

## Calculus Ch. 5.7 Notes Integrals of Inverse Trig Functions

Recall Rules for Inverse Trig Derivatives:

$$\frac{d}{dx} \arcsin u = \frac{u'}{\sqrt{1-u^2}}$$

$$\frac{d}{dx} \arctan u = \frac{u'}{1+u^2}$$

$$\frac{d}{dx} \operatorname{arcsec} u = \frac{u'}{|u|\sqrt{u^2-1}}$$

$$\frac{d}{dx} \arccos u = -\frac{u'}{\sqrt{1-u^2}}$$

$$\frac{d}{dx} \operatorname{arc cot} u = -\frac{u'}{1+u^2}$$

$$\frac{d}{dx} \operatorname{arc csc} u = -\frac{u'}{|u|\sqrt{u^2-1}}$$

### Inverse Trig Integral Rules:

\*a is a constant\*

$$1. \int \frac{du}{\sqrt{a^2-u^2}} = \arcsin\left(\frac{u}{a}\right) + C$$

$$2. \int \frac{du}{a^2+u^2} = \frac{1}{a} \arctan\left(\frac{u}{a}\right) + C$$

$$3. \int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a} \operatorname{arcsec}\left(\frac{|u|}{a}\right) + C$$

Ex. 1:  $\int \frac{5}{x\sqrt{x^2-9}} dx$

Ex. 2:  $\int \frac{1}{4+(x-1)^2} dx$

Ex. 3:  $\int \frac{1}{\sqrt{7-16x^2}} dx$

## Inverse Trig Integral Rules

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### Completing the Square Steps:

1. Write in standard form:  $x^2 + bx + c$
2. Add spaces  $x^2 + bx + \underline{\quad} + c - \underline{\quad}$
3. Put  $\left(\frac{b}{2}\right)^2$  into the spaces
4. Factor expression

**Ex. 4:**  $\int \frac{dx}{x^2 - 6x + 13}$

**Ex. 5:**  $\int \frac{2x-5}{x^2 + 2x + 2} dx$

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Key

Recall Rules for Inverse Trig Derivatives:

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Inverse Trig Integral Rules:

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3.  $\int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a} \operatorname{arcsec}\left(\frac{|u|}{a}\right) + C$

\*u is variable

Ex. 1:  $\int \frac{5}{x\sqrt{x^2-9}} dx$      $u=x$      $a=3$

$5 \cdot \frac{1}{a} \operatorname{arcsec}\left(\frac{|u|}{a}\right) + C$

$$\begin{aligned} \frac{du}{dx} &= 1 & \int \frac{5}{u\sqrt{u^2-a^2}} du &= 5 \int \frac{du}{u\sqrt{u^2-9}} = \\ \frac{dx}{du} &= 1 & &= 5 \cdot \frac{1}{3} \operatorname{arcsec}\left(\frac{|x|}{3}\right) + C \end{aligned}$$

Ex. 2:  $\int \frac{1}{4+(x-1)^2} dx$

$$\int \frac{1}{a^2+u^2} du = \frac{1}{a} \arctan\left(\frac{u}{a}\right) + C$$

$$\int \frac{1}{(2)^2+(x-1)^2} dx$$

$$= \frac{1}{2} \arctan\left(\frac{x-1}{2}\right) + C$$

$a=2$      $\frac{du}{dx}=1$

$u=x-1$      $dx=du$

Ex. 3:  $\int \frac{1}{\sqrt{7-16x^2}} dx$

$$\int \frac{1}{\sqrt{(7)^2-(4x)^2}} dx$$

$$\int \frac{1}{\sqrt{a^2-u^2}} \cdot \frac{du}{4}$$

$a=\sqrt{7}$      $\frac{du}{dx}=4$

$u=4x$      $dx=\frac{du}{4}$

$$\frac{1}{4} \int \frac{du}{\sqrt{a^2-u^2}} = \frac{1}{4} \arcsin\left(\frac{u}{a}\right) + C$$

$$\frac{1}{4} \arcsin\left(\frac{4x}{\sqrt{7}}\right) + C$$

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### Completing the Square Steps:

1. Write in standard form:  $x^2 + bx + c$

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4. Factor expression

\*Usually Arc Trig integral Rules work best when there are no variables in numerator

$$\left(\frac{-6}{2}\right)^2 = 9 \quad \text{Ex. 4: } \int \frac{dx}{x^2 - 6x + 13}$$

$$x^2 - 6x + 13$$

$$\underbrace{x^2 - 6x + \underline{9}}_{(x-3)^2 + 4} + 13 - \underline{9}$$

$$\int \frac{dx}{(x-3)^2 + (2)^2}$$

$$u = x-3$$

$$\frac{du}{dx} = 1 \quad dx = du$$

$$a = 2$$

u-sub

$$\int \frac{du}{u^2 + a^2} = \frac{1}{a} \arctan\left(\frac{u}{a}\right) + C$$

$$\boxed{\frac{1}{2} \arctan\left(\frac{x-3}{2}\right) + C}$$

arc trig Rule

$$\text{Ex. 5: } \int \frac{2x-5}{x^2+2x+2} dx = \int \frac{2x+2}{x^2+2x+2} dx + \int \frac{-7}{x^2+2x+2} dx \quad | \quad u = x+1 \quad a = 1$$

$$u = x^2 + 2x + 2$$

$$\frac{du}{dx} = 2x+2$$

$$dx = \frac{du}{2x+2}$$

$$\int \frac{2x+2}{u} \cdot \frac{du}{2x+2}$$

$$\int \frac{1}{u} du$$

$$\int \frac{-7}{x^2+2x+2} dx$$

$$x^2 + 2x + \underline{1} + 2 - \underline{1}$$

$$\frac{du}{dx} = 1$$

$$-7 \int \frac{1}{u^2 + a^2} du$$

$$-7 \cdot \frac{1}{a} \arctan\left(\frac{u}{a}\right)$$

$$\ln|u|$$

$$\boxed{\ln(x^2 + 2x + 2) - 7 \arctan\left(\frac{x+1}{1}\right) + C}$$