Key

Calculus Ch. 5.7 Notes Integrals of Inverse Trig Functions

a = constant

u = variable expression

Recall Rules for Inverse Trig Derivatives:

$\frac{d}{dx}\arcsin u = \frac{u'}{\sqrt{1-u^2}}$	$\frac{d}{dx}\arccos u = -\frac{u'}{\sqrt{1-u^2}}$	**Inverse Trig Integral Rules**: *a is a constant*
$\frac{d}{dx}\arctan u = \frac{u'}{1+u^2}$	$\frac{d}{dx}arc\cot u = -\frac{u'}{1+u^2}$	1. $\int \frac{du}{\sqrt{a^2-u^2}} = \arcsin\left(\frac{u}{a}\right)+C$
$\frac{d}{dx}arc\sec u = \frac{u'}{\lvert u\rvert\sqrt{u^2-1}}$	$\frac{d}{dx}arc\csc u = -\frac{u'}{\lvert u\rvert\sqrt{u^2-1}}$	2. $\int \frac{du}{a^2+u^2} = \frac{1}{a}\arctan\left(\frac{u}{a}\right)+C$
		3. $\int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a}arc\sec\left(\frac{\lvert u\rvert}{a}\right)+C$

Ex. 1: $\int \frac{5}{x\sqrt{x^2-9}}dx$

$\int \frac{5}{x\sqrt{(x)^2-(3)^2}}dx$

$a=3$

$u=x$

$\frac{du}{dx}=1$

$dx=du$

$5\int \frac{1}{u\sqrt{u^2-a^2}}du \rightarrow 5\cdot\frac{1}{3}\text{arcsec}\left(\frac{|x|}{3}\right)+C$

$\boxed{\frac{5}{3}\text{arcsec}\left(\frac{|x|}{3}\right)+C}$

Ex. 2: $\int \frac{1}{4+(x-1)^2}dx$

$* \int \frac{du}{a^2+u^2} = \frac{1}{a}\arctan\left(\frac{u}{a}\right)+C$

$\int \frac{1}{(2)^2+(x-1)^2}dx$

$a=2$

$u=x-1$

$\frac{du}{dx}=1$

$dx=du$

$\int \frac{1}{a^2+u^2}du = \boxed{\frac{1}{2}\arctan\left(\frac{x-1}{2}\right)+C}$

Ex. 3: $\int \frac{1}{\sqrt{7-16x^2}}dx$

$* \int \frac{du}{\sqrt{a^2-u^2}} = \arcsin\left(\frac{u}{a}\right)+C$

$\int \frac{1}{\sqrt{(\sqrt{7})^2-(4x)^2}}dx$

$a=\sqrt{7}$

$u=4x$

$\frac{du}{dx}=4$

$4dx=du$

$dx=du/4$

$\int \frac{1}{\sqrt{a^2-u^2}}\frac{du}{4} \rightarrow \frac{1}{4}\int \frac{1}{\sqrt{a^2-u^2}}du$

$\boxed{\frac{1}{4}\cdot\arcsin\left(\frac{4x}{\sqrt{7}}\right)+C}$

Inverse Trig Integral Rules

a is a constant 1. $\int \frac{du}{\sqrt{a^2-u^2}} = \arcsin\left(\frac{u}{a}\right)+C$ 2. $\int \frac{du}{a^2+u^2} = \frac{1}{a}\arctan\left(\frac{u}{a}\right)+C$ 3. $\int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a}\operatorname{arcsec}\left(\frac{|u|}{a}\right)+C$

Completing the Square Steps:

1. Write in standard form: $x^2 + bx + c$
2. Add spaces $x^2 + bx + ___ + c - ___$
3. Put $\left(\frac{b}{2}\right)^2$ into the spaces
4. Factor expression

Ex. 4: $\int \frac{dx}{x^2-6x+13}$

$\left(\frac{b}{2}\right)^2 \rightarrow \left(\frac{-6}{2}\right)^2 = 9$

$x^2-6x+13$

$x^2-6x+\underline{9}+13-\underline{9}$

$(x-3)(x-3)$

$(x-3)^2+4$

$\int \frac{dx}{(x-3)^2+4} \rightarrow \int \frac{dx}{(x-3)^2+(2)^2}$

$a=2$

$u=x-3$

$\frac{du}{dx}=1$

$dx=du$

$\int \frac{du}{u^2+a^2} = \frac{1}{2}\arctan\left(\frac{x-3}{2}\right)+C$

$\boxed{\frac{1}{2}\arctan\left(\frac{x-3}{2}\right)+C}$

Ex. 5: $\int \frac{2x-5}{x^2+2x+2}dx$

41) $\int \frac{x}{x^4+2x^2+2}dx$

Complete the square:

$\left(\frac{b}{2}\right)^2 \rightarrow \left(\frac{2}{2}\right)^2 = 1$

x^4+2x^2+2

$x^4+2x^2+\underline{1}+2-\underline{1}$

$(x^2+1)(x^2+1)$

$(x^2+1)^2+1$

$\int \frac{x}{(x^2+1)^2+1}dx \rightarrow \int \frac{x}{(x^2+1)^2+(1)^2}dx$

$\int \frac{du}{a^2+u^2} = \frac{1}{a}\arctan\left(\frac{u}{a}\right)+C$

$a=1$

$u=x^2+1$

$\frac{du}{dx}=2x$

$2xdx=du$

$dx=\frac{du}{2x}$

$\int \frac{\cancel{x}}{u^2+a^2}\cdot\frac{du}{2\cancel{x}}$

$\frac{1}{2}\int \frac{du}{u^2+a^2} =$

$\frac{1}{2}\cdot\frac{1}{1}\arctan\left(\frac{x^2+1}{1}\right)+C$

$\boxed{\frac{1}{2}\arctan(x^2+1)+C}$