

5.2-5.5 Logs/Exponential Integrals Review WS #4

$$\int \frac{1}{u} du = \ln|u| + C \quad \left| \begin{array}{l} \text{Power Rule:} \\ \int u^n du = \frac{u^{n+1}}{n+1} + C \end{array} \right. \quad \left| \int e^u du = e^u + C \right. \quad \left| \frac{d}{dx}[\ln u] = \frac{u'}{u} \right. \quad \left| \frac{d}{dx}[e^u] = e^u u' \right.$$

Integrals Checklist Order: **1)**Expand/Power Rule **2)** U-Sub/Change of Variable **3)**Long Division/Synthetic Division

Possible u-value locations: **1)** exponent **2)** value inside parentheses **3)** denominator

Find the Indefinite Integral of functions below

1. $\int \frac{2x^3 - 4x + 3}{x-2} dx$

2. $\int 2 \tan\left(\frac{ex}{4}\right) - \sin(5x) dx$

3. $\int \frac{7\pi}{2-9x} dx$

4. $\int \frac{\sqrt{x^5 - 3x^2}}{x^3} dx$

5. $\int \frac{2\sqrt{(\ln x)^9}}{3x} dx$

$$\int \tan u \, du = -\ln|\cos u| + C$$

$$\int \cot u \, du = \ln|\sin u| + C$$

$$\int \sec u \, du = \ln|\sec u + \tan u| + C$$

$$\int \csc u \, du = -\ln|\csc u + \cot u| + C$$

$$\int \cos u \, du = \sin u + C \quad \left| \quad \frac{d}{dx}[\sin u] = (\cos u)u' \right.$$

$$\int a^u \, du = \left(\frac{1}{\ln a}\right)a^u + C$$

$$\int \sin u \, du = -\cos u + C$$

$$\frac{d}{dx} \cos u = -(\sin u)u'$$

$$6. \int (7 - 14x)6^{x-x^2} dx$$

$$7. \int \frac{3e^{2x}}{\sqrt[5]{(11 - 5e^{2x})}} dx$$

$$8. \int \frac{6e^{5x} - 3e^{2x} - 1}{e^{2x}} dx$$

$$9. \int 3e^{x-\pi x} \cos(e^{x-\pi x}) dx$$

$$10. \int e^{\cos(1-\pi x)} \sin(1-\pi x) dx$$

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Key

Power Rule: $\int \frac{1}{u} du = \ln|u| + C$ | $\int u^n du = \frac{u^{n+1}}{n+1} + C$ | $\int e^u du = e^u + C$ | $\frac{d}{dx}[\ln u] = \frac{u'}{u}$ | $\frac{d}{dx}[e^u] = e^u \cdot u'$

Integrals Checklist Order: 1) Expand/Power Rule 2) U-Sub/Change of Variable 3) Long Division/Synthetic Division

Possible u-value locations: 1) exponent 2) value inside parentheses 3) denominator

Find the Indefinite Integral of functions below

<p>1. $\int \frac{2x^3 - 4x + 3}{x-2} dx$</p> <p><i>*Long Division</i></p> $\begin{array}{r} 2x^2 + 4x + 4 + \frac{11}{x-2} \\ x-2 \overline{) 2x^3 - 4x + 3} \\ \underline{2x^3 - 4x^2 + 8} \\ 4x^2 - 4x + 3 \\ \underline{4x^2 + 8x} \\ 4x + 3 \\ \underline{4x + 8} \\ 11 \end{array}$ <p>$\int (2x^2 + 4x + 4 + \frac{11}{x-2}) dx$</p> $\frac{2x^3}{3} + \frac{4x^2}{2} + 4x + 11 \ln x-2 + C \rightarrow \frac{2}{3}x^3 + 2x^2 + 4x + 11 \ln x-2 + C$	<p>2. $\int 2 \tan\left(\frac{ex}{4}\right) - \sin(5x) dx$</p> <p><i>*Synthetic division</i></p> $\begin{array}{r} 2 \quad 0 \quad -4 \quad 3 \\ \downarrow \quad 4 \quad 8 \quad 8 \\ \hline 2 \quad 4 \quad 4 \quad 11 \end{array}$ <p>$2x^2 + 4x + 4 + \frac{11}{x-2}$</p> <p>$\int (2x^2 + 4x + 4 + \frac{11}{x-2}) dx$</p>
<p>$\int 2x^2 + 4x + 4 dx + 11 \int \frac{1}{x-2} dx$</p>	<p>$2 \int \tan\left(\frac{ex}{4}\right) dx - \int \sin(5x) dx$</p> <p>$u = \frac{ex}{4}$ $\frac{du}{dx} = \frac{e}{4}$ $e dx = 4 du$ $dx = \frac{4}{e} du$</p> <p>$2 \int \tan u \cdot \frac{4}{e} du$</p> <p>$u = 5x$ $\frac{du}{dx} = 5$ $dx = \frac{du}{5}$</p> <p>$\int \sin u \cdot \frac{du}{5}$</p> <p>$\frac{8}{e} \ln \cos u - \frac{1}{5} \cos u + C$</p>
<p>$\frac{2x^3}{3} + \frac{4x^2}{2} + 4x + 11 \ln x-2 + C$</p>	<p>$\frac{8}{e} \ln \cos(\frac{ex}{4}) - \frac{1}{5} \cos(5x) + C$</p>

<p>3. $\int \frac{7\pi}{2-9x} dx$</p> <p>$u = 2-9x$ $\frac{du}{dx} = -9$ $-9 dx = du$ $dx = \frac{du}{-9}$</p> <p>$\int \frac{7\pi}{u} \cdot \frac{du}{-9}$</p> <p>$-\frac{7\pi}{9} \ln u + C$</p> <p>$-\frac{7\pi}{9} \ln 2-9x + C$</p>	<p>4. $\int \frac{\sqrt{x^5 - 3x^2}}{x^3} dx$</p> <p><i>*expand</i></p> <p>$\int (x^{5/2} - 3x^2) x^{-3} dx$</p> <p>$\int x^{-1/2} - 3x^{-1} dx$</p> <p>$\int x^{-1/2} dx - 3 \int \frac{1}{x} dx$</p> <p>$\frac{x^{1/2}}{1/2} - 3 \ln x + C$</p> <p>$2x^{1/2} - 3 \ln x + C$</p>
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5. $\int \frac{2\sqrt{(\ln x)^9}}{3x} dx$

$u = \ln x$
 $\frac{du}{dx} = \frac{1}{x}$
 $dx = x du$

$\int \frac{2 \cdot u^{9/2}}{3x} \cdot x du$

$\frac{2}{3} \int u^{9/2} du$

$\frac{2}{3} \cdot \frac{2}{11} \cdot u^{11/2} + C$

$\frac{4}{33} (\ln x)^{11/2} + C$

$$\int \tan u \, du = -\ln|\cos u| + C$$

$$\int \cot u \, du = \ln|\sin u| + C$$

$$\int \sec u \, du = \ln|\sec u + \tan u| + C$$

$$\int \csc u \, du = -\ln|\csc u + \cot u| + C$$

$$\int \cos u \, du = \sin u + C \quad \left| \quad \frac{d}{dx}[\sin u] = (\cos u)u' \right.$$

$$\int a^u \, du = \left(\frac{1}{\ln a}\right)a^u + C$$

$$\int \sin u \, du = -\cos u + C$$

$$\frac{d}{dx} \cos u = -(\sin u)u'$$

6. $\int (7 - 14x)6^{x-x^2} dx$

$$u = x - x^2$$

$$\frac{du}{dx} = 1 - 2x$$

$$dx(1 - 2x) = du$$

$$dx = \frac{du}{1 - 2x}$$

$$\int (7 - 14x) \cdot 6^u \cdot \frac{du}{1 - 2x}$$

$$\int 7(1 - 2x) \cdot 6^u \cdot \frac{du}{(1 - 2x)}$$

$$7 \int 6^u du$$

$$7 \cdot \frac{1}{\ln 6} \cdot 6^u + C$$

$$\boxed{\frac{7}{\ln 6} \cdot 6^{x-x^2} + C}$$

7. $\int \frac{3e^{2x}}{\sqrt[5]{(11 - 5e^{2x})}} dx$

$$\int \frac{3e^{2x}}{(11 - 5e^{2x})^{1/5}} dx$$

$$u = 11 - 5e^{2x}$$

$$\frac{du}{dx} = -5e^{2x} \cdot 2$$

$$\frac{du}{dx} = -10e^{2x}$$

$$dx(-10e^{2x}) = du$$

$$dx = \frac{du}{-10e^{2x}}$$

$$\int \frac{3e^{2x}}{u^{1/5}} \cdot \frac{du}{-10e^{2x}}$$

$$-\frac{3}{10} \int u^{-1/5} du$$

$$-\frac{3}{10} \cdot \frac{u^{4/5}}{4/5} + C$$

$$-\frac{3}{10} \cdot \frac{5}{4} u^{4/5} + C$$

$$\boxed{-\frac{3}{8} (11 - 5e^{2x})^{4/5} + C}$$

8. $\int \frac{6e^{5x} - 3e^{2x} - 1}{e^{2x}} dx$

*expand

$$\int (6e^{5x} - 3e^{2x} - 1)e^{-2x} dx$$

$$\int 6e^{3x} - 3 - 1e^{-2x} dx$$

$$6 \int e^{3x} dx - 3 \int 1 dx - 1 \int e^{-2x} dx$$

$$\frac{u=3x}{\frac{du}{dx}=3} \left| 6 \int e^u \cdot \frac{du}{3} \right. \quad \left. \frac{u=-2x}{\frac{du}{dx}=-2} \left| \int e^u \cdot \frac{du}{-2} \right. \right.$$

$$\frac{du}{dx} = 3 \quad \left| \quad \frac{du}{dx} = -2 \right.$$

$$dx = \frac{du}{3} \quad \left| \quad dx = \frac{du}{-2} \right.$$

9. $\int 3e^{x-\pi x} \cos(e^{x-\pi x}) dx$

$$u = e^{x-\pi x}$$

$$\frac{du}{dx} = e^{x-\pi x} \cdot (1-\pi)$$

$$dx[e^{x-\pi x}(1-\pi)] = du$$

$$dx = \frac{du}{e^{x-\pi x}(1-\pi)}$$

$$\int 3e^{x-\pi x} \cos u \cdot \frac{du}{e^{x-\pi x}(1-\pi)}$$

$$\frac{3}{1-\pi} \int \cos u \, du$$

$$\frac{3}{1-\pi} \cdot \sin u + C$$

$$\boxed{\frac{3}{1-\pi} \sin(e^{x-\pi x}) + C}$$

10. $\int e^{\cos(1-\pi x)} \sin(1-\pi x) dx$

$$u = \cos(1-\pi x)$$

$$\frac{du}{dx} = -\sin(1-\pi x) \cdot -\pi$$

$$dx(\pi \sin(1-\pi x)) = du$$

$$dx = \frac{du}{\pi \sin(1-\pi x)}$$

$$\int e^u \cdot \sin(1-\pi x) \cdot \frac{du}{\pi \sin(1-\pi x)}$$

$$\frac{1}{\pi} \int e^u du$$

$$\frac{1}{\pi} e^u + C$$

$$\boxed{\frac{1}{\pi} e^{\cos(1-\pi x)} + C}$$