5.4 Exercises

See CalcChat.com for tutorial help and worked-out solutions to odd-numbered exercises.

Solving an Exponential or Logarithmic Equation In Exercises 1–16, solve for x accurate to three decimal places.

1. $e^{\ln x} = 4$
2. $e^{\ln 3x} = 24$
3. $e^x = 12$
4. $5e^x = 36$
5. $9 - 2e^x = 7$
6. $8e^x - 12 = 7$
7. $50e^{-x} = 30$
8. $100e^{-2x} = 35$
9. $\dfrac{800}{100 - e^{x/2}} = 50$
10. $\dfrac{5000}{1 + e^{2x}} = 2$
11. $\ln x = 2$
12. $\ln x^2 = 10$
13. $\ln(x - 3) = 2$
14. $\ln 4x = 1$
15. $\ln\sqrt{x + 2} = 1$
16. $\ln(x - 2)^2 = 12$

Sketching a Graph In Exercises 17–22, sketch the graph of the function.

17. $y = e^{-x}$
18. $y = \frac{1}{2}e^x$
19. $y = e^x + 2$
20. $y = e^{x-1}$
21. $y = e^{-x^2}$
22. $y = e^{-x/2}$

23. **Comparing Graphs** Use a graphing utility to graph $f(x) = e^x$ and the given function in the same viewing window. How are the two graphs related?

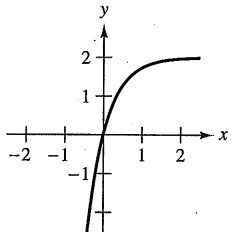
(a) $g(x) = e^{x-2}$ (b) $h(x) = -\frac{1}{2}e^x$ (c) $q(x) = e^{-x} + 3$

24. **Asymptotes** Use a graphing utility to graph the function. Use the graph to determine any asymptotes of the function.

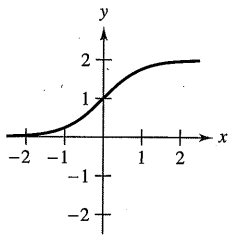
(a) $f(x) = \dfrac{8}{1 + e^{-0.5x}}$ (b) $g(x) = \dfrac{8}{1 + e^{-0.5/x}}$

Matching In Exercises 25–28, match the equation with the correct graph. Assume that a and C are positive real numbers. [The graphs are labeled (a), (b), (c), and (d).]

(a)



(b)



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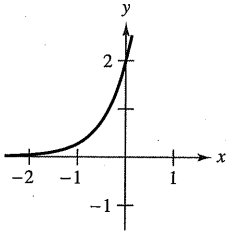
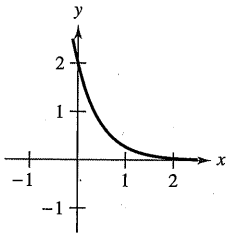


(d)



25. $y = Ce^{ax}$
26. $y = Ce^{-ax}$
27. $y = C(1 - e^{-ax})$
28. $y = \dfrac{C}{1 + e^{-ax}}$

Inverse Functions In Exercises 29–32, illustrate that the functions are inverses of each other by graphing both functions on the same set of coordinate axes.

29. $f(x) = e^{2x}$, $g(x) = \ln\sqrt{x}$
30. $f(x) = e^{x/3}$, $g(x) = \ln x^3$
31. $f(x) = e^x - 1$, $g(x) = \ln(x + 1)$
32. $f(x) = e^{x-1}$, $g(x) = 1 + \ln x$

Finding a Derivative In Exercises 33–54, find the derivative.

33. $f(x) = e^{2x}$
34. $y = e^{-8x}$
35. $y = e^{\sqrt{x}}$
36. $y = e^{-2x^3}$
37. $y = e^{x-4}$
38. $y = 5e^{x^2+5}$
39. $y = e^x \ln x$
40. $y = xe^{4x}$
41. $y = x^3e^x$
42. $y = x^2e^{-x}$
43. $g(t) = (e^{-t} + e^t)^3$
44. $g(t) = e^{-3/t^2}$
45. $y = \ln(1 + e^{2x})$
46. $y = \ln\left(\dfrac{1 + e^x}{1 - e^x}\right)$
47. $y = \dfrac{2}{e^x + e^{-x}}$
48. $y = \dfrac{e^x - e^{-x}}{2}$
49. $y = \dfrac{e^x + 1}{e^x - 1}$
50. $y = \dfrac{e^{2x}}{e^{2x} + 1}$
51. $y = e^x(\sin x + \cos x)$
52. $y = e^{2x} \tan 2x$
53. $F(x) = \displaystyle\int_{\pi}^{\ln x} \cos e^t\,dt$
54. $F(x) = \displaystyle\int_{0}^{e^{2x}} \ln(t + 1)\,dt$

Finding an Equation of a Tangent Line In Exercises 55–62, find an equation of the tangent line to the graph of the function at the given point.

55. $f(x) = e^{3x}$, $(0, 1)$
56. $f(x) = e^{-2x}$, $(0, 1)$
57. $f(x) = e^{1-x}$, $(1, 1)$
58. $y = e^{-2x+x^2}$, $(2, 1)$
59. $f(x) = e^{-x} \ln x$, $(1, 0)$
60. $y = \ln\dfrac{e^x + e^{-x}}{2}$, $(0, 0)$
61. $y = x^2e^x - 2xe^x + 2e^x$, $(1, e)$
62. $y = xe^x - e^x$, $(1, 0)$

Implicit Differentiation In Exercises 63 and 64, use implicit differentiation to find dy/dx.

63. $xe^y - 10x + 3y = 0$
64. $e^{xy} + x^2 - y^2 = 10$

Finding the Equation of a Tangent Line In Exercises 65 and 66, find an equation of the tangent line to the graph of the function at the given point.

65. $xe^y + ye^x = 1$, $(0, 1)$
66. $1 + \ln xy = e^{x-y}$, $(1, 1)$

Finding a Second Derivative **In Exercises 67 and 68, find the second derivative of the function.**

67. $f(x) = (3 + 2x)e^{-3x}$

68. $g(x) = \sqrt{x} + e^x \ln x$

Differential Equation **In Exercises 69 and 70, show that the function $y = f(x)$ is a solution of the differential equation.**

69. $y = 4e^{-x}$
$y'' - y = 0$

70. $y = e^{3x} + e^{-3x}$
$y'' - 9y = 0$

Finding Extrema and Points of Inflection **In Exercises 71–78, find the extrema and the points of inflection (if any exist) of the function. Use a graphing utility to graph the function and confirm your results.**

71. $f(x) = \dfrac{e^x + e^{-x}}{2}$

72. $f(x) = \dfrac{e^x - e^{-x}}{2}$

73. $g(x) = \dfrac{1}{\sqrt{2\pi}} e^{-(x-2)^2/2}$

74. $g(x) = \dfrac{1}{\sqrt{2\pi}} e^{-(x-3)^2/2}$

75. $f(x) = x^2 e^{-x}$

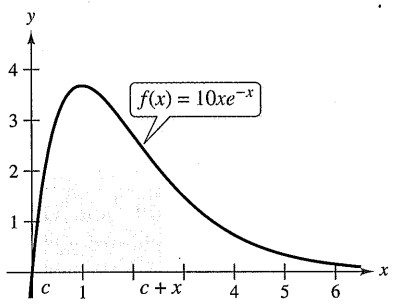
76. $f(x) = xe^{-x}$

77. $g(t) = 1 + (2 + t)e^{-t}$

78. $f(x) = -2 + e^{3x}(4 - 2x)$

79. **Area** Find the area of the largest rectangle that can be inscribed under the curve $y = e^{-x^2}$ in the first and second quadrants.

80. **Area** Perform the following steps to find the maximum area of the rectangle shown in the figure.



(a) Solve for c in the equation $f(c) = f(c + x)$.

(b) Use the result in part (a) to write the area A as a function of x. [*Hint:* $A = xf(c)$]

(c) Use a graphing utility to graph the area function. Use the graph to approximate the dimensions of the rectangle of maximum area. Determine the maximum area.

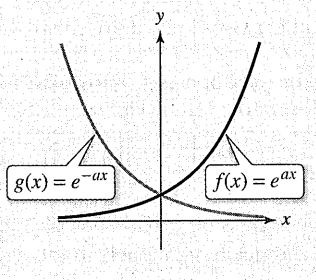
(d) Use a graphing utility to graph the expression for c found in part (a). Use the graph to approximate

$$\lim_{x \to 0^+} c \quad \text{and} \quad \lim_{x \to \infty} c.$$

Use this result to describe the changes in dimensions and position of the rectangle for $0 < x < \infty$.

81. **Finding an Equation of a Tangent Line** Find a point on the graph of the function $f(x) = e^{2x}$ such that the tangent line to the graph at that point passes through the origin. Use a graphing utility to graph f and the tangent line in the same viewing window.

82. **HOW DO YOU SEE IT?** The figure shows the graphs of f and g, where a is a positive real number. Identify the open interval(s) on which the graphs of f and g are (a) increasing or decreasing, and (b) concave upward or concave downward.



83. **Depreciation** The value V of an item t years after it is purchased is $V = 15{,}000e^{-0.6286t}$, $0 \le t \le 10$.

(a) Use a graphing utility to graph the function.

(b) Find the rates of change of V with respect to t when $t = 1$ and $t = 5$.

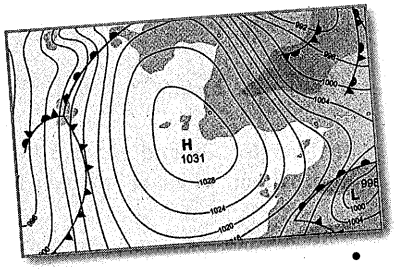
(c) Use a graphing utility to graph the tangent lines to the function when $t = 1$ and $t = 5$.

84. **Harmonic Motion** The displacement from equilibrium of a mass oscillating on the end of a spring suspended from a ceiling is $y = 1.56e^{-0.22t} \cos 4.9t$, where y is the displacement (in feet) and t is the time (in seconds). Use a graphing utility to graph the displacement function on the interval $[0, 10]$. Find a value of t past which the displacement is less than 3 inches from equilibrium.

85. **Atmospheric Pressure**

A meteorologist measures the atmospheric pressure P (in kilograms per square meter) at altitude h (in kilometers). The data are shown below.

h	0	5	10	15	20
P	10,332	5583	2376	1240	517



(a) Use a graphing utility to plot the points $(h, \ln P)$. Use the regression capabilities of the graphing utility to find a linear model for the revised data points.

(b) The line in part (a) has the form $\ln P = ah + b$. Write the equation in exponential form.

(c) Use a graphing utility to plot the original data and graph the exponential model in part (b).

(d) Find the rate of change of the pressure when $h = 5$ and $h = 18$.

86. Modeling Data The table lists the approximate values V of a mid-sized sedan for the years 2006 through 2012. The variable t represents the time (in years), with $t = 6$ corresponding to 2006.

t	6	7	8	9
V	\$23,046	\$20,596	\$18,851	\$17,001

t	10	11	12
V	\$15,226	\$14,101	\$12,841

(a) Use the regression capabilities of a graphing utility to fit linear and quadratic models to the data. Plot the data and graph the models.

(b) What does the slope represent in the linear model in part (a)?

(c) Use the regression capabilities of a graphing utility to fit an exponential model to the data.

(d) Determine the horizontal asymptote of the exponential model found in part (c). Interpret its meaning in the context of the problem.

(e) Use the exponential model to find the rate of decrease in the value of the sedan when $t = 7$ and $t = 11$.

Linear and Quadratic Approximation **In Exercises 87 and 88, use a graphing utility to graph the function. Then graph**

$$P_1(x) = f(0) + f'(0)(x - 0) \quad \text{and}$$

$$P_2(x) = f(0) + f'(0)(x - 0) + \tfrac{1}{2}f''(0)(x - 0)^2$$

in the same viewing window. Compare the values of f, P_1, P_2, and their first derivatives at $x = 0$.

87. $f(x) = e^x$ **88.** $f(x) = e^{x/2}$

Stirling's Formula **For large values of n,**

$$n! = 1 \cdot 2 \cdot 3 \cdot 4 \cdot \cdots (n - 1) \cdot n$$

can be approximated by Stirling's Formula,

$$n! \approx \left(\frac{n}{e}\right)^n \sqrt{2\pi n}.$$

In Exercises 89 and 90, find the exact value of $n!$, and then approximate $n!$ using Stirling's Formula.

89. $n = 12$ **90.** $n = 15$

Finding an Indefinite Integral **In Exercises 91–108, find the indefinite integral.**

91. $\int e^{5x}(5)\,dx$ **92.** $\int e^{-x^4}(-4x^3)\,dx$

93. $\int e^{2x-1}\,dx$ **94.** $\int e^{1-3x}\,dx$

95. $\int x^2 e^{x^3}\,dx$ **96.** $\int e^x(e^x + 1)^2\,dx$

97. $\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx$ **98.** $\int \frac{e^{1/x^2}}{x^3}\,dx$

99. $\int \frac{e^{-x}}{1 + e^{-x}}\,dx$ **100.** $\int \frac{e^{2x}}{1 + e^{2x}}\,dx$

101. $\int e^x\sqrt{1 - e^x}\,dx$ **102.** $\int \frac{e^x - e^{-x}}{e^x + e^{-x}}\,dx$

103. $\int \frac{e^x + e^{-x}}{e^x - e^{-x}}\,dx$ **104.** $\int \frac{2e^x - 2e^{-x}}{(e^x + e^{-x})^2}\,dx$

105. $\int \frac{5 - e^x}{e^{2x}}\,dx$ **106.** $\int \frac{e^{2x} + 2e^x + 1}{e^x}\,dx$

107. $\int e^{-x}\tan(e^{-x})\,dx$

108. $\int e^{2x}\csc(e^{2x})\,dx$

Evaluating a Definite Integral **In Exercises 109–118, evaluate the definite integral. Use a graphing utility to verify your result.**

109. $\int_0^1 e^{-2x}\,dx$ **110.** $\int_1^2 e^{5x-3}\,dx$

111. $\int_0^1 xe^{-x^2}\,dx$ **112.** $\int_{-2}^0 x^2e^{x^3/2}\,dx$

113. $\int_1^3 \frac{e^{3/x}}{x^2}\,dx$ **114.** $\int_0^{\sqrt{2}} xe^{-(x^2/2)}\,dx$

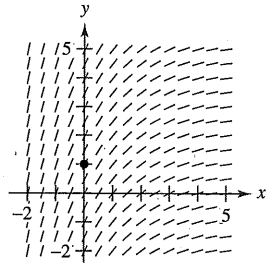
115. $\int_0^3 \frac{2e^{2x}}{1 + e^{2x}}\,dx$ **116.** $\int_0^1 \frac{e^x}{5 - e^x}\,dx$

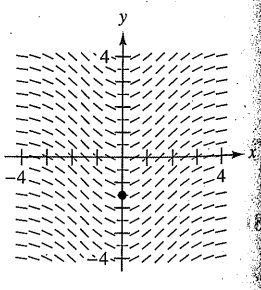
117. $\int_0^{\pi/2} e^{\sin \pi x}\cos \pi x\,dx$

118. $\int_{\pi/3}^{\pi/2} e^{\sec 2x}\sec 2x\tan 2x\,dx$

Slope Field **In Exercises 119 and 120, a differential equation, a point, and a slope field are given. (a) Sketch two approximate solutions of the differential equation on the slope field, one of which passes through the given point. (b) Use integration to find the particular solution of the differential equation and use a graphing utility to graph the solution. Compare the result with the sketches in part (a). To print an enlarged copy of the graph, go to *MathGraphs.com*.**

119. $\frac{dy}{dx} = 2e^{-x/2}$, $(0, 1)$ **120.** $\frac{dy}{dx} = xe^{-0.2x^2}$, $\left(0, -\frac{3}{2}\right)$





Differential Equation **In Exercises 121 and 122, solve the differential equation.**

121. $\frac{dy}{dx} = xe^{ax^2}$ **122.** $\frac{dy}{dx} = (e^x - e^{-x})^2$

Differential Equation In Exercises 123 and 124, find the particular solution that satisfies the initial conditions.

123. $f''(x) = \frac{1}{2}(e^x + e^{-x})$, $f(0) = 1, f'(0) = 0$

124. $f''(x) = \sin x + e^{2x}$, $f(0) = \frac{1}{4}, f'(0) = \frac{1}{2}$

Area In Exercises 125–128, find the area of the region bounded by the graphs of the equations. Use a graphing utility to graph the region and verify your result.

125. $y = e^x, y = 0, x = 0, x = 5$

126. $y = e^{-2x}, y = 0, x = -1, x = 3$

127. $y = xe^{-x^2/4}, y = 0, x = 0, x = \sqrt{6}$

128. $y = e^{-2x} + 2, y = 0, x = 0, x = 2$

Numerical Integration In Exercises 129 and 130, approximate the integral using the Midpoint Rule, the Trapezoidal Rule, and Simpson's Rule with $n = 12$. Use a graphing utility to verify your results.

129. $\int_0^4 \sqrt{x}e^x\,dx$

130. $\int_0^2 2xe^{-x}\,dx$

131. **Probability** A car battery has an average lifetime of 48 months with a standard deviation of 6 months. The battery lives are normally distributed. The probability that a given battery will last between 48 months and 60 months is

$$0.0065\int_{48}^{60} e^{-0.0139(t-48)^2}\,dt.$$

Use the integration capabilities of a graphing utility to approximate the integral. Interpret the resulting probability.

132. **Probability** The median waiting time (in minutes) for people waiting for service in a convenience store is given by the solution of the equation

$$\int_0^x 0.3e^{-0.3t}\,dt = \frac{1}{2}.$$

What is the median waiting time?

133. **Using the Area of a Region** Find the value of a such that the area bounded by $y = e^{-x}$, the x-axis, $x = -a$, and $x = a$ is $\frac{8}{3}$.

134. **Modeling Data** A valve on a storage tank is opened for 4 hours to release a chemical in a manufacturing process. The flow rate R (in liters per hour) at time t (in hours) is given in the table.

t	0	1	2	3	4
R	425	240	118	71	36

(a) Use the regression capabilities of a graphing utility to find a linear model for the points $(t, \ln R)$. Write the resulting equation of the form $\ln R = at + b$ in exponential form.

(b) Use a graphing utility to plot the data and graph the exponential model.

(c) Use the definite integral to approximate the number of liters of chemical released during the 4 hours.

WRITING ABOUT CONCEPTS

135. **Properties of the Natural Exponential Function** In your own words, state the properties of the natural exponential function.

136. **A Function and Its Derivative** Is there a function f such that $f(x) = f'(x)$? If so, identify it.

137. **Choosing a Function** Without integrating, state the integration formula you can use to integrate each of the following.

(a) $\int \frac{e^x}{e^x + 1}\,dx$

(b) $\int xe^{x^2}\,dx$

138. **Analyzing a Graph** Consider the function

$$f(x) = \frac{2}{1 + e^{1/x}}.$$

(a) Use a graphing utility to graph f.

(b) Write a short paragraph explaining why the graph has a horizontal asymptote at $y = 1$ and why the function has a nonremovable discontinuity at $x = 0$.

139. **Deriving an Inequality** Given $e^x \geq 1$ for $x \geq 0$, it follows that

$$\int_0^x e^t\,dt \geq \int_0^x 1\,dt.$$

Perform this integration to derive the inequality

$$e^x \geq 1 + x$$

for $x \geq 0$.

140. **Solving an Equation** Find, to three decimal places, the value of x such that $e^{-x} = x$. (Use Newton's Method or the *zero* or *root* feature of a graphing utility.)

141. **Horizontal Motion** The position function of a particle moving along the x-axis is $x(t) = Ae^{kt} + Be^{-kt}$, where A, B, and k are positive constants.

(a) During what times t is the particle closest to the origin?

(b) Show that the acceleration of the particle is proportional to the position of the particle. What is the constant of proportionality?

142. **Analyzing a Function** Let $f(x) = \frac{\ln x}{x}$.

(a) Graph f on $(0, \infty)$ and show that f is strictly decreasing on (e, ∞).

(b) Show that if $e \leq A < B$, then $A^B > B^A$.

(c) Use part (b) to show that $e^\pi > \pi^e$.

143. **Finding the Maximum Rate of Change** Verify that the function

$$y = \frac{L}{1 + ae^{-x/b}}, \quad a > 0, \quad b > 0, \quad L > 0$$

increases at a maximum rate when $y = L/2$.