

5.2-5.5 Logs/Exponential Integrals Review WS #2

$\int \frac{1}{u} du = \ln u + C$	Power Rule: $\int u^n du = \frac{u^{n+1}}{n+1} + C$	$\int e^u du = e^u + C$	$\frac{d}{dx}[\ln u] = \frac{u'}{u}$	$\frac{d}{dx}[e^u] = e^u u'$
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Integrals Checklist Order: **1)** Expand/Power Rule **2)** U-Sub/Change of Variable **3)** Long Division/Synthetic Division

Possible u-value locations: **1)** exponent **2)** value inside parentheses **3)** denominator

Find the Indefinite Integral of functions below

1. $\int \frac{\sqrt{x^5 - 3x^2}}{x^3} dx$

2. $\int \frac{e}{7 - 3x} dx$

3. $\int \frac{2x^3 - 3x^2 + 5}{x+2} dx$

4. $\int 4 \sin(\pi x) - \cot\left(\frac{x}{3}\right) dx$

5. $\int \frac{3}{x\sqrt{(\ln x)}} dx$

$$\int \tan u \, du = -\ln|\cos u| + C$$

$$\int \cot u \, du = \ln|\sin u| + C$$

$$\int \sec u \, du = \ln|\sec u + \tan u| + C$$

$$\int \csc u \, du = -\ln|\csc u + \cot u| + C$$

$$\int \cos u \, du = \sin u + C$$

$$\frac{d}{dx}[\sin u] = (\cos u)u'$$

$$\int a^u \, du = \left(\frac{1}{\ln a}\right)a^u + C$$

$$\int \sin u \, du = -\cos u + C$$

$$\frac{d}{dx}\cos u = -(\sin u)u'$$

$$6. \int (10 - 4x)e^{5x-x^2} dx$$

$$7. \int \frac{e^{2x}}{\sqrt[3]{17 - 3e^{2x}}} dx$$

$$8. \int \frac{e^{3x}-5e^x-6}{e^x} dx$$

$$9. \int 3e^{\pi x} \tan(e^{\pi x}) \, dx$$

$$10. \int 4^{\cos(4x)} \sin(4x) dx$$

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$\int \frac{1}{u} du = \ln u + C$	Power Rule: $\int u^n du = \frac{u^{n+1}}{n+1} + C$	$\int e^u du = e^u + C$	$\frac{d}{dx}[\ln u] = \frac{u'}{u}$	$\frac{d}{dx}[e^{u'}] = e^{u'} u'$
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Key

Integrals Checklist Order: 1) Expand/Power Rule 2) U-Sub/Change of Variable 3) Long Division/Synthetic Division

Possible u-value locations: 1) exponent 2) value inside parentheses 3) denominator

Find the Indefinite Integral of functions below

1. $\int \frac{\sqrt{x^5 - 3x^2}}{x^3} dx$

*expand

$$\int (x^{5/2} - 3x^2)x^{-3} dx$$

$$\int x^{-1/2} - 3x^{-1} dx$$

$$\begin{aligned} & \int x^{-1/2} dx - 3 \int \frac{1}{x} dx \\ & \frac{x^{1/2}}{1/2} - 3 \ln|x| + C \\ & \frac{2}{1} x^{1/2} - 3 \ln|x| + C \\ & \boxed{2x^{1/2} - 3 \ln|x| + C} \end{aligned}$$

2. $\int \frac{e}{7-3x} dx$

*u-sub
 $u = 7-3x$
 $\frac{du}{dx} = -3$
 $-3dx = du$
 $dx = \frac{du}{-3}$

$$\int \frac{e}{u} \cdot \frac{du}{-3}$$

$$\frac{e}{-3} \int \frac{1}{u} du$$

$$-\frac{e}{3} \ln|u| + C$$

$-\frac{e}{3} \ln|7-3x| + C$

3. $\int \frac{2x^3 - 3x^2 + 5}{x+2} dx$

*long division

$$\begin{array}{r} 2x^3 - 3x^2 + 5 \\ \underline{-2} \quad \quad \quad 0 \quad 5 \\ 2x^3 - 7x^2 + 14 \\ \underline{-2x^3 + 4x^2} \\ -7x^2 + 5 \\ \underline{+7x^2 + 14x} \\ +14x + 5 \\ \underline{-14x - 28} \\ -23 \end{array}$$

$$\int 2x^2 - 7x + 14 - \frac{23}{x+2} dx$$

$$\begin{aligned} & \int 2x^2 - 7x + 14 dx \\ & \quad u = x+2 \\ & \quad \frac{du}{dx} = 1 \\ & \quad du = dx \\ & \int 23 \int \frac{1}{u} du \\ & \boxed{\frac{2x^3}{3} - \frac{7x^2}{2} + 14x - 23 \ln|x+2| + C} \end{aligned}$$

4. $\int 4 \sin(\pi x) - \cot\left(\frac{x}{3}\right) dx$

$u = \pi x$
 $\frac{du}{dx} = \pi$
 $dx = \frac{du}{\pi}$

$u = \frac{1}{3}x$
 $\frac{du}{dx} = \frac{1}{3}$
 $dx = 3du$

$$\int 4 \sin u \cdot \frac{du}{\pi}$$

$$4 \cdot \frac{1}{\pi} \int \sin u du - 3 \int \cot u du$$

$$\boxed{-\frac{4}{\pi} \cos(\pi x) - 3(\ln|\sin(\frac{x}{3})|) + C}$$

5. $\int \frac{3}{x\sqrt{(\ln x)}} dx$

$$\int \frac{3}{x(\ln x)^{1/2}} dx$$

$$u = \ln x \quad x du = dx$$

$$\frac{du}{dx} = \frac{1}{x}$$

$$\int \frac{3}{x \cdot u^{1/2}} \cdot x du$$

$$\int 3u^{-1/2} du$$

$$\begin{aligned} & \frac{3u^{1/2}}{1/2} + C \\ & \frac{2}{1} \cdot 3u^{1/2} + C \\ & \boxed{6(\ln x)^{1/2} + C} \end{aligned}$$

$$\int \tan u \, du = -\ln|\cos u| + C$$

$$\int \cot u \, du = \ln|\sin u| + C$$

$$\int \sec u \, du = \ln|\sec u + \tan u| + C$$

$$\int \csc u \, du = -\ln|\csc u + \cot u| + C$$

$$\int \cos u \, du = \sin u + C$$

$$\frac{d}{dx}[\sin u] = (\cos u)u'$$

$$6. \int (10 - 4x)e^{5x-x^2} dx$$

$$\begin{aligned} u &= 5x - x^2 \\ \frac{du}{dx} &= 5 - 2x \\ dx(5-2x) &= du \\ \frac{dx}{5-2x} &= \frac{du}{5-2x} \\ &\left[\begin{array}{l} \int (10-4x) \cdot e^u \cdot \frac{du}{5-2x} \\ \int 2(5-2x) \cdot e^u \cdot \frac{du}{(5-2x)} \\ 2 \int e^u du \\ = 2e^u + C \end{array} \right] \\ &= 2e^{5x-x^2} + C \end{aligned}$$

$$8. \int \frac{e^{3x}-5e^x-6}{e^x} dx$$

*expand

$$\int (e^{3x}-5e^x-6)e^{-x} dx \quad \left[\frac{1}{2}e^{2x} - 5x - 6(-e^{-x}) + C \right]$$

$$\int e^{2x} - 5e^x - 6e^{-x} dx$$

$$\begin{aligned} &\int e^{2x} dx - 5 \int e^x dx - 6 \int e^{-x} dx \\ u &= 2x \\ \frac{du}{dx} &= 2 \\ dx &= \frac{du}{2} \\ \int e^{\frac{du}{2}} \frac{du}{2} &= \frac{1}{2} \int e^u du \end{aligned}$$

$$\begin{aligned} u &= -x \\ \frac{du}{dx} &= -1 \\ dx &= -du \\ \int e^{-x} (-du) &= - \int e^u du \end{aligned}$$

$$10. \int 4^{\cos(4x)} \sin(4x) dx$$

$$\int 4^{\cos(4x)} \cdot \sin(4x) dx$$

$$u = \cos(4x)$$

$$\frac{du}{dx} = -\sin(4x) \cdot 4$$

$$dx(-4\sin(4x)) = du$$

$$dx = \frac{du}{-4\sin(4x)}$$

$$\int 4^u \cdot \sin(4x) \cdot \frac{du}{-4\sin(4x)}$$

$$-\frac{1}{4} \int 4^u du$$

$$-\frac{1}{4} \cdot \frac{1}{\ln 4} \cdot 4^u + C$$

$$-\frac{1}{4 \ln 4} \cdot 4^{\cos(4x)} + C$$