

Calculus Ch. 5.2 Natural Log Integrals

What would happen if we attempted to apply power rule for this problem?

$$\int \frac{1}{x} dx$$

Recall Derivative Rule:

$$\frac{d}{dx} \ln u = \frac{u'}{u}$$

Natural Log Integral Rule

$$\int \frac{1}{u} du = \ln|u| + C$$

Example 1: $\int \frac{2x}{x^2 + 1} dx$

Example 2: $\int \frac{1}{x \ln x} dx$

Example 3: $\int \frac{x^2 + x + 1}{x^2 + 1} dx$

1

2

Ch. 5.2 More Trig Integral Rules:

1) $\int \tan u \, du = -\ln|\cos u| + C$

2) $\int \cot u \, du = \ln|\sin u| + C$

3) $\int \sec u \, du = \ln|\sec u + \tan u| + C$

4) $\int \csc u \, du = -\ln|\csc u + \cot u| + C$

Example 4:

a) $\int \tan x \, dx$

b) $\int \cot x \, dx$

Example 5: (method 1) long division

$$\int \frac{x^3 - 6x - 20}{x + 5} \, dx$$

(Example 5: Method 2) synthetic division

$$\int \frac{x^3 - 6x - 20}{x + 5} \, dx$$

Calculus Ch. 5.4 Notes Integrals of e^x

Integral Exponential Rule (base e): $\int e^u du = e^u + C$

Recall: $\frac{d}{dx} e^u = e^u \times u'$

Ex. 1: Find $\int e^{3x+1} dx$

Ex. 2: Find $\int \cos x \cdot e^{\sin x} dx$

Ex. 3: Find $\int \frac{e^x}{2 + e^x} dx$

(4)

(4)

Ex. 4: Find $\int e^x \cos(e^x) dx$

Ex. 5: Find $\int \frac{e^{2x}}{e^{2x} + 1} dx$

Ex. 6: Find $\int \frac{2e^x - 2e^{-x}}{e^x + e^{-x}} dx$

Ch. 5.5 Notes

Exponential Rule (base a)

Recall Rules: $\frac{d}{dx} e^u = \int e^u du =$

$$\frac{d}{dx} a^u = \int a^u du =$$

*Remember: ln a is a constant

Ex. 1: $\int 2^x dx =$

Ex. 2: $\int 3^{4x} dx =$

Ex. 3: $\int 5^{\tan x} \sec^2 x dx =$

Ex. 4: $\int (3-x)7^{(3-x)^2} dx$

Ex. 5: $\int \frac{2(7^{2x^2-5})}{5-7^{2x^2-5}} x dx$

Ex. 6: $\int_6^{-12} 4^{\frac{x}{3}} dx$

(6)

5.2-5.5 Mixed Review Worksheet

(6)

1.

$$\int \frac{6x+5}{3x^2+5x-2} dx$$

2.

$$\int \frac{\cos 3x}{5+2\sin 3x} dx$$

3.

$$\int (2t+1)e^{5t^2+5t} dt$$

4.

$$\int \frac{12x+10}{9x^2+15x-6} dx$$

5.

$$\int \frac{e}{x^2} \cot\left(\frac{7}{x}\right) dx$$

6.

Evaluate $\int_e^4 \frac{5}{x\sqrt{\ln x}} dx$

7.

$$\int_0^1 \frac{1+e^{3x}}{e^{3x}+3x} dx$$

8.

$$\int \frac{\ln^3 3x}{3x} dx$$

9.

$$\int \frac{2x^3+5x^2-12}{x+3} dx$$

(Chapter 5) Derivative & Integral Rules Reference Sheet

Derivative Rules:

Power Rule:

$$\frac{d}{dx} x^n = nx^{n-1}$$

Trig Derivatives:

$\frac{d}{dx} \sin u = \cos u * u'$	$\frac{d}{dx} \cos u = -\sin u * u'$
$\frac{d}{dx} \tan u = \sec^2 u * u'$	$\frac{d}{dx} \cot u = -\csc^2 u * u'$
$\frac{d}{dx} \sec u = \sec u \tan u * u'$	$\frac{d}{dx} \csc u = -\csc u \cot u * u'$

$$\frac{d}{dx} e^u = e^u * u'$$

$$\frac{d}{dx} \ln u = \frac{u'}{u}$$

$$\frac{d}{dx} a^u = \ln a * a^u * u'$$

$$\frac{d}{dx} \log_a u = \frac{1}{\ln a} * \frac{u'}{u}$$

Integral Rules:

Power Rule:

$$\int u^n du = \frac{u^{n+1}}{n+1} + C$$

Trig Integrals:

$\int \sin u du = -\cos u + C$	$\int \cos u du = \sin u + C$
$\int \sec^2 u du = \tan u + C$	$\int \sec u \tan u du = \sec u + C$
$\int \csc^2 u du = -\cot u + C$	$\int \csc u \cot u du = -\csc u + C$

$$\int \frac{1}{u} du = \ln|u| + C$$

$$\int e^u du = e^u + C$$

$$\int a^u du = \left(\frac{1}{\ln a}\right) a^u + C$$

More Trig Integral Rules:

$$\int \tan u du = -\ln|\cos u| + C$$

$$\int \sec u du = \ln|\sec u + \tan u| + C$$

$$\int \cot u du = \ln|\sin u| + C$$

$$\int \csc u du = -\ln|\csc u + \cot u| + C$$

Ch. 5 Integration Technique Checklist

1) Power Rule (Can you rearrange problem to rely on just power rule?)

*Some examples include: $\int (3-x)^2 \left(\frac{2}{\sqrt{x}}\right) dx$ and $\int \frac{2x(5-3x+x^4)}{3(\sqrt{x^7})} dx$

- a) convert radicals to rational exponential form (example: $\sqrt{x^5} = x^{\frac{5}{2}}$)
- b) move denominator variable to numerator
- c) resolve parentheses, separate terms.

*typically, if there are multiple terms in denominator separated by addition or subtraction, power rule alone will not be enough to make progress. Proceed to Option #2

2) If unable to rely on just power rule, then explore **U-Substitution** options.

- a) Big picture: We want to choose a u-value that will lead to a match with a **known Integral rule**. (Needs to be a perfect match outside of coefficient terms, and with no x-variables remaining)
- b) If expression can be rewritten using parentheses, the u-value is usually the expression inside the set of parentheses.
- c) u-value is more than just replacing an "x", and may involve replacing a significant portion of the expression.
- d) For fractional expressions, the u-value usually comes from the denominator.

(potential notable exceptions are log functions like $\ln x$ and radical expressions like \sqrt{x})

- e) u-value are typically higher degree expressions when choosing between 2 expressions with different degrees.

2b) U-Substitution (using **change of variable**)

- a) If the initial round of u-substitution is not enough to remove the remaining x's in the integrand, then explore option of rearranging the expression assigned to u, and solving for x.
- b) Once we make that second set of substitutions, the problem is now purely in terms of u, and with all x's removed and replaced.

3) Rewrite rational expression using **Long Division** (synthetic division)

- a) Condition needed to apply **long division** is the **numerator degree \geq denominator degree**.

(example: $\int \frac{2x^3-4x+1}{x^2+3} dx$)

- b) For long division problems, we can apply **synthetic division** only if denominator degree is = 1 (linear degree) (example: $\int \frac{4x^3-7x+2}{x-5} dx$)
- c) Once our rewrite is complete, we can typically find the antiderivative by using a combination of power rule and u-substitution across the different terms.

Review Integral Rules:

$\int u^n du =$	$\int \frac{1}{u} du =$	$\int e^u du =$
$\int \tan u du =$	$\int \cot u du =$	$\int \sec u du =$
$\int \csc u du =$	$\int a^u du =$	

1. $\int \frac{dx}{2x+3}$

2. $\int \frac{x}{4x^2+1} dx$

3. $\int \frac{2x-5}{x} dx$

4. $\int \frac{x}{x+1} dx$

5. $\int \frac{(\ln x)^2}{x} dx$

6. $\int \frac{dx}{\sqrt{x}(1+\sqrt{x})}$

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$$7. \int \frac{x^2 - 4x + 8}{x-3} dx$$

$$8. \int \frac{x^3 - 5x^2 + x - 2}{x+1} dx$$

$$9. \int \frac{dx}{(2x+3)^2}$$

$$10. \int 5^{\sec x} \sec x \tan x dx$$

$$11. \int_0^{\sqrt{2}} xe^{-\frac{1}{2}x^2} dx$$

$$12. \int_3^4 e^{3-x} dx$$

$$13. \int (\tan x + \sec x + \cot x + \csc x) dx$$

$$14. \int (5-2x)^2 7^{(5-2x)^3} dx$$

Review Integral Rules:

$\int u^n du =$	$\int \frac{1}{u} du =$	$\int e^u du =$
$\int \tan u du =$	$\int \cot u du =$	$\int \sec u du =$
$\int \csc u du =$	$\int a^u du =$	

1. $\int \frac{dx}{x^{2/3}(1+x^{1/3})}$

2. $\int \frac{x^4 + x - 4}{x^2 + 2} dx$

3. $\int \frac{\sec x \tan x}{\sec x - 1} dx$

4. $\int \frac{e^{2x} + 2e^x + 1}{e^x} dx$

12

$$5. \int_1^3 \frac{e^{3/x}}{x^2} dx$$



$$6. \int \frac{2x}{1-3x} dx$$

$$7. \int \sec\left(\frac{x}{2}\right) dx$$



$$8. \int \frac{4^{3x}}{1-4^{3x}} dx$$

$$9. \int \frac{\cos x}{2^{\sin x}} dx$$



AP CALCULUS AB

Morning Quiz Review on 5-2, 5-4, 5-5

$$1. \int \frac{(\ln x)^2}{5x} dx$$

$$2. \int \frac{7x^3 + 21x^2 + 7x}{x^4 + 4x^3 + 2x^2} dx =$$

$$3. \int \frac{x^4 + 3x^3 - x^2 - 5}{x + 2} dx =$$

$$4. \int_4^9 \frac{dx}{\sqrt{x}(1 + \sqrt{x})} =$$

$$5. \int 2\cot(3x) - \sec(4x) dx$$

$$6. \int \frac{7^x}{x^2} dx$$

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AP Calculus AB

Quiz Review #1 for 5.2, 5.4, 5.5

Review Integral Rules:

$$\int u^n du = \frac{u^{n+1}}{n+1} + C$$

$$\int \tan u du = -\ln |\cos u| + C \quad \int \cot u du = \ln |\sin u| + C \quad \int \sec u du = \ln |\sec u + \tan u| + C$$

$$\int \csc u du = -\ln |\csc u + \cot u| + C \quad \int a^x du = \frac{1}{\ln a} \cdot a^u + C$$

Key

1. $\int \frac{dx}{2x+3}$ $\left| \begin{array}{l} \frac{1}{2} \cdot \frac{du}{u} = \frac{1}{2} \int \frac{1}{u} du \\ u=2x+3 \\ \frac{du}{dx} = 2 \end{array} \right. \quad \int \frac{1}{u} du = \frac{1}{2} \int \frac{1}{u} \frac{du}{\frac{du}{dx}} = \frac{1}{2} \int \frac{1}{u} \frac{du}{2} = \frac{1}{8} \int \frac{1}{u} du$

2. $\int \frac{x^{-1}}{x-5} dx = \int \frac{1}{x-5} dx$ $\left| \begin{array}{l} u=x-5 \\ du=dx \end{array} \right. \quad \int \frac{1}{u} du = \frac{1}{2} \int \frac{1}{u} du$

3. $\int 2x^{-5} dx = \int (2x-5)x^{-1} dx$ $\left| \begin{array}{l} u=2x-5 \\ \frac{du}{dx}=2 \end{array} \right. \quad \int (2x-5)x^{-1} dx = \int \frac{1}{x} \cdot du$

4. $\int \frac{x}{x+1} dx = \int \frac{x+1-1}{x+1} dx = \int \frac{x+1}{x+1} dx - \int \frac{1}{x+1} dx$ $\left| \begin{array}{l} u=x+1 \\ \frac{du}{dx}=1 \end{array} \right. \quad \int \frac{x+1}{x+1} dx = \int 1 du = \int \frac{1}{u} du$

5. $\int \frac{(\ln x)^2 dx}{x}$ $\left| \begin{array}{l} u=\ln x \\ \frac{du}{dx}=\frac{1}{x} \end{array} \right. \quad \int u^2 du = \int u^2 \frac{du}{\frac{du}{dx}} = \int u^2 \frac{du}{u} = \int u du$

6. $\int \frac{dx}{\sqrt{x+1-x}} = \int \frac{dx}{\sqrt{2x}} \quad \left| \begin{array}{l} u=1+\sqrt{x} \\ u=1+x^{\frac{1}{2}} \\ \frac{du}{dx}=\frac{1}{2}x^{-\frac{1}{2}} \end{array} \right. \quad \int \frac{1}{u} du = \int \frac{1}{\sqrt{u}} du$

7. $\int \frac{x^2-4x+8}{x-3} dx$ $\left| \begin{array}{l} u=x-3 \\ du=dx \end{array} \right. \quad \int \frac{x-1+\frac{5}{x-3}}{x-3} dx = \int \frac{u-1+\frac{5}{u}}{u-3} du$

8. $\int \frac{x^2-5x^2+x-2}{x+1} dx$ $\left| \begin{array}{l} u=x+1 \\ du=dx \end{array} \right. \quad \int \frac{x^2-6x+7}{x+1} dx = \int \frac{u^2-6u+7}{u} du = \int u-6+\frac{7}{u} du$

9. $\int \frac{dx}{(2x+3)^2}$ $\left| \begin{array}{l} u=2x+3 \\ du=2dx \end{array} \right. \quad \int \frac{1}{(2x+3)^2} dx = \int \frac{1}{u^2} \frac{du}{2} = \frac{1}{2} \int u^{-2} du$

10. $\int 5 \sec x \tan x dx$ $\left| \begin{array}{l} u=\sec x \\ du=\sec x dx \\ du=\frac{du}{\sec x} \end{array} \right. \quad \int 5 \sec x \tan x dx = \int 5 \sec x \frac{du}{\sec x} = \int 5 du = 5u + C$

11. $\int xe^{\frac{1}{2}x^2} dx$ $\left| \begin{array}{l} u=-\frac{1}{2}x^2 \\ du=-x dx \end{array} \right. \quad \int xe^{\frac{1}{2}x^2} dx = \int xe^{-u} \cdot \frac{du}{-x} = -e^u \Big|_0^{\frac{1}{2}u^2} = -e^{-\frac{1}{2}x^2} + 1$

12. $\int e^{3-x} dx$ $\left| \begin{array}{l} u=3-x \\ du=-dx \end{array} \right. \quad \int e^{3-x} dx = \int e^u du = -e^u \Big|_0^{\frac{1}{2}u^2} = -e^{-\frac{1}{2}(3-x)} + 1$

13. $\int (\tan x + \sec x + \csc x) dx$ $\left| \begin{array}{l} u=\ln |\csc x + \cot x| \\ du=\csc x + \cot x dx \end{array} \right. \quad \int (\tan x + \sec x + \csc x) dx = -\ln |\csc x + \cot x| + C$

14. $\int (5-2x)^2 \cdot 7^{(5-2x)^3} dx$ $\left| \begin{array}{l} u=(5-2x)^3 \\ du=3(5-2x)^2(-2) \\ du=-6(5-2x)^2 dx \end{array} \right. \quad \int (5-2x)^2 \cdot 7^{(5-2x)^3} dx = \int (5-2x)^2 \cdot 7^u \cdot \frac{du}{-6(5-2x)^2} = \frac{1}{6} \int 7^u du = \frac{1}{6} \cdot \frac{1}{\ln 7} \cdot 7^u + C$

Review Integral Rules:

$$\int u^n du = \frac{u^{n+1}}{n+1} + C$$

$$\int \cot u du = -\ln |\sin u| + C$$

$$\int \csc u du = -\ln |\csc u + \cot u| + C$$

$$\text{If } x=1, u=\frac{3}{x}, \frac{du}{dx}=-\frac{3}{x^2}, \frac{dx}{du}=-\frac{x^2}{3}$$

$$\text{If } x=3, u=\frac{3}{x}, \frac{du}{dx}=-\frac{1}{x^2}$$

$$5. \int \frac{e^{3/x}}{x^2} dx$$

$u=3/x$	$\begin{cases} 3du = x^2 du \\ du = -\frac{x^2}{3} du \end{cases}$	$\left[-\frac{1}{3} e^u \right]_3^1$
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$$\begin{aligned} u &= 3/x \\ du &= -3x^{-2} dx \\ \frac{du}{dx} &= -3x^{-2} \\ \frac{du}{dx} &= -\frac{3}{x^2} \end{aligned}$$

$\int \frac{e^u}{x^2} \cdot -\frac{x^2}{3} du$	$-\frac{1}{3} e^u - \left(-\frac{1}{3} e^3 \right)$
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$$6. \int \frac{2x}{1-3x} dx$$

$u=1-3x$	$\int \frac{d(1-u)}{u} \cdot \frac{du}{-3}$	$7. \int \sec\left(\frac{x}{2}\right) dx$
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$$\begin{aligned} \frac{du}{dx} &= -3 \\ du &= -3dx \\ dx &= -\frac{du}{3} \\ \int \frac{2x}{1-3x} \cdot \frac{du}{-3} &= \int \frac{2}{3} \left(\frac{1-u}{u} \right) \frac{du}{-3} = -\frac{2}{9} \int \frac{1-u}{u} du \\ &= -\frac{2}{9} \int \frac{1}{u} du - \int du \\ u &= 1-3x \\ X &= \frac{1-u}{3} \end{aligned}$$

$\int \sec u \cdot du$	$\left[\frac{1}{2} \ln \left \sec\left(\frac{u}{2}\right) + \tan\left(\frac{u}{2}\right) \right + C \right]$
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$$8. \int \frac{4^{3x}}{1-4^{3x}} dx$$

$u=-4^{3x}$	$\int \frac{-1}{3 \cdot 4^{3x}} du$	$9. \int \frac{\cos x}{2^{\sin x}} dx$
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$$\begin{aligned} \frac{du}{dx} &= -12 \cdot 4^{3x} \cdot 3 \\ du &= -36 \cdot 4^{3x} dx \\ dx &= \frac{du}{-36 \cdot 4^{3x}} \\ \int \frac{4^{3x}}{1-4^{3x}} \cdot \frac{du}{-36 \cdot 4^{3x}} &= \int \frac{1}{u} du \end{aligned}$$

$\int 2^u \cdot \cos x \cdot \frac{du}{-36 \cdot 4^{3x}}$	$\left[\frac{1}{36} \ln 1-4^{3x} + C \right]$
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$$10. \int \frac{u}{\ln u} du$$

$u=\ln x$	$\int \frac{1}{u} du$	$11. \int \frac{2}{\ln x} dx$
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$$\begin{aligned} \frac{du}{dx} &= \frac{1}{x} \\ du &= \frac{1}{x} dx \\ dx &= x du \\ \int \frac{u}{\ln u} \cdot \frac{1}{x} du &= \int u du \\ u &= \ln x \\ x &= e^u \end{aligned}$$

$\int x \cdot \frac{1}{e^u} du$	$\left[-\frac{1}{e^u} \right]_{\ln x}^{\infty}$
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$$12. \int \frac{x^4+x-4}{x^2+2} dx$$

$u=1+\frac{1}{x^2}$	$\int x^2-2+\frac{x}{x^2+2} dx$	$13. \int \frac{3x^3}{x^2+2} dx$
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$$\begin{aligned} \frac{du}{dx} &= -2x^{-3} \\ du &= -2x^{-3} dx \\ dx &= -\frac{du}{2x^3} \\ \int \frac{3x^3}{x^2+2} \cdot -\frac{du}{2x^3} &= \int \frac{x^2-2}{x^2+2} dx \\ &= \int \frac{x^2}{x^2+2} dx - \int \frac{2}{x^2+2} dx \\ &= \int \frac{x^2}{x^2+1} dx - \int \frac{2}{x^2+1} dx \\ &= \int \frac{x^2}{x^2+1} dx - 2 \arctan x + C \\ &= \frac{1}{2} \ln |1+x^2| - \arctan x + C \\ X &= \frac{1-u}{3} \end{aligned}$$

$\int \frac{1}{u} du$	$\left[\frac{1}{2} \ln 1-3x - (1-3x) + C \right]$
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$$14. \int \frac{e^{2x}+2e^x+1}{e^x} dx$$

$u=\sec x - 1$	$\int (\sec x + 2e^x + 1) e^{-x} dx$	$15. \int \frac{\cos x}{2^{\sin x}} dx$
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$$\begin{aligned} \frac{du}{dx} &= \sec x \tan x \\ du &= \sec x \tan x dx \\ dx &= \frac{du}{\sec x \tan x} \\ \int e^x + \cancel{2} + e^{-x} \cancel{du} &= \int e^x + \cancel{2} + e^{-x} du \\ &= \int e^x + \cancel{2} + e^{-x} du \\ &= e^x + 2x - e^{-x} + C \end{aligned}$$

$\int 2^u \cdot \cos x \cdot \frac{du}{-36 \cdot 4^{3x}}$	$\left[\frac{1}{36} \ln 1-4^{3x} + C \right]$
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AP CALCULUS AB

Morning

Quiz Review on 5-2, 5-4, 5-5

$$1. \int \frac{(\ln x)^2}{5x} dx = u = \ln x \quad du = x dx$$

$$\frac{du}{dx} = \frac{1}{x} \quad \int \frac{u^2}{5x} \cdot x du = \frac{1}{5} \int u^2 du = \frac{1}{5} \frac{u^3}{3} = \frac{u^3}{15} + C$$

$$= \frac{(\ln x)^3}{15} + C$$

$$2. \int \frac{7x^3 + 21x^2 + 7x}{x^4 + 4x^3 + 2x^2} dx = u = x^4 + 4x^3 + 2x^2$$

$$\frac{du}{dx} = 4x^3 + 12x^2 + 4x = 4(x^3 + 3x^2 + x) \quad dx = \frac{du}{4(x^3 + 3x^2 + x)}$$

$$\int \frac{7(x^3 + 3x^2 + x)}{u} \cdot \frac{du}{4(x^3 + 3x^2 + x)} = \frac{7}{4} \int \frac{1}{u} du = \frac{7}{4} \ln|u| + C$$

$$= \frac{7}{4} \ln|x^4 + 4x^3 + 2x^2| + C$$

$$3. \int \frac{x^4 + 3x^3 - x^2 - 5}{x+2} dx = \int x^3 + x^2 - 3x + 6 - \frac{17}{x+2} dx = \frac{7}{4} \ln|x^4 + 4x^3 + 2x^2| + C$$

$$\begin{array}{r} 1 & 3 & -1 & 0 & -5 \\ \downarrow & -2 & -2 & 6 & -12 \\ 1 & 1 & -3 & 6 & -17 \end{array}$$

$$\downarrow \quad \begin{array}{l} u = x+2 \quad du = dx \\ \frac{du}{dx} = 1 \quad \int \frac{17}{u} du = 17 \ln|u| = 17 \ln|x+2| \end{array}$$

$$= \frac{7}{4} \ln|x^4 + 4x^3 + 2x^2| + C$$

$$4. \int_{\frac{4}{4}}^{\frac{9}{9}} \frac{dx}{\sqrt{x}(1+\sqrt{x})} = \begin{array}{l} \text{if } x=4, u=1+\sqrt{4}=3 \\ \text{if } x=9, u=1+\sqrt{9}=4 \end{array}$$

$$u = 1+x^{\frac{1}{2}} \quad \frac{du}{dx} = \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$$

$$dx = 2\sqrt{x} du \quad \int \frac{1}{\sqrt{x} \cdot u} \cdot 2\sqrt{x} du = 2 \int \frac{1}{u} du = 2 \ln|u| \Big|_3^4 = 2 \ln 4 - 2 \ln 3$$

$$= 2(\ln 4 - \ln 3)$$

$$= 2 \ln\left(\frac{4}{3}\right)$$

$$5. \int 2\cot(3x) - \sec(4x) dx = \int 2\cot 3x dx - \int \sec 4x dx$$

$$u = 3x \quad \frac{du}{dx} = 3 \quad du = 3dx \quad u = 4x \quad \frac{du}{dx} = 4 \quad dx = \frac{du}{4}$$

$$\frac{du}{3} = \frac{1}{3} du \quad \int \sec u \cdot \frac{du}{4} = \frac{1}{4} \int \sec u du$$

$$2 \int \cot u \cdot \frac{du}{3} = \frac{2}{3} \ln|\sin(u)| \quad \frac{1}{4} \int \sec u du = \frac{1}{4} \ln|\sec u + \tan u| + C$$

$$= \frac{2}{3} \ln|\sin(3x)| - \frac{1}{4} \ln|\sec(4x) + \tan(4x)| + C$$

$$6. \int \frac{7^x}{x^2} dx = \int \frac{7^u}{x^2} \cdot \frac{x^2 du}{-2} = -\frac{1}{2} \int 7^u du = -\frac{1}{2} \cdot \frac{7^u}{\ln 7} + C$$

$$u = 2x^{-1} \quad \frac{du}{dx} = -2x^{-2} = -\frac{2}{x^2} \quad dx = \frac{x^2 du}{-2}$$

$$= \frac{-7^{2x}}{2 \ln 7} + C$$