

5.2-5.5 Logs/Exponential Integrals Review WS #1

$\int \frac{1}{u} du = \ln u + C$	Power Rule: $\int u^n du = \frac{u^{n+1}}{n+1} + C$	$\int e^u du = e^u + C$	$\frac{d}{dx}[\ln u] = \frac{u'}{u}$	$\frac{d}{dx}[e^u] = e^u u'$
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Integrals Checklist Order: 1) Expand/Power Rule 2) U-Sub/Change of Variable 3) Long Division/Synthetic Division

Possible u-value locations: 1) exponent 2) value inside parentheses 3) denominator

Find the Indefinite Integral of functions below

1. $\int \frac{10}{3-2x} dx$

2. $\int \frac{5\sqrt{x} - 9x}{x^2} dx$

3. $\int \frac{3x^3 - 5x^2 + 10}{x^2 - 2} dx$

4. $\int \cos(2x) - \tan(3x) dx$

5. $\int \frac{(\ln x)^3}{5x} dx$

$$\int \tan u \, du = -\ln|\cos u| + C$$

$$\int \cot u \, du = \ln|\sin u| + C$$

$$\int \sec u \, du = \ln|\sec u + \tan u| + C$$

$$\int \csc u \, du = -\ln|\csc u + \cot u| + C$$

$$\int \cos u \, du = \sin u + C \quad \left| \frac{d}{dx}[\sin u] = (\cos u)u' \right.$$

$$\int a^u \, du = \left(\frac{1}{\ln a} \right) a^u + C$$

$$6. \int (3 - 6x)e^{x-x^2} dx$$

$$7. \int \frac{e^{5x}}{7 - 2e^{5x}} dx$$

$$8. \int \frac{e^{4x} - 5e^{2x} - 6}{e^x} dx$$

$$9. \int 2e^{5x} \sec(e^{5x}) dx$$

$$10. \int 4^{\sin(2x)} \cos(2x) dx$$

5.2-5.5 Logs/Exponential Integrals Review WS #1

Power Rule:	$\int \frac{1}{u} du = \ln u + C$	$\int u^n du = \frac{u^{n+1}}{n+1} + C$	$\int e^u du = e^u + C$	$\frac{d}{dx}[\ln u] = \frac{1}{u}$	$\frac{d}{dx}[e^u] = e^u u'$
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Key

Integrals Checklist Order: 1) Expand/Power Rule 2) U-Sub/Change of Variable 3) Long Division/Synthetic Division

Possible u-value locations: 1) exponent 2) value inside parentheses 3) denominator

Find the Indefinite Integral of functions below

*u-sub 1. $\int \frac{10}{3-2x} dx$

$$\begin{aligned} u &= 3-2x \\ \frac{du}{dx} &= -2 \\ dx &= \frac{du}{-2} \\ u &= 3-2x \\ \frac{du}{-2} &= dx \\ \int \frac{10}{u} \cdot \frac{du}{-2} &= -\frac{10}{2} \int \frac{1}{u} du \\ -5 \int \frac{1}{u} du &= -5 \ln|u| + C \\ -5 \ln|3-2x| + C & \end{aligned}$$

2. $\int \frac{5\sqrt{x} - 9x}{x^2} dx$

$$\begin{aligned} &\text{*Expand} \\ &\int (5x^{1/2} - 9x)x^{-2} dx \\ &\int 5x^{-3/2} - 9x^{-1} dx \\ &\int 5x^{-3/2} - \frac{9}{x} dx \end{aligned}$$

$$\frac{5x^{-1/2}}{-1/2} - 9 \ln|x| + C$$

$$-2 \cdot 5x^{-1/2} - 9 \ln|x| + C$$

$$-10x^{-1/2} - 9 \ln|x| + C$$

$$\frac{-10}{\sqrt{x}} - 9 \ln|x| + C$$

3. $\int \frac{3x^3 - 5x^2 + 10}{x^2 - 2} dx$

$$\begin{aligned} &\text{*Long Division:} \\ &\begin{array}{r} 3x^2 - 5x + 3 \\ \hline x^2 - 2 \end{array} \begin{array}{l} 6x \\ \hline 3x^3 - 5x^2 + 10 \\ \hline 3x^3 + 6x \\ \hline -5x^2 + 10 \\ \hline +5x^2 + 10 \\ \hline 0 \end{array} \end{aligned}$$

$$\begin{aligned} &\int 3x - 5 + \frac{6x}{x^2 - 2} dx \\ u &= x^2 - 2 \\ \frac{du}{dx} &= 2x \\ du &= 2x dx \end{aligned}$$

4. $\int \cos(2x) - \tan(3x) dx$

$$u = 2x \quad u = 3x$$

$$\int \cos u du - \int \tan u du$$

$$\frac{1}{2} \sin(2x) - \left(-\frac{1}{3} \ln|\cos u| \right) + C$$

$$\frac{1}{2} \sin(2x) + \frac{1}{3} \ln|\cos(3x)| + C$$

5. $\int \frac{(\ln x)^3}{5x} dx$

$$\begin{aligned} u &= \ln x \\ \frac{du}{dx} &= \frac{1}{x} \\ dx &= x du \end{aligned}$$

$$\frac{1}{5} \cdot \frac{u^4}{4} + C$$

$$\boxed{\frac{1}{20} (\ln x)^4 + C}$$

$$\frac{1}{20} u^4 + C$$

$$\int \tan u \, du = -\ln|\cos u| + C$$

$$\int \cot u \, du = \ln|\sin u| + C$$

$$\int \sec u \, du = \ln|\sec u + \tan u| + C$$

$$\int \csc u \, du = -\ln|\csc u + \cot u| + C$$

$$\int \cos u \, du = \sin u + C \quad \frac{d}{dx}[\sin u] = (\cos u)u'$$

$$6. \int (3 - 6x)e^{x-x^2} dx$$

$$\begin{aligned} u &= x - x^2 & \int (3-6x)e^u \cdot \frac{du}{1-2x} \\ \frac{du}{dx} &= 1-2x & \int 3(1-2x) \cdot e^u \cdot \frac{du}{(1-2x)} \\ (1-2x)dx &= du & \int e^u du = 3e^u + C \\ dx &= \frac{du}{1-2x} & = 3e^{x-x^2} + C \end{aligned}$$

$$8. \int \frac{e^{4x}-5e^{2x}-6}{e^x} dx$$

$$\begin{aligned} &\int (e^{4x}-5e^{2x}-6)e^{-x} dx & \frac{1}{3}e^{3x} - 5e^x - (-6e^{-x}) + C \\ &\int e^{3x} - 5e^x - 6e^{-x} dx & = \frac{1}{3}e^{3x} - 5e^x + 6e^{-x} + C \\ &\int e^{3x} dx - \int 5e^x dx - 6 \int e^{-x} dx \end{aligned}$$

$$10. \int 4^{\sin(2x)} \cos(2x) dx$$

$$\begin{aligned} u &= \sin(2x) \\ \frac{du}{dx} &= \cos(2x) \cdot 2 \end{aligned}$$

$$dx \cdot 2\cos(2x) = du$$

$$dx = \frac{du}{2\cos(2x)}$$

$$\int 4^u \cdot \cos(2x) \cdot \frac{du}{2\cos(2x)}$$

$$\frac{1}{2} \int 4^u du$$

$$* \int a^u du = \frac{1}{\ln a} \cdot a^u + C$$

$$\frac{1}{2} \int 4^u du = \frac{1}{2 \ln 4} \cdot 4^u + C$$

$$= \boxed{\frac{4^{\sin(2x)}}{2 \ln 4} + C}$$

$$7. \int \frac{e^{5x}}{7-2e^{5x}} dx$$

$$\begin{aligned} u &= 7-2e^{5x} & dx = \frac{du}{-10e^{5x}} \\ \frac{du}{dx} &= -2e^{5x} \cdot 5 & \int \frac{e^{5x}}{u} \cdot \frac{du}{-10e^{5x}} \\ \frac{du}{dx} &= -10e^{5x} & = -\frac{1}{10} \int \frac{1}{u} du \\ -10e^{5x} dx &= du & = -\frac{1}{10} \ln|u| + C \end{aligned}$$

$$9. \int 2e^{5x} \sec(e^{5x}) dx$$

$$\begin{aligned} u &= e^{5x} & \int 2e^{5x} \cdot \sec(u) \cdot \frac{du}{5e^{5x}} \\ \frac{du}{dx} &= e^{5x} \cdot 5 & = \frac{2}{5} \int \sec u du \\ 5e^{5x} dx &= du & = \frac{2}{5} \ln|\sec u + \tan u| + C \\ dx &= \frac{du}{5e^{5x}} & = \boxed{\frac{2}{5} \ln|\sec(e^{5x}) + \tan(e^{5x})| + C} \end{aligned}$$