AB Calculus Ch. 5.4 – 5.5 Morning Quiz Review

For each function, find $\frac{dy}{dx}$.

Derivative Rules:

a) $\frac{d}{dx} \ln u =$

b) $\frac{d}{dx} \log_a u =$

c) $\frac{d}{dx} e^u =$

d) $\frac{d}{dx} a^u =$

1. $y = 2\ln\left(\frac{\sqrt[4]{(3x-2x^4)^3}}{2x^3}\right)$

2. $y = x^3 e^{5x^2+3x}$

3. $y = 2\log_8\sqrt{x - e^x}$

4. $f(x) = 2^{3x}\left(\log(2-\sqrt{x})\right)$

5. $f(x) = \log_2\left(\frac{\sqrt{1-3x}}{(x-5x^2)}\right)$

6. $f(x) = 11^{\sqrt[3]{4x-5x^3-3e^2}}$

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For each function, find $\frac{dy}{dx}$.

Log properties

$\log(ab) = \log a + \log b$

$\log\left(\frac{a}{b}\right) = \log a - \log b$

$\log a^n = n \log a$

Derivative Rules:

a) $\frac{d}{dx} \ln u = \frac{u'}{u}$

b) $\frac{d}{dx} \log_a u = \frac{1}{\ln a} \cdot \frac{u'}{u}$

c) $\frac{d}{dx} e^u = e^u \cdot u'$

d) $\frac{d}{dx} a^u = \ln a \cdot a^u \cdot u'$

Key

1. $y = 2\ln\left(\frac{\sqrt[4]{(3x-2x^4)^3}}{2x^3}\right)$ *Expand first

$y = 2\ln(3x-2x^4)^{3/4} - 2\ln(2x^3)$

$y = 2 \cdot \frac{3}{4}\ln(3x-2x^4) - 2\ln(2x^3)$

$y' = \frac{3}{2}\left(\frac{3-8x^3}{3x-2x^4}\right) - 2\left(\frac{6x^2}{2x^3}\right)$

$$\boxed{\frac{dy}{dx} = \frac{3(3-8x^3)}{2(3x-2x^4)} - \frac{6}{x}}$$

2. $y = x^3 e^{5x^2+3x}$ *product rule $f'g + fg'$

$y' = 3x^2 e^{5x^2+3x} + x^3 \cdot e^{5x^2+3x}(10x+3)$

$$\boxed{\frac{dy}{dx} = x^2 e^{5x^2+3x}\left[3 + x(10x+3)\right]}$$

3. $y = 2\log_8\sqrt{x - e^x}$

$y = 2 \cdot \frac{\ln(x-e^x)^{1/2}}{\ln 8} = \frac{2}{\ln 8}\ln(x-e^x)^{1/2}$

$y = \frac{2}{\ln 8} \cdot \frac{1}{2}\ln(x-e^x) = \frac{1}{\ln 8}\ln(x-e^x)$

$$\frac{dy}{dx} = \frac{1}{\ln 8}\left(\frac{1-e^x}{x-e^x}\right) = \boxed{\frac{1-e^x}{\ln 8(x-e^x)}}$$

4. $f(x) = 2^{3x}(\log(2-\sqrt{x}))$ *product rule

$\log_{10}(2-\sqrt{x})$

$\frac{\ln(2-\sqrt{x})}{\ln 10}$

$= \frac{1}{\ln 10}\ln(2-\sqrt{x})$

$f(x) = 2^{3x} \cdot \frac{1}{\ln 10}\ln(2-\sqrt{x})$

$f'(x) = \underbrace{(\ln 2 \cdot 2^{3x} \cdot 3)}_{f'}\underbrace{\left(\frac{1}{\ln 10}\ln(2-\sqrt{x})\right)}_{g} + \underbrace{(2^{3x})}_{f}\underbrace{\left(\frac{1}{\ln 10}\right)\left(\frac{-\frac{1}{2}x^{-1/2}}{2-\sqrt{x}}\right)}_{g'}$

$$\boxed{f'(x) = 3\ln 2(2^{3x})\frac{1}{\ln 10}\ln(2-\sqrt{x}) + 2^{3x} \cdot \left(\frac{-1}{2\ln 10\sqrt{x}(2-\sqrt{x})}\right)}$$

5. $f(x) = \log_2\left(\frac{\sqrt{1-3x}}{(x-5x^2)}\right)$

$f(x) = \frac{\ln\left(\frac{(1-3x)^{1/2}}{x-5x^2}\right)}{\ln 2} = \frac{1}{\ln 2}\ln\left[\frac{(1-3x)^{1/2}}{x-5x^2}\right]$

$f(x) = \frac{1}{\ln 2}\left[\frac{1}{2}\ln(1-3x) - \ln(x-5x^2)\right]$

$$\boxed{f'(x) = \frac{1}{\ln 2}\left[\frac{1}{2}\left(\frac{-3}{1-3x}\right) - \left(\frac{1-10x}{x-5x^2}\right)\right]}$$

6. $f(x) = 11^{\sqrt[3]{4x-5x^3-3e^2}} = 11^{(4x-5x^3-3e^2)^{1/3}}$

$f'(x) = \ln 11 \cdot 11^{\sqrt[3]{4x-5x^3-3e^2}} \cdot \frac{1}{3}(4x-5x^3-3e^2)^{-2/3}(4-15x^2)$

$f'(x) = \ln 11 \cdot 11^{\sqrt[3]{4x-5x^3-3e^2}} \cdot \frac{1}{3}(4x-5x^3-3e^2)^{-2/3}(4-15x^2)$

$$\boxed{f'(x) = \frac{(\ln 11)\left(11^{\sqrt[3]{4x-5x^3-3e^2}}\right)(4-15x^2)}{3(4x-5x^3-3e^2)^{2/3}}}$$