

## 5.2 Exercises

See [CalcChat.com](http://CalcChat.com) for tutorial help and worked-out solutions to odd-numbered exercises.**Finding an Indefinite Integral** In Exercises 1–26, find the indefinite integral.

1.  $\int \frac{5}{x} dx$
2.  $\int \frac{10}{x} dx$
3.  $\int \frac{1}{x+1} dx$
4.  $\int \frac{1}{x-5} dx$
5.  $\int \frac{1}{2x+5} dx$
6.  $\int \frac{9}{5-4x} dx$
7.  $\int \frac{x}{x^2-3} dx$
8.  $\int \frac{x^2}{5-x^3} dx$
9.  $\int \frac{4x^3+3}{x^4+3x} dx$
10.  $\int \frac{x^2-2x}{x^3-3x^2} dx$
11.  $\int \frac{x^2-4}{x} dx$
12.  $\int \frac{x^3-8x}{x^2} dx$
13.  $\int \frac{x^2+2x+3}{x^3+3x^2+9x} dx$
14.  $\int \frac{x^2+4x}{x^3+6x^2+5} dx$
15.  $\int \frac{x^2-3x+2}{x+1} dx$
16.  $\int \frac{2x^2+7x-3}{x-2} dx$
17.  $\int \frac{x^3-3x^2+5}{x-3} dx$
18.  $\int \frac{x^3-6x-20}{x+5} dx$
19.  $\int \frac{x^4+x-4}{x^2+2} dx$
20.  $\int \frac{x^3-4x^2-4x+20}{x^2-5} dx$
21.  $\int \frac{(\ln x)^2}{x} dx$
22.  $\int \frac{1}{x \ln x^3} dx$
23.  $\int \frac{1}{\sqrt{x}(1-3\sqrt{x})} dx$
24.  $\int \frac{1}{x^{2/3}(1+x^{1/3})} dx$
25.  $\int \frac{2x}{(x-1)^2} dx$
26.  $\int \frac{x(x-2)}{(x-1)^3} dx$

**Finding an Indefinite Integral by  $u$ -Substitution** In Exercises 27–30, find the indefinite integral by  $u$ -substitution. (Hint: Let  $u$  be the denominator of the integrand.)

27.  $\int \frac{1}{1+\sqrt{2x}} dx$
28.  $\int \frac{1}{1+\sqrt{3x}} dx$
29.  $\int \frac{\sqrt{x}}{\sqrt{x}-3} dx$
30.  $\int \frac{\sqrt[3]{x}}{\sqrt[3]{x}-1} dx$

**Finding an Indefinite Integral of a Trigonometric Function** In Exercises 31–40, find the indefinite integral.

31.  $\int \cot \frac{\theta}{3} d\theta$
32.  $\int \tan 5\theta d\theta$

33.  $\int \csc 2x dx$
34.  $\int \sec \frac{x}{2} dx$
35.  $\int (\cos 3\theta - 1) d\theta$
36.  $\int \left(2 - \tan \frac{\theta}{4}\right) d\theta$
37.  $\int \frac{\cos t}{1+\sin t} dt$
38.  $\int \frac{\csc^2 t}{\cot t} dt$
39.  $\int \frac{\sec x \tan x}{\sec x - 1} dx$
40.  $\int (\sec 2x + \tan 2x) dx$

**Differential Equation** In Exercises 41–44, solve the differential equation. Use a graphing utility to graph three solutions, one of which passes through the given point.

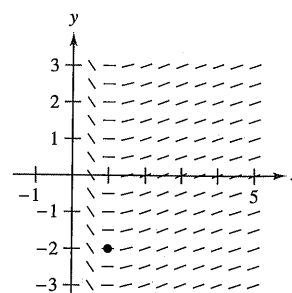
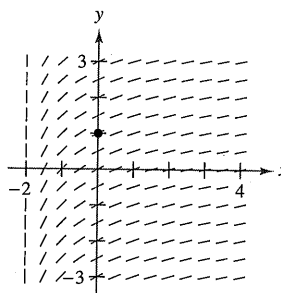
41.  $\frac{dy}{dx} = \frac{3}{2-x}, (1, 0)$
42.  $\frac{dy}{dx} = \frac{x-2}{x}, (-1, 0)$
43.  $\frac{dy}{dx} = \frac{2x}{x^2-9x}, (0, 4)$
44.  $\frac{dr}{dt} = \frac{\sec^2 t}{\tan t + 1}, (\pi, 4)$

**Finding a Particular Solution** In Exercises 45 and 46, find the particular solution that satisfies the differential equation and the initial equations.

45.  $f''(x) = \frac{2}{x^2}, f'(1) = 1, f(1) = 1, x > 0$
46.  $f''(x) = -\frac{4}{(x-1)^2} - 2, f'(2) = 0, f(2) = 3, x > 1$

**Slope Field** In Exercises 47 and 48, a differential equation, a point, and a slope field are given. (a) Sketch two approximate solutions of the differential equation on the slope field, one of which passes through the given point. (b) Use integration to find the particular solution of the differential equation and use a graphing utility to graph the solution. Compare the result with the sketches in part (a). To print an enlarged copy of the graph, go to [MathGraphs.com](http://MathGraphs.com).

47.  $\frac{dy}{dx} = \frac{1}{x+2}, (0, 1)$
48.  $\frac{dy}{dx} = \frac{\ln x}{x}, (1, -2)$

**Evaluating a Definite Integral** In Exercises 49–56, evaluate the definite integral. Use a graphing utility to verify your result.

49.  $\int_0^4 \frac{5}{3x+1} dx$
50.  $\int_{-1}^1 \frac{1}{2x+3} dx$

51.  $\int_1^e \frac{(1 + \ln x)^2}{x} dx$

52.  $\int_e^{e^2} \frac{1}{x \ln x} dx$

53.  $\int_0^2 \frac{x^2 - 2}{x + 1} dx$

54.  $\int_0^1 \frac{x - 1}{x + 1} dx$

55.  $\int_1^2 \frac{1 - \cos \theta}{\theta - \sin \theta} d\theta$

56.  $\int_{\pi/8}^{\pi/4} (\csc 2\theta - \cot 2\theta) d\theta$

**Using Technology to Find an Integral** In Exercises 57–62, use a computer algebra system to find or evaluate the integral.

57.  $\int \frac{1}{1 + \sqrt{x}} dx$

58.  $\int \frac{1 - \sqrt{x}}{1 + \sqrt{x}} dx$

59.  $\int \frac{\sqrt{x}}{x - 1} dx$

60.  $\int \frac{x^2}{x - 1} dx$

61.  $\int_{\pi/4}^{\pi/2} (\csc x - \sin x) dx$

62.  $\int_{-\pi/4}^{\pi/4} \frac{\sin^2 x - \cos^2 x}{\cos x} dx$

**Finding a Derivative** In Exercises 63–66, find  $F'(x)$ .

63.  $F(x) = \int_1^x \frac{1}{t} dt$

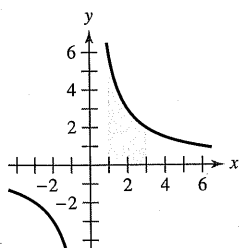
64.  $F(x) = \int_0^x \tan t dt$

65.  $F(x) = \int_1^{3x} \frac{1}{t} dt$

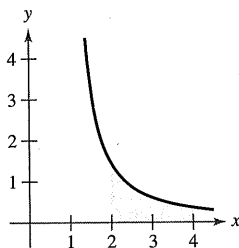
66.  $F(x) = \int_1^{x^2} \frac{1}{t} dt$

**Area** In Exercises 67–70, find the area of the given region. Use a graphing utility to verify your result.

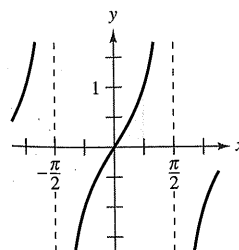
67.  $y = \frac{6}{x}$



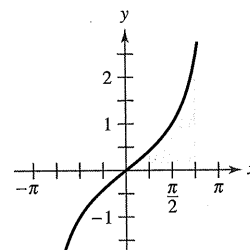
68.  $y = \frac{2}{x \ln x}$



69.  $y = \tan x$



70.  $y = \frac{\sin x}{1 + \cos x}$



**Area** In Exercises 71–74, find the area of the region bounded by the graphs of the equations. Use a graphing utility to verify your result.

71.  $y = \frac{x^2 + 4}{x}, x = 1, x = 4, y = 0$

72.  $y = \frac{5x}{x^2 + 2}, x = 1, x = 5, y = 0$

73.  $y = 2 \sec \frac{\pi x}{6}, x = 0, x = 2, y = 0$

74.  $y = 2x - \tan 0.3x, x = 1, x = 4, y = 0$

**Numerical Integration** In Exercises 75–78, use the Trapezoidal Rule and Simpson's Rule to approximate the value of the definite integral. Let  $n = 4$  and round your answer to four decimal places. Use a graphing utility to verify your result.

75.  $\int_1^5 \frac{12}{x} dx$

76.  $\int_0^4 \frac{8x}{x^2 + 4} dx$

77.  $\int_2^6 \ln x dx$

78.  $\int_{-\pi/3}^{\pi/3} \sec x dx$

### WRITING ABOUT CONCEPTS

**Choosing a Formula** In Exercises 79–82, state the integration formula you would use to perform the integration. Do not integrate.

79.  $\int \sqrt[3]{x} dx$

80.  $\int \frac{x}{(x^2 + 4)^3} dx$

81.  $\int \frac{x}{x^2 + 4} dx$

82.  $\int \frac{\sec^2 x}{\tan x} dx$

**Approximation** In Exercises 83 and 84, determine which value best approximates the area of the region between the  $x$ -axis and the graph of the function over the given interval. (Make your selection on the basis of a sketch of the region, not by performing any calculations.)

83.  $f(x) = \sec x, [0, 1]$

- (a) 6 (b) -6 (c)  $\frac{1}{2}$  (d) 1.25 (e) 3

84.  $f(x) = \frac{2x}{x^2 + 1}, [0, 4]$

- (a) 3 (b) 7 (c) -2 (d) 5 (e) 1

**85. Finding a Value** Find a value of  $x$  such that

$$\int_1^x \frac{3}{t} dt = \int_{1/4}^x \frac{1}{t} dt.$$

**86. Finding a Value** Find a value of  $x$  such that

$$\int_1^x \frac{1}{t} dt$$

is equal to (a)  $\ln 5$  and (b) 1.

**87. Proof** Prove that

$$\int \cot u du = \ln|\sin u| + C.$$

**88. Proof** Prove that

$$\int \csc u du = -\ln|\csc u + \cot u| + C.$$

**Using Properties of Logarithms and Trigonometric Identities** In Exercises 89–92, show that the two formulas are equivalent.

89.  $\int \tan x \, dx = -\ln|\cos x| + C$

$$\int \tan x \, dx = \ln|\sec x| + C$$

90.  $\int \cot x \, dx = \ln|\sin x| + C$

$$\int \cot x \, dx = -\ln|\csc x| + C$$

91.  $\int \sec x \, dx = \ln|\sec x + \tan x| + C$

$$\int \sec x \, dx = -\ln|\sec x - \tan x| + C$$

92.  $\int \csc x \, dx = -\ln|\csc x + \cot x| + C$

$$\int \csc x \, dx = \ln|\csc x - \cot x| + C$$

**Finding the Average Value of a Function** In Exercises 93–96, find the average value of the function over the given interval.

93.  $f(x) = \frac{8}{x^2}, [2, 4]$

94.  $f(x) = \frac{4(x+1)}{x^2}, [2, 4]$

95.  $f(x) = \frac{2 \ln x}{x}, [1, e]$

96.  $f(x) = \sec \frac{\pi x}{6}, [0, 2]$

**97. Population Growth** A population of bacteria  $P$  is changing at a rate of

$$\frac{dP}{dt} = \frac{3000}{1 + 0.25t}$$

where  $t$  is the time in days. The initial population (when  $t = 0$ ) is 1000. Write an equation that gives the population at any time  $t$ . Then find the population when  $t = 3$  days.

**98. Sales** The rate of change in sales  $S$  is inversely proportional to time  $t$  ( $t > 1$ ), measured in weeks. Find  $S$  as a function of  $t$  when the sales after 2 and 4 weeks are 200 units and 300 units, respectively.

### 99. Heat Transfer

Find the time required for an object to cool from 300°F to 250°F by evaluating

$$t = \frac{10}{\ln 2} \int_{250}^{300} \frac{1}{T - 100} dT$$

where  $t$  is time in minutes.



**100. Average Price** The demand equation for a product is

$$p = \frac{90,000}{400 + 3x}$$

where  $p$  is the price (in dollars) and  $x$  is the number of units (in thousands). Find the average price  $p$  on the interval  $40 \leq x \leq 50$ .

**101. Area and Slope** Graph the function

$$f(x) = \frac{x}{1 + x^2}$$

on the interval  $[0, \infty)$ .

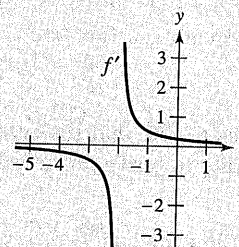
(a) Find the area bounded by the graph of  $f$  and the line  $y = \frac{1}{2}x$ .

(b) Determine the values of the slope  $m$  such that the line  $y = mx$  and the graph of  $f$  enclose a finite region.

(c) Calculate the area of this region as a function of  $m$ .



**102. HOW DO YOU SEE IT?** Use the graph of  $f'$  shown in the figure to answer the following.



(a) Approximate the slope of  $f$  at  $x = -1$ . Explain.

(b) Approximate any open intervals in which the graph of  $f$  is increasing and any open intervals in which it is decreasing. Explain.

**True or False?** In Exercises 103–106, determine whether the statement is true or false. If it is false, explain why or give an example that shows it is false.

103.  $(\ln x)^{1/2} = \frac{1}{2} \ln x$

104.  $\int \ln x \, dx = (1/x) + C$

105.  $\int \frac{1}{x} dx = \ln|cx|, c \neq 0$

106.  $\int_{-1}^2 \frac{1}{x} dx = \left[ \ln|x| \right]_{-1}^2 = \ln 2 - \ln 1 = \ln 2$

**107. Napier's Inequality** For  $0 < x < y$ , show that

$$\frac{1}{y} < \frac{\ln y - \ln x}{y - x} < \frac{1}{x}$$

**108. Proof** Prove that the function

$$F(x) = \int_x^{2x} \frac{1}{t} dt$$

is constant on the interval  $(0, \infty)$ .