$

$dx=\frac{6\,du}{\pi}$

$\int 2\sec u\cdot\frac{6}{\pi}\,du$

$=\frac{12}{\pi}\int\sec u\,du$

$=\frac{12}{\pi}\ln|\sec u+\tan u|$

if $x=0$, $u=\frac{\pi(0)}{6}=0$

if $x=2$, $u=\frac{\pi(2)}{6}=\pi/3$

$=\frac{12}{\pi}\ln|\sec u+\tan u|\Big]_0^{\pi/3}$

$=\frac{12}{\pi}\ln|\sec(\pi/3)+\tan(\pi/3)|-\frac{12}{\pi}\ln|\sec 0+\tan 0|$

$\boxed{=\frac{12}{\pi}\ln|2+\sqrt{3}|-0 \approx 5.0304}$

5.4 Notes Integrals of e^x

Exponential Rule (Base): $\boxed{\int e^u\,du = e^u + C}$

Recall: $\frac{d}{dx}e^u = e^u \cdot u'$

$\boxed{\text{Ex. 1}}$ $\int e^{3x+1}\,dx$

$u = 3x+1$ $\quad dx = \frac{du}{3}$

$\frac{du}{dx} = 3$

$\int e^u \cdot \frac{du}{3}$

$\frac{1}{3}\int e^u\,du = \frac{1}{3}e^u + C = \boxed{\frac{1}{3}e^{3x+1} + C}$

2) $\int \cos x \cdot e^{\sin x}\,dx$

$u = \sin x$

$\frac{du}{dx} = \cos x$

$dx = \frac{du}{\cos x}$

$\int \cancel{\cos x} \cdot e^u \cdot \frac{du}{\cancel{\cos x}}$

$\int e^u\,du$

$= e^u + C$

$= \boxed{e^{\sin x} + C}$

3) $\int \frac{e^x}{2+e^x}\,dx$

$u = 2+e^x$

$\frac{du}{dx} = e^x$

$dx = \frac{du}{e^x}$

$\int \frac{\cancel{e^x}}{u} \cdot \frac{du}{\cancel{e^x}}$

$\int \frac{1}{u}\,du$

$= \ln|u| + C = \boxed{\ln|2+e^x| + C}$

4) $\int e^x \cos(e^x)\,dx$

$u = e^x \qquad dx = \frac{du}{e^x}$

$\frac{du}{dx} = e^x$

$\int \cancel{e^x} \cos u \cdot \frac{du}{\cancel{e^x}}$

$\int \cos u\,du = \sin u + c = \boxed{\sin(e^x) + C}$

5) $\int \frac{e^{2x}}{1+e^{2x}}\,dx$

$u = 1 + e^{2x}$

$\frac{du}{dx} = e^{2x} \cdot 2$

$dx = \frac{du}{2e^{2x}}$

$\int \frac{\cancel{e^{2x}}}{u} \cdot \frac{du}{2\cancel{e^{2x}}}$

$\frac{1}{2}\int \frac{1}{u}\,du$

$= \frac{1}{2}\ln|u| + c$

$= \boxed{\frac{1}{2}\ln|1+e^{2x}| + C}$

6) $\int \frac{2e^x - 2e^{-x}}{e^x + e^{-x}}\,dx$

$u = e^x + e^{-x}$

$\frac{du}{dx} = e^x + e^{-x} \cdot (-1)$

$\frac{du}{dx} = e^x - e^{-x}$

$dx = \frac{du}{e^x - e^{-x}}$

$\int \frac{2\cancel{(e^x - e^{-x})}}{u} \cdot \frac{du}{\cancel{e^x - e^{-x}}}$

$2\int \frac{1}{u}\,du$

$= 2\ln|u| + c = \boxed{2\ln|e^x + e^{-x}| + C}$

5.4 Homework p.358-359 #85-109 odd, 113, 115

Integrals of e^x

87) $\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx$

$u = \sqrt{x} = x^{1/2}$

$\frac{du}{dx} = \frac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{x}}$

$dx = 2\sqrt{x}\,du$

$\int \frac{e^u}{\cancel{\sqrt{x}}} \cdot 2\cancel{\sqrt{x}}\,du$

$2\int e^u\,du$

$= 2e^u + C = \boxed{2e^{\sqrt{x}} + C}$

93) $\int \frac{e^x + e^{-x}}{e^x - e^{-x}}$

$u = e^x - e^{-x}$

$\frac{du}{dx} = e^x - e^{-x}(-1)$

$\frac{du}{dx} = e^x + e^{-x}$

$dx = \frac{du}{e^x + e^{-x}}$

$\int \frac{\cancel{e^x + e^{-x}}}{u} \cdot \frac{du}{\cancel{e^x + e^{-x}}}$

$\int \frac{1}{u}\,du = \ln|u| + C = \ln|e^x - e^{-x}| + C$

$= \ln\left|e^x - \frac{1}{e^x}\right| + C$

95) $\int \frac{5 - e^x}{e^{2x}}\,dx = \int \frac{5}{e^{2x}} - \frac{e^x}{e^{2x}}\,dx = 5\int e^{-2x}\,dx - \int e^{-x}\,dx$

$u = -x$

$\frac{du}{dx} = -1$

$dx = -du$

$\int e^u \cdot (-du) = -e^u + C$

$u = -2x$

$\frac{du}{dx} = -2$

$dx = \frac{du}{-2}$

$5\int e^u \cdot \frac{du}{-2}$

$= \frac{5}{-2}\int e^u\,du = \boxed{-\frac{5}{2}e^{-2x} + e^{-x} + C}$

97) $\int e^{-x}\tan(e^{-x})\,dx$ $\quad \int \cancel{e^{-x}}\tan u \cdot \frac{du}{-\cancel{e^{-x}}}$

$u = e^{-x}$

$\frac{du}{dx} = e^{-x}(-1)$ $\quad = -\int \tan u\,du$

$dx = \frac{du}{-e^{-x}}$ $\quad = -(-\ln|\cos u|) + C = \boxed{\ln|\cos(e^{-x})| + C}$

105) $\int_0^{\pi/2} e^{\sin \pi x}\cos \pi x\,dx$

$u = \sin(\pi x)$

$\frac{du}{dx} = \cos(\pi x)\cdot \pi$

$dx = \frac{du}{\pi\cos(\pi x)}$

$\int e^u \cdot \cancel{\cos(\pi x)} \cdot \frac{du}{\pi \cancel{\cos(\pi x)}}$

if $x=0$, $u = \sin(0\pi) = 0$

if $x = \pi/2$, $u = \sin(\pi^2/2)$

$\frac{1}{\pi}\int e^u\,du = \frac{1}{\pi}e^u\Big]_0^{\sin(\pi^2/2)} = \frac{1}{\pi}e^{\sin(\pi^2/2)} - \frac{1}{\pi}e^0$

$= \boxed{\frac{1}{\pi}e^{\sin(\pi^2/2)} - \frac{1}{\pi}(1)}$

107) $\frac{dy}{dx} = xe^{ax^2}$

$\int dy = \int xe^{ax^2}dx$

$y = \int xe^{ax^2}dx$

$u = ax^2$

$\frac{du}{dx} = a\cdot 2x$

$dx = \frac{du}{2ax}$

$y = \int \cancel{x}e^u \cdot \frac{du}{2a\cancel{x}}$

$y = \frac{1}{2a}\int e^u\,du$

$= \frac{1}{2a}e^u + C$

$= \boxed{\frac{1}{2a}e^{ax^2} + C}$

109) $f''(x) = \frac{1}{2}(e^x + e^{-x})$ $\quad f(0) = 1$ $\quad f'(0) = 0$

$f'(x) = \int \frac{1}{2}(e^x + e^{-x})dx$

$f'(x) = \frac{1}{2}e^x - \frac{1}{2}e^{-x} + C$

$0 = \frac{1}{2}e^0 - \frac{1}{2}e^0 + C$

$0 = C$

$f'(x) = \frac{1}{2}e^x - \frac{1}{2}e^{-x}$

$f(x) = \int \frac{1}{2}e^x - \frac{1}{2}e^{-x}dx$

$f(x) = \frac{1}{2}e^x + \frac{1}{2}e^{-x} + k$

$1 = \frac{1}{2}e^0 + \frac{1}{2}e^0 + k$

$1 = \frac{1}{2} + \frac{1}{2} + k$

$0 = k$

$\boxed{f(x) = \frac{1}{2}e^x + \frac{1}{2}e^{-x}}$

Key

Ch. 5.5 Notes **Exponential Rule (base a)**

Recall Rules: $\frac{d}{dx}e^u = e^u \cdot u'$ $\int e^u du = e^u + C$

$\frac{d}{dx}a^u = \ln a \cdot a^u \cdot u'$ $\int a^u du = \frac{1}{\ln a}a^u + C$

*Remember: ln a is a **constant**

Ex. 1: $\int 2^x dx = \boxed{\frac{1}{\ln 2}\cdot 2^x + C \text{ or } \frac{2^x}{\ln 2} + C}$

$a = 2$

Ex. 2: $\int 3^{4x} dx =$

$u = 4x$

$\frac{du}{dx} = 4$

$4dx = du$

$dx = \frac{du}{4}$

$= \int 3^u \cdot \frac{du}{4}$

$= \frac{1}{4}\int 3^u du$

$\frac{1}{4}\cdot\frac{1}{\ln 3}\cdot 3^u + C$

$\boxed{\frac{1}{4\ln 3}\cdot 3^{4x} + C}$

Ex. 3: $\int 5^{\tan x}\sec^2 x\, dx =$

$u = \tan x$

$\frac{du}{dx} = \sec^2 x$

$\sec^2 x dx = du$

$dx = \frac{du}{\sec^2 x}$

$\int 5^u \cdot \cancel{\sec^2 x} \cdot \frac{du}{\cancel{\sec^2 x}}$

$\int 5^u du = \frac{1}{\ln 5}\cdot 5^u + C$

$= \boxed{\frac{1}{\ln 5}\cdot 5^{\tan x} + C}$

Ex. 4: $\int (3-x)7^{(3-x)^2}\, dx$

$u = (3-x)^2$

$\frac{du}{dx} = 2(3-x)^1(-1)$

$-2(3-x)dx = du$

$dx = \frac{du}{-2(3-x)}$

$\int \cancel{(3-x)} \cdot 7^u \cdot \frac{du}{-2\cancel{(3-x)}}$

$-\frac{1}{2}\int 7^u du = -\frac{1}{2}\cdot\frac{1}{\ln 7}\cdot 7^u + C$

$= \boxed{\frac{-1}{2\ln 7}\cdot 7^{(3-x)^2} + C}$

Ex. 5: $\int \frac{2(7^{2x^2-5})}{5-7^{2x^2-5}}x\, dx$

$*\frac{d}{dx}a^u = \ln a \cdot a^u \cdot u'$

$u = 5 - 7^{2x^2-5}$

$\frac{du}{dx} = -\ln 7 \cdot 7^{2x^2-5}\cdot(4x)$

$dx = \frac{du}{(-\ln 7)(4x)\cdot 7^{2x^2-5}}$

$\int \frac{2\cdot 7^{2x^2-5}}{u} \cancel{x} \cdot \frac{du}{(-\ln 7)(4\cancel{x})7^{2x^2-5}}$

$\frac{-2}{\ln 7(4)}\int \frac{1}{u}du$

$\frac{-1}{2\ln 7}\ln|u| + C$

$= \boxed{\frac{-1}{2\ln 7}\ln\left|5 - 7^{2x^2-5}\right| + C}$

Ex. 6: $\int_6^{-12} 4^{\frac{x}{3}} dx$

$u = \frac{1}{3}x$ $\frac{du}{dx} = \frac{1}{3}$ $dx = 3du$

$\int 4^u \cdot 3du = 3\int 4^u du = 3\cdot\frac{1}{\ln 4}\cdot 4^u\Big]_2^{-4}$

Convert bounds:
if $x = 6$, $u = \frac{1}{3}(6) = 2$
if $x = -12$, $u = \frac{1}{3}(-12) = -4$

$= \frac{3}{\ln 4}(4^{-4}) - \frac{3}{\ln 4}(4^2)$

$= \boxed{\frac{3}{(4^4)\ln 4} - \frac{48}{\ln 4}}$

5.2-5.5 Mixed Review Worksheet

Key

1.

$$\int \frac{6x+5}{3x^2+5x-2}dx$$

$u=3x^2+5x-2$

$\frac{du}{dx}=6x+5$

$dx=\frac{du}{6x+5}$

$\int \frac{\cancel{6x+5}}{u}\cdot\frac{du}{\cancel{6x+5}}$

$\int \frac{1}{u}du=\ln|u|+C$

$=\boxed{\ln|3x^2+5x-2|+C}$

2.

$$\int \frac{\cos 3x}{5+2\sin 3x}dx$$

$u=5+2\sin(3x)$

$\frac{du}{dx}=2\cos(3x)\cdot 3$

$dx=\frac{du}{6\cos(3x)}$

$\int \frac{\cancel{\cos(3x)}}{u}\cdot\frac{du}{6\cancel{\cos(3x)}}$

$=\frac{1}{6}\int\frac{1}{u}du$

$=\frac{1}{6}\ln|u|+C$

$=\boxed{\frac{1}{6}\ln|5+2\sin(3x)|+C}$

3.

$\int(2t+1)e^{5t^2+5t}dt$ $\quad u=5t^2+5t$

$\frac{du}{dt}=10t+5$ $\quad dt=\frac{du}{10t+5}$

$\int(2t+1)\cdot e^u\cdot\frac{du}{10t+5}$

$\int\cancel{(2t+1)}e^u\cdot\frac{du}{5\cancel{(2t+1)}}$

$\frac{1}{5}\int e^u du=\frac{1}{5}e^u+C=\boxed{\frac{1}{5}e^{5t^2+5t}+C}$

4.

$$\int \frac{12x+10}{9x^2+15x-6}dx$$

$u=9x^2+15x-6$

$\frac{du}{dx}=18x+15$

$dx=\frac{du}{18x+15}$

$\int\frac{12x+10}{u}\cdot\frac{du}{18x+15}$

$\int\frac{2\cancel{(6x+5)}}{u}\cdot\frac{du}{3\cancel{(6x+5)}}$

$=\frac{2}{3}\int\frac{1}{u}du=\frac{2}{3}\ln|u|+C$

$=\boxed{\frac{2}{3}\ln|9x^2+15x-6|+C}$

5.

$$\int \frac{2^{\sin x}}{3+2^{\sin x}}\cos x\,dx$$

$u=3+2^{\sin x}$

$\frac{du}{dx}=\ln 2\cdot 2^{\sin x}\cdot\cos x$

$dx=\frac{du}{\ln 2\cdot 2^{\sin x}\cdot\cos x}$

$\int\frac{\cancel{2^{\sin x}}}{u}\cancel{\cos x}\cdot\frac{du}{\ln 2\cdot\cancel{2^{\sin x}}\cdot\cancel{\cos x}}$

$\frac{1}{\ln 2}\int\frac{1}{u}du$

$\frac{1}{\ln 2}\ln|u|+C$

$\boxed{\frac{1}{\ln 2}\ln|3+2^{\sin x}|+C}$

6.

Evaluate $\int_e^{e^4}\frac{5}{x\sqrt{\ln x}}dx$ $\quad \int\frac{5}{x(\ln x)^{1/2}}dx$

$u=\ln x$ $\quad \frac{du}{dx}=\frac{1}{x}$ $\quad dx=x\,du$

$\int\frac{5}{x\cdot u^{1/2}}\cdot x\,du=5\int\frac{1}{u^{1/2}}du$

$5\int u^{-1/2}du=\frac{5u^{1/2}}{1/2}=10u^{1/2}\Big]_1^4$

if $x=e$, $u=\ln e=1$

if $x=e^4$, $u=\ln e^4=4$

$=10(4)^{1/2}-10(1)^{1/2}$

$=20-10=\boxed{10}$

7.

$$\int_0^1\frac{1+e^{3x}}{e^{3x}+3x}dx$$

$\int\frac{1+e^{3x}}{u}\cdot\frac{du}{3e^{3x}+3}$

$u=e^{3x}+3x$

$\frac{du}{dx}=e^{3x}(3)+3$

$dx=\frac{du}{3e^{3x}+3}$

$\int\frac{\cancel{1+e^{3x}}}{u}\cdot\frac{du}{3\cancel{(e^{3x}+1)}}$

$\frac{1}{3}\int\frac{1}{u}du=\frac{1}{3}\ln|u|\Big]_1^{e^3+3}$

if $x=0$, $u=1$

if $x=1$, $u=e^3+3$

$\frac{1}{3}\ln|e^3+3|-0$

$=\boxed{\frac{1}{3}\ln|e^3+3|}$

8.

$$\int\frac{\ln^3 3x}{3x}dx=\int\frac{[\ln(3x)]^3}{3x}dx$$

$u=\ln(3x)$

$\frac{du}{dx}=\frac{3}{3x}=\frac{1}{x}$

$dx=x\,du$

$\int\frac{u^3}{\cancel{3x}}\cdot\cancel{x}\,du$

$\frac{1}{3}\int u^3du$

$=\frac{1}{3}\left(\frac{u^4}{4}\right)+C$

$=\boxed{\frac{1}{12}(\ln(3x))^4+C}$

9.

$$\int\frac{2x^3+5x^2-12}{x+3}dx$$

$$\begin{array}{r} 2x^2-x+3-\frac{21}{x+3} \\ x+3\overline{)2x^3+5x^2-12} \\ -2x^3-6x^2 \\ \hline -x^2-12 \\ +x^2+3x \\ \hline 3x-12 \\ -3x-9 \\ \hline -21 \end{array}$$

OR synthetic division

$$\begin{array}{r|rrrr} -3 & 2 & 5 & 0 & -12 \\ & \downarrow & -6 & 3 & -9 \\ \hline & 2 & -1 & 3 & -21 \end{array}$$

$2x^2-x+3-\frac{21}{x+3}$

$u=x+3$ $\quad \frac{du}{dx}=1$

$21\int\frac{1}{u}du=21\ln|u|$

$\int 2x^2-x+3-\frac{21}{x+3}dx$

$\boxed{\frac{2x^3}{3}-\frac{x^2}{2}+3x-21\ln|x+3|+C}$

5.5 Notes Exponential Rule (base a)

Exponential Rule (not base e)

$$\boxed{\int a^u\,dx = \frac{1}{\ln a}\cdot a^u + C}$$

Recall: $\frac{d}{dx}a^u = \ln a \cdot a^u \cdot u'$

Remember: $\ln a$ is a constant!

$\boxed{\text{Ex. 1}}$ $\int 2^x\,dx = \frac{1}{\ln 2}\cdot 2^x + C = \boxed{\frac{2^x}{\ln 2} + C}$

2) $\int 2^{3x}\,dx$

$u = 3x \quad dx = \frac{du}{3}$

$\frac{du}{dx} = 3$

$= \int 2^u \cdot \frac{du}{3}$

$= \frac{1}{3}\int 2^u\,du$

$= \frac{1}{3}\cdot\frac{1}{\ln 2}\cdot 2^u + C$

$= \boxed{\frac{2^{3x}}{3\ln 2} + C}$

3) $\int (3-x)\,7^{(3-x)^2}\,dx$

$u = (3-x)^2$

$\frac{du}{dx} = 2(3-x)\cdot(-1)$

$dx = \frac{du}{-2(3-x)}$

$\int \cancel{(3-x)}\cdot 7^u \cdot \frac{du}{-2\cancel{(3-x)}}$

$= -\frac{1}{2}\int 7^u\,du$

$= -\frac{1}{2}\cdot\frac{1}{\ln 7}\cdot 7^u + C$

$= \boxed{-\frac{7^{(3-x)^2}}{2\ln 7} + C}$

5.5 Homework p.367 #61–71 all

Integrals of exponentials (base a)

63) $\int x(5^{-x^2})dx$

$u=-x^2$

$\frac{du}{dx}=-2x$

$dx=\frac{du}{-2x}$

$=\int \cancel{x}\cdot 5^u\cdot\frac{du}{-2\cancel{x}}$

$-\frac{1}{2}\int 5^u du$

$-\frac{1}{2}\cdot\frac{1}{\ln 5}\cdot 5^u+C$

$\boxed{\frac{-5^{-x^2}}{2\ln 5}+C}$

65) $\int\frac{3^{2x}}{1+3^{2x}}dx$

$u=1+3^{2x}$

$\frac{du}{dx}=\ln 3\cdot 3^{2x}\cdot 2$

*Recall: $\frac{d}{dx}a^u=\ln a\cdot a^u\cdot u'$

$dx=\frac{du}{2\ln 3\cdot 3^{2x}}$

$\int\frac{\cancel{3^{2x}}}{u}\cdot\frac{du}{2\ln 3\cdot\cancel{3^{2x}}}$

$=\frac{1}{2\ln 3}\int\frac{1}{u}du$

$\frac{1}{2\ln 3}\cdot\ln|u|+C$

$\frac{1}{2\ln 3}\cdot\ln|1+3^{2x}|+C$

$\boxed{\frac{\ln|1+3^{2x}|}{2\ln 3}+C}$

66) $\int 2^{\sin x}\cos x\,dx$

$u=\sin x$

$\frac{du}{dx}=\cos x$

$dx=\frac{du}{\cos x}$

$\int 2^u\cdot\cancel{\cos x}\cdot\frac{du}{\cancel{\cos x}}$

$=\frac{1}{\ln 2}\cdot 2^u+C$

$=\boxed{\frac{2^{\sin x}}{\ln 2}+C}$

5.5 HW (continued)

68) $\int_{-2}^{2} 4^{x/2}\,dx$

$u = \frac{x}{2}$

$\frac{du}{dx} = \frac{1}{2}$

$dx = 2du$

$\int 4^u \cdot 2du$

if $x=-2$, $u = \frac{-2}{2} = -1$

if $x = 2$, $u = \frac{2}{2} = 1$

$2\int_{-1}^{1} 4^u\,du$

$2 \cdot \frac{1}{\ln 4} \cdot 4^u \Big]_{-1}^{1}$

$\frac{2}{\ln 4}(4^1) - \frac{2}{\ln 4}(4^{-1})$

$= \frac{2}{\ln 4}\left(4 - \frac{1}{4}\right) = \frac{\cancel{2}}{\ln 4}\left(\frac{15}{\cancel{4}}\right)_2$

$= \boxed{\frac{15}{2\ln 4} = \frac{15}{2\ln 2^2} = \frac{15}{2 \cdot 2\ln 2} = \frac{15}{4\ln 2}}$

70) $\int_{1}^{e} 6^x - 2^x\,dx$

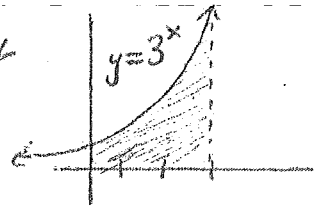
$\frac{6^x}{\ln 6} - \frac{2^x}{\ln 2}\Big]_{1}^{e}$

$= \left(\frac{6^e}{\ln 6} - \frac{2^e}{\ln 2}\right) - \left(\frac{6^1}{\ln 6} - \frac{2^1}{\ln 2}\right)$

$= \boxed{\frac{6^e}{\ln 6} - \frac{2^e}{\ln 2} - \frac{6}{\ln 6} + \frac{2}{\ln 2}}$

71) Find area of region bounded by

$y = 3^x$, $y = 0$, $x = 0$, $x = 3$



$A = \int_{0}^{3} 3^x\,dx = \frac{3^x}{\ln 3}\Big]_{0}^{3} = \frac{3^3}{\ln 3} - \frac{3^0}{\ln 3} = \frac{27}{\ln 3} - \frac{1}{\ln 3} = \boxed{\frac{26}{\ln 3}}$

A.P. Calculus AB **Quiz Review #1 for 5-2, 5-4, 5-5**

Key

Review Integral Rules:

$\int u^n du = \frac{u^{n+1}}{n+1} + C$	$\int \frac{1}{u} du = \ln\lvert u\rvert + c$	$\int e^u du = e^u + C$
$\int \tan u\, du = -\ln\lvert\cos u\rvert + c$	$\int \cot u\, du = \ln\lvert\sin u\rvert + c$	$\int \sec u\, du = \ln\lvert\sec u + \tan u\rvert + c$
$\int \csc u\, du = -\ln\lvert\csc u + \cot u\rvert + c$	$\int a^u du = \frac{1}{\ln a} \cdot a^u + C$	

1. $\int \frac{dx}{2x+3}$

$u = 2x+3$

$\frac{du}{dx} = 2$

$dx = \frac{du}{2}$

$\int \frac{1}{u} \cdot \frac{du}{2} = \frac{1}{2}\int \frac{1}{u} du$

$\frac{1}{2}\ln\lvert u\rvert + c$

$\boxed{\frac{1}{2}\ln\lvert 2x+3\rvert + c}$

2. $\int \frac{x}{4x^2+1} dx$

$u = 4x^2+1$

$\frac{du}{dx} = 8x$

$dx = \frac{du}{8x}$

$\int \frac{\cancel{x}}{u} \cdot \frac{du}{8\cancel{x}} = \frac{1}{8}\int \frac{1}{u} du$

$\frac{1}{8}\ln\lvert u\rvert + c$

$\boxed{\frac{1}{8}\ln\lvert 4x^2+1\rvert + c}$

3. $\int \frac{2x-5}{x} dx$

$\int (2x-5)x^{-1} dx$

$\int 2 - 5x^{-1} dx = \int 2dx - 5\int \frac{1}{x} dx$

$\boxed{2x - 5\ln\lvert x\rvert + c}$

4. $\int \frac{x}{x+1} dx$

$u = x+1$

$\frac{du}{dx} = 1$

$dx = du$

$\int \frac{x}{u} \cdot du$

$x = u-1$

$\int \frac{u-1}{u} du$

$\int 1 - \frac{1}{u} du$

$u - \ln\lvert u\rvert + c$

$x+1 - \ln\lvert x+1\rvert + c$

$x - \ln\lvert x+1\rvert + 1 + c$

$\boxed{x - \ln\lvert x+1\rvert + c}$

5. $\int \frac{(\ln x)^2 dx}{x}$

$u = \ln x$

$\frac{du}{dx} = \frac{1}{x}$

$dx = x du$

$\int \frac{u^2}{\cancel{x}} \cdot \cancel{x} du = \int u^2 du$

$\frac{u^3}{3} + c = \boxed{\frac{(\ln x)^3}{3} + c}$

6. $\int \frac{dx}{\sqrt{x}(1+\sqrt{x})}$

$u = 1+\sqrt{x}$

$u = 1 + x^{1/2}$

$\frac{du}{dx} = \frac{1}{2}x^{-1/2}$

$\frac{du}{dx} = \frac{1}{2\sqrt{x}}$

$dx = 2\sqrt{x}\, du$

$\int \frac{2\cancel{\sqrt{x}}\, du}{\cancel{\sqrt{x}} \cdot u}$

$2\int \frac{1}{u} du$

$2\ln\lvert u\rvert + c$

$\boxed{2\ln\lvert 1+\sqrt{x}\rvert + c}$

7. $\int \frac{x^2-4x+8}{x-3}dx$

$$\begin{array}{r} x-1+\frac{5}{x-3} \\ x-3\,\overline{)\,x^2-4x+8} \\ \ominus x^2 \oplus 3x \\ \hline -1x+8 \\ \oplus 1x \ominus 3 \\ \hline 5 \end{array}$$

OR

$$\begin{array}{r|rrr} & 1 & -4 & 8 \\ 3 & & 3 & -3 \\ \hline & 1 & -1 & 5 \end{array}$$

$x-1+\frac{5}{x-3}$

$\int x-1+\frac{5}{x-3}\,dx$

$u=x-3$

$\frac{du}{dx}=1$

$dx=du$

$5\int \frac{1}{u}du$

$=5\ln|u|$

$\boxed{\frac{x^2}{2}-x+5\ln|x-3|+C}$

8. $\int \frac{x^3-5x^2+x-2}{x+1}dx$

$$\begin{array}{r} x^2-6x+7-\frac{9}{x+1} \\ x+1\,\overline{)\,x^3-5x^2+x-2} \\ \ominus x^3 \ominus x^2 \\ \hline -6x^2+x-2 \\ \oplus 6x^2 \oplus 6x \\ \hline 7x-2 \\ \ominus 7x \ominus 7 \\ \hline -9 \end{array}$$

OR

$$\begin{array}{r|rrrr} -1 & 1 & -5 & 1 & -2 \\ & \downarrow & -1 & 6 & -7 \\ \hline & 1 & -6 & 7 & \boxed{-9} \end{array}$$

$\int x^2-6x+7-\frac{9}{x+1}dx$

$u=x+1$

$\frac{du}{dx}=1$

$dx=du$

$\frac{x^3}{3}-\frac{6x^2}{2}+7x-9\ln|u|-9\int\frac{1}{u}du$

$\boxed{\frac{x^3}{3}-3x^2+7x-9\ln|x+1|+C}$

9. $\int \frac{dx}{(2x+3)^2}$ $\quad \int \frac{1}{(2x+3)^2}dx$

$u=2x+3$

$\frac{du}{dx}=2$

$dx=\frac{du}{2}$

$\int \frac{1}{u^2}\cdot\frac{du}{2}$

$\frac{1}{2}\int\frac{1}{u^2}du$

$\frac{1}{2}\int u^{-2}du$

$\frac{1}{2}\frac{u^{-1}}{-1}+C$

$\frac{-1}{2u}+C=\boxed{\frac{-1}{2(2x+3)}+C}$

10. $\int 5^{\sec x}\sec x\tan x\,dx$

$u=\sec x$

$\frac{du}{dx}=\sec x\tan x$

$dx=\frac{du}{\sec x\tan x}$

$\int 5^u\cdot\cancel{\sec x\tan x}\cdot\frac{du}{\cancel{\sec x\tan x}}$

$\int 5^u du=\frac{5^u}{\ln 5}=\boxed{\frac{5^{\sec x}}{\ln 5}+C}$

11. $\int_0^{\sqrt{2}} xe^{-\frac{1}{2}x^2}dx$

If $x=0$, $u=-\frac{1}{2}(0)=0$

If $x=\sqrt{2}$, $u=-\frac{1}{2}(\sqrt{2})^2=-1$

$u=-\frac{1}{2}x^2$

$\frac{du}{dx}=-x$

$dx=\frac{du}{-x}$

$\int \cancel{x}e^u\cdot\frac{du}{-\cancel{x}}$

$-\int e^u du$

$=-e^u\Big]_0^{-1}=-e^{-1}-(-e^0)$

$=\boxed{\frac{-1}{e}+1}$

12. $\int_3^4 e^{3-x}dx$

If $x=3$, $u=3-3=0$

If $x=4$, $u=3-4=-1$

$u=3-x$

$\frac{du}{dx}=-1$

$dx=-du$

$\int e^u(-du)=-e^u\Big]_0^{-1}=-e^{-1}-(-e^0)$

$=\boxed{\frac{-1}{e}+1}$

13. $\int(\tan x+\sec x+\cot x+\csc x)dx$

$\boxed{-\ln|\cos x|+\ln|\sec x+\tan x|+\ln|\sin x|-\ln|\csc x+\cot x|+C}$

14. $\int(5-2x)^2\,7^{(5-2x)^3}dx$

$u=(5-2x)^3$

$\frac{du}{dx}=3(5-2x)^2(-2)$

$dx=\frac{du}{-6(5-2x)^2}$

$\int\cancel{(5-2x)^2}\cdot 7^u\cdot\frac{du}{-6\cancel{(5-2x)^2}}$

$-\frac{1}{6}\int 7^u du$

$-\frac{1}{6}\cdot\frac{1}{\ln 7}\cdot 7^u+C$

$\boxed{\frac{-7^{(5-2x)^3}}{6\ln 7}+C}$

A.P. Calculus AB **Ch. 5-2, 5-4, 5-5 Quiz Review #2**

Key

Review Integral Rules:

$\int u^n du = \frac{u^{n+1}}{n+1} + C$	$\int \frac{1}{u} du = \ln\lvert u\rvert + C$	$\int e^u du = e^u + C$
$\int \tan u\, du = -\ln\lvert\cos u\rvert + C$	$\int \cot u\, du = \ln\lvert\sin u\rvert + C$	$\int \sec u\, du = \ln\lvert\sec u + \tan u\rvert + C$
$\int \csc u\, du = -\ln\lvert\csc u + \cot u\rvert + C$	$\int a^u du = \frac{1}{\ln a}\cdot a^u + C$ or $\frac{a^u}{\ln a} + C$	

1. $\int \frac{dx}{x^{2/3}(1+x^{1/3})}$

$u = 1 + x^{1/3}$

$\frac{du}{dx} = \frac{1}{3}x^{-2/3}$

$\frac{du}{dx} = \frac{1}{3x^{2/3}}$

$dx = 3x^{2/3}du$

$\int \frac{3\cancel{x^{2/3}}du}{\cancel{x^{2/3}}\cdot u}$

$3\int \frac{1}{u}du$

$3\ln\lvert u\rvert + C$

$\boxed{3\ln\lvert 1+x^{1/3}\rvert + C}$

2. $\int \frac{x^4+x-4}{x^2+2}dx$

$$\begin{array}{r} x^2 - 2 + \frac{x}{x^2+2} \\ x^2+2 \overline{\smash{)}\, x^4 + x - 4} \\ \underline{-x^4 \mp 2x^2} \\ -2x^2 + x - 4 \\ \underline{\pm 2x^2 \pm 4} \\ x \end{array}$$

$\int x^2 - 2 + \frac{x}{x^2+2}dx$

$u = x^2+2$

$\frac{du}{dx} = 2x$

$dx = \frac{du}{2x}$

$\int \frac{\cancel{x}}{u}\cdot\frac{du}{2\cancel{x}}$

$\frac{1}{2}\int \frac{1}{u}du$

$\boxed{\frac{x^3}{3} - 2x + \frac{1}{2}\ln\lvert x^2+2\rvert + C}$

3. $\int \frac{\sec x \tan x}{\sec x - 1}dx$

$u = \sec x - 1$

$\frac{du}{dx} = \sec x \tan x$

$dx = \frac{du}{\sec x \tan x}$

$\int \frac{\cancel{\sec x \tan x}}{u}\cdot\frac{du}{\cancel{\sec x \tan x}}$

$\int \frac{1}{u}du$

$\ln\lvert u\rvert + C$

$\boxed{\ln\lvert \sec x - 1\rvert + C}$

4. $\int \frac{e^{2x}+2e^x+1}{e^x}dx$

$\int (e^{2x}+2e^x+1)e^{-x}dx$

$\int e^x + 2 + e^{-x}dx$

$u = -x$ $\frac{du}{dx} = -1$

$dx = -du$

$-\int e^u du$

$\boxed{e^x + 2x - e^{-x} + C}$

5. $\int_1^3 \frac{e^{3/x}}{x^2}dx$

If $x=1$, $u=3/1=3$
If $x=3$, $u=3/3=1$

$u=3/x$

$u=3x^{-1}$

$\frac{du}{dx}=-3x^{-2}$

$\frac{du}{dx}=\frac{-3}{x^2}$

$-3dx=x^2du$

$dx=-\frac{x^2}{3}du$

$\int \frac{e^u}{\cancel{x^2}}\cdot\frac{-\cancel{x^2}}{3}du$

$-\frac{1}{3}\int e^u du$

$-\frac{1}{3}e^u\Big]_3^1$

$-\frac{1}{3}e^1-\left(-\frac{1}{3}e^3\right)$

$\boxed{-\frac{1}{3}e+\frac{1}{3}e^3}$

6. $\int \frac{2x}{1-3x}dx$

$u=1-3x$

$\frac{du}{dx}=-3$

$dx=\frac{du}{-3}$

$\int \frac{2x}{u}\cdot\frac{du}{-3}$

$u=1-3x$

$3x=1-u$

$x=\frac{1-u}{3}$

$\int \frac{2\left(\frac{1-u}{3}\right)}{u}\cdot\frac{du}{-3}$

$\int \frac{2}{3}\left(\frac{1-u}{u}\right)\frac{du}{-3}=\frac{-2}{9}\int\frac{1-u}{u}du$

$\frac{-2}{9}\int\frac{1}{u}-1\,du$

$\frac{-2}{9}\ln|u|-u+c$

$\boxed{\frac{-2}{9}\ln|1-3x|-(1-3x)+C}$

7. $\int \sec\left(\frac{x}{2}\right)dx$

$u=x/2$

$\frac{du}{dx}=\frac{1}{2}$

$dx=2du$

$\int \sec u\cdot 2du$

$2\int \sec u\,du$

$2\ln|\sec u+\tan u|+C$

$\boxed{2\ln\left|\sec(x/2)+\tan(x/2)\right|+C}$

8. $\int \frac{4^{3x}}{1-4^{3x}}dx$

$u=1-4^{3x}$

$\frac{du}{dx}=-\ln 4\cdot 4^{3x}\cdot 3$

$dx=\frac{du}{-3\ln 4\cdot 4^{3x}}$

$\int \frac{\cancel{4^{3x}}}{u}\cdot\frac{du}{-3\ln 4\cdot\cancel{4^{3x}}}$

$\frac{-1}{3\ln 4}\int\frac{1}{u}du$

$\frac{-1}{3\ln 4}\ln|u|+C$

$\boxed{\frac{-1}{3\ln 4}\ln|1-4^{3x}|+C}$

9. $\int \frac{\cos x}{2^{\sin x}}dx$

$\int 2^{-\sin x}\cos x\,dx$

$u=-\sin x$

$\frac{du}{dx}=-\cos x$

$dx=\frac{du}{-\cos x}$

$\int 2^u\cdot\cancel{\cos x}\cdot\frac{du}{-\cancel{\cos x}}$

$-\int 2^u du$

$-\frac{1}{\ln 2}\cdot 2^u+C=\boxed{\frac{-2^{-\sin x}}{\ln 2}+C}$

AP CALCULUS AB Morning **Quiz Review on 5-2, 5-4, 5-5**

1. $\int \frac{(\ln x)^2}{5x}\,dx =$

$u = \ln x \qquad dx = x\,du$

$\frac{du}{dx} = \frac{1}{x}$

$\int \frac{u^2}{5\cancel{x}} \cdot x\,du = \frac{1}{5}\int u^2\,du = \frac{1}{5}\frac{u^3}{3} = \frac{u^3}{15} + C$

$= \boxed{\frac{(\ln x)^3}{15} + C}$

2. $\int \frac{7x^3 + 21x^2 + 7x}{x^4 + 4x^3 + 2x^2}\,dx =$

$\int \frac{7\cancel{(x^3+3x^2+x)}}{u} \cdot \frac{du}{4\cancel{(x^3+3x^2+x)}}$

$u = x^4 + 4x^3 + 2x^2$

$\frac{du}{dx} = 4x^3 + 12x^2 + 4x = 4(x^3 + 3x^2 + x) \qquad dx = \frac{du}{4(x^3+3x^2+x)}$

$\frac{7}{4}\int \frac{1}{u}\,du$

$= \frac{7}{4}\ln|u| + C$

$= \boxed{\frac{7}{4}\ln|x^4 + 4x^3 + 2x^2| + C}$

3. $\int \frac{x^4 + 3x^3 - x^2 - 5}{x+2}\,dx = \int x^3 + x^2 - 3x + 6 - \frac{17}{x+2}\,dx$

$u = x+2 \qquad dx = du$

$\frac{du}{dx} = 1 \qquad \int \frac{17}{u}\,du = 17\ln|u| = 17\ln|x+2|$

-2	1	3	-1	0	-5
	↓	-2	-2	6	-12
	1	1	-3	6	-17

$\boxed{\frac{x^4}{4} + \frac{x^3}{3} - \frac{3x^2}{2} + 6x - 17\ln|x+2| + C}$

4. $\int_4^9 \frac{dx}{\sqrt{x}(1+\sqrt{x})} =$

if $x = 4$ $u = 1 + \sqrt{4} = 3$

if $x = 9$ $u = 1 + \sqrt{9} = 4$

$u = 1 + x^{1/2}$

$\frac{du}{dx} = \frac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{x}}$

$dx = 2\sqrt{x}\,du$

$\int \frac{1}{\cancel{\sqrt{x}}\,u} \cdot 2\cancel{\sqrt{x}}\,du = 2\int_3^4 \frac{1}{u}\,du = 2\ln|u|\Big]_3^4 = 2\ln 4 - 2\ln 3$

$= 2(\ln 4 - \ln 3)$

$= \boxed{2\ln(4/3)}$

5. $\int 2\cot(3x) - \sec(4x)\,dx = \int 2\cot 3x\,dx - \int \sec 4x\,dx$

$u = 4x \qquad \frac{du}{dx} = 4 \qquad dx = \frac{du}{4}$

$\int \sec u \cdot \frac{du}{4} = \frac{1}{4}\int \sec u\,du$

$u = 3x$

$\frac{du}{dx} = 3$

$dx = \frac{du}{3}$

$2\int \cot u \cdot \frac{du}{3}$

$\frac{2}{3}\int \cot u\,du$

$\boxed{\frac{2}{3}\ln|\sin(3x)| - \frac{1}{4}\ln|\sec(4x) + \tan(4x)| + C}$

6. $\int \frac{7^{\frac{2}{x}}}{x^2}\,dx =$

$\int \frac{7^u}{\cancel{x^2}} \cdot \frac{\cancel{x^2}\,du}{-2} = \frac{-1}{2}\int 7^u\,du = \frac{-1}{2} \cdot \frac{7^u}{\ln 7} + C$

$u = 2x^{-1}$

$\frac{du}{dx} = -2x^{-2} = \frac{-2}{x^2} \qquad dx = \frac{x^2\,du}{-2}$

$= \boxed{\frac{-7^{2/x}}{2\ln 7} + C}$