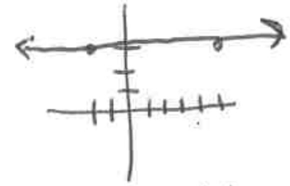
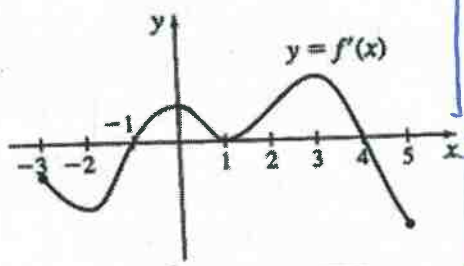
5.2 AP Practice Problems (p.330)



1. A function f is continuous on the closed interval $[-2, 5]$, differentiable on the open interval $(-2, 5)$, and $f(-2) = f(5) = 3$. Which of the following statements must be true?
   (A) There is a number c in the interval $(-2, 5)$ for which $f'(c) = 0$.
   (B) $f'(x) > 0$ for all numbers in the interval $(-2, 5)$.
   (C) $f'(x) = 3$ for all numbers in the interval $(-2, 5)$.
   $\boxed{\text{(D)}}$ None of the above

* there could be more than a c-value where $f'(c) = 0$ if $f(x)$ is a horizontal line

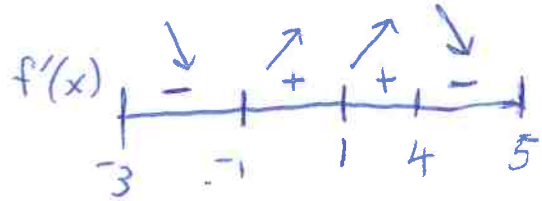
2. The graph of f' for the interval $[-3, 5]$ is shown below. On what interval(s) is(are) f decreasing?



positive slope

slope = 0

negative slope



(A) $[-3, -2]$, $[0, 1]$ and $[3, 5]$ $\boxed{\text{(B)}}$ $[-3, -1]$ and $[4, 5]$

(C) $[-3, -2]$ and $[3, 5]$ (D) $[-3, -2]$ and $[4, 5]$

3. For the function $f(x) = \sqrt{x}$, find the value(s) of c that satisfy the conclusion of the Mean Value Theorem on the interval $[0, 4]$.

(A) 0 (B) $\frac{1}{2}$ $\boxed{\text{(C)}}$ 1 (D) 2

*set $f'(x) = $ slope

$f'(x) = \frac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{x}}$

$f(x)$ continuous on $[0, 4]$

$f(x)$ is differentiable on $(0, 4)$

$f(0) = \sqrt{0} = 0$

$f(4) = \sqrt{4} = 2$

slope: $\frac{2-0}{4-0} = \frac{1}{2}$

$\frac{1}{2\sqrt{x}} = \frac{1}{2}$

$2\sqrt{x} = 2$

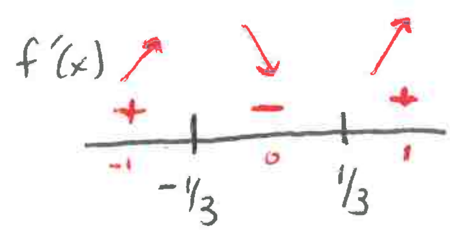
$\sqrt{x} = \frac{2}{2} = 1$

$x = 1$

4. On what interval(s) is the function $f(x) = e^{3x^3 - x}$ increasing?

(A) $(-\infty, \infty)$ (B) $[0, \infty)$

$\boxed{\text{(C)}}$ $\left(-\infty, -\frac{1}{3}\right]$ and $\left[\frac{1}{3}, \infty\right)$ (D) $\left[-\frac{1}{3}, \infty\right)$



$f'(x) = e^{3x^3 - x} \cdot (9x^2 - 1)$

$0 = (e^{3x^3 - x})(9x^2 - 1)$

$9x^2 - 1 = 0$

$9x^2 = 1$

$x^2 = \frac{1}{9}$

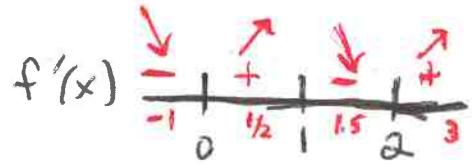
$x = \pm\frac{1}{3}$

$\boxed{(-\infty, -1/3), (1/3, \infty)}$

5. For which values of x is the function $f(x) = x^4 - 4x^3 + 4x^2 + 1$ decreasing?

$\boxed{\text{(A)}}$ $x < 0$ or $1 < x < 2$ (B) $x < 0$ or $x > 2$

(C) $0 < x < 2$ (D) $x > 2$ only

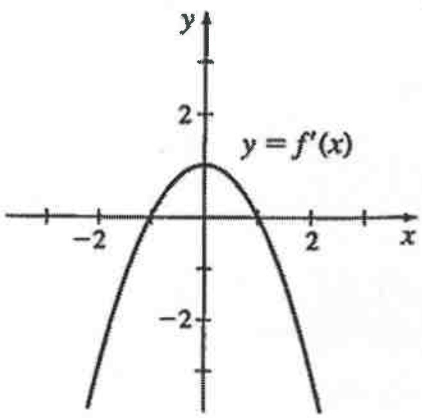


$\boxed{(-\infty, 0), (1, 2)}$

$f'(x) = 4x^3 - 12x^2 + 8x$ | $0 = 4x(x-2)(x-1)$

$0 = 4x(x^2 - 3x + 2)$ | $x = 0, 2, 1$

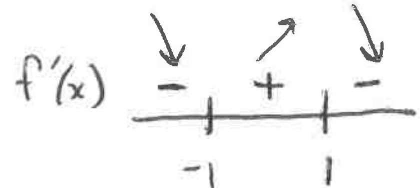
6. The graph of the derivative of f is shown. Which of the following can be the graph of f?

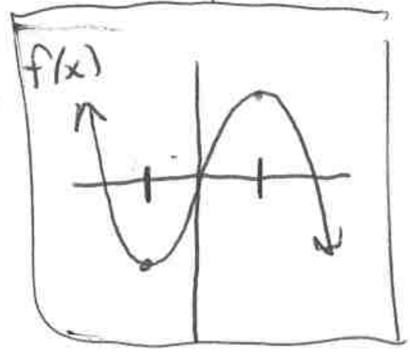


pos. slope

← slope = 0

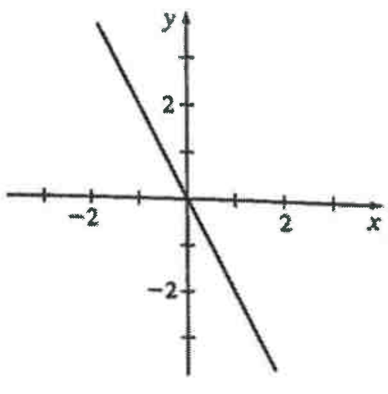
neg. slope



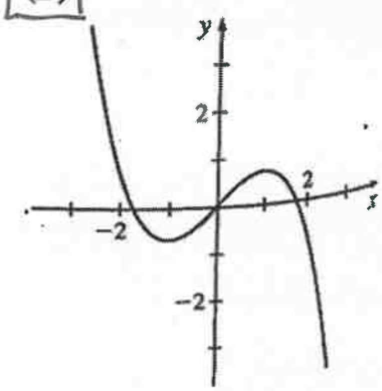


$\boxed{\text{(B)}}$

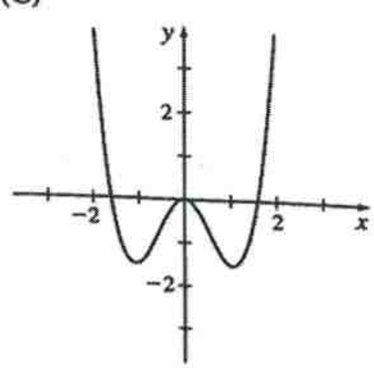
(A)



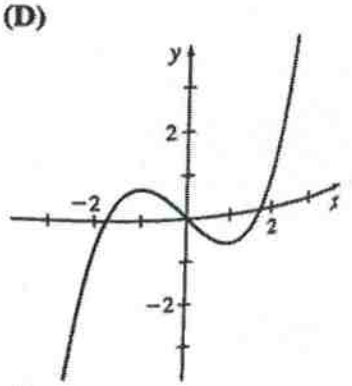
(B)



(C)



(D)



7. Suppose f and g are differentiable functions for which

- $f(x) > 0$ for all real numbers
- $g(0) = 4$

If $h(x) = f(x)g(x)$ and $h'(x) = f'(x)g(x)$ for all real numbers, then $g(x)$ equals

(A) $f(x)$ (B) $g'(x)$ (C) 0 $\boxed{\text{(D)}}$ 4

* find first $h'(x)$ using product Rule

$$h(x) = f(x)g(x)$$

$$h'(x) = f'(x)g(x) + f(x)g'(x)$$

$$\boxed{g(x) = 4}$$

Since $h'(x) = f'(x)g(x)$ only, then $f(x)g'(x) = 0$

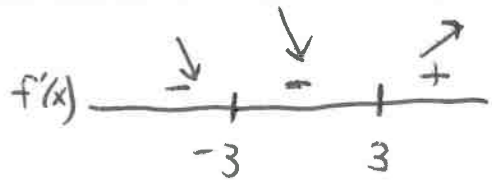
$g(x)$ must be a constant in order for $g'(x)$ to be equal to zero

8. The domain of the function f is the set of all real numbers. If $f'(x) = \dfrac{|x^2 - 9|}{x - 3}$, then f is increasing on the interval

(A) $(-\infty, \infty)$. (B) $[-3, 3]$.

(C) $[3, 9]$. $\boxed{\text{(D)}}$ $[3, \infty)$.

$$f'(x) = \frac{|(x-3)(x+3)|}{(x-3)}$$



9. An object moves along the x-axis so that its distance from the origin at any time $\boxed{t \geq 0}$ is given by $x(t) = t^3 + \frac{3}{2}t^2 - 18t + 4$. At what times t is the object at rest?

(A) 2 and -3 $\quad$ $\boxed{\text{(B)}}$ 2 only $\quad$ (C) 3 only $\quad$ (D) 0 and 2

$v(t) = 3t^2 + \frac{3}{2}\cdot 2t - 18$

$0 = 3t^2 + 3t - 18$

$0 = 3(t^2 + t - 6)$

$0 = 3(t+3)(t-2)$

$t = 2, \cancel{-3}$

$v(t)$: 0 $\quad$ $-$ (0) $\quad$ 2 $\quad$ $+$ (3)

10. Which of the following statements is true for the function

$$f(x) = \frac{\ln x}{x} \qquad x > 0$$

(A) f is increasing on the interval $(0, \infty)$.
(B) f is increasing on the interval $[e, \infty)$.
(C) f is decreasing on the interval $[1, e]$.
$\boxed{\text{(D)}}$ f is decreasing on the interval $[e, \infty)$.

$f'(x) = \dfrac{\left(\frac{1}{x}\right)(x) - (\ln x)(1)}{x^2}$

$f'(x) = \dfrac{1 - \ln x}{x^2}$

$1 - \ln x = 0$

$1 = \ln x$

$x = e$

$f'(x)$: 0 $\quad$ $+$ (1) $\quad$ e $\quad$ $-$ (e^2)

$f(x)$ decreasing on (e, ∞) b/c $f'(x) < 0$