5.7 Exercises

See CalcChat.com for tutorial help and worked-out solutions to odd-numbered exercises.

Finding an Indefinite Integral In Exercises 1–20, find the indefinite integral.

1. $\int \frac{dx}{\sqrt{9-x^2}}$

2. $\int \frac{dx}{\sqrt{1-4x^2}}$

3. $\int \frac{1}{x\sqrt{4x^2-1}}\,dx$

4. $\int \frac{12}{1+9x^2}\,dx$

5. $\int \frac{1}{\sqrt{1-(x+1)^2}}\,dx$

6. $\int \frac{1}{4+(x-3)^2}\,dx$

7. $\int \frac{t}{\sqrt{1-t^4}}\,dt$

8. $\int \frac{1}{x\sqrt{x^4-4}}\,dx$

9. $\int \frac{t}{t^4+25}\,dt$

10. $\int \frac{1}{x\sqrt{1-(\ln x)^2}}\,dx$

11. $\int \frac{e^{2x}}{4+e^{4x}}\,dx$

12. $\int \frac{2}{x\sqrt{9x^2-25}}\,dx$

13. $\int \frac{\sec^2 x}{\sqrt{25-\tan^2 x}}\,dx$

14. $\int \frac{\sin x}{7+\cos^2 x}\,dx$

15. $\int \frac{1}{\sqrt{x}\sqrt{1-x}}\,dx$

16. $\int \frac{3}{2\sqrt{x}(1+x)}\,dx$

17. $\int \frac{x-3}{x^2+1}\,dx$

18. $\int \frac{x^2+3}{x\sqrt{x^2-4}}\,dx$

19. $\int \frac{x+5}{\sqrt{9-(x-3)^2}}\,dx$

20. $\int \frac{x-2}{(x+1)^2+4}\,dx$

Evaluating a Definite Integral In Exercises 21–32, evaluate the definite integral.

21. $\int_0^{1/6} \frac{3}{\sqrt{1-9x^2}}\,dx$

22. $\int_0^{\sqrt{2}} \frac{1}{\sqrt{4-x^2}}\,dx$

23. $\int_0^{\sqrt{3}/2} \frac{1}{1+4x^2}\,dx$

24. $\int_{\sqrt{3}}^{3} \frac{1}{x\sqrt{4x^2-9}}\,dx$

25. $\int_3^6 \frac{1}{25+(x-3)^2}\,dx$

26. $\int_1^4 \frac{1}{x\sqrt{16x^2-5}}\,dx$

27. $\int_0^{\ln 5} \frac{e^x}{1+e^{2x}}\,dx$

28. $\int_{\ln 2}^{\ln 4} \frac{e^{-x}}{\sqrt{1-e^{-2x}}}\,dx$

29. $\int_{\pi/2}^{\pi} \frac{\sin x}{1+\cos^2 x}\,dx$

30. $\int_0^{\pi/2} \frac{\cos x}{1+\sin^2 x}\,dx$

31. $\int_0^{1/\sqrt{2}} \frac{\arcsin x}{\sqrt{1-x^2}}\,dx$

32. $\int_0^{1/\sqrt{2}} \frac{\arccos x}{\sqrt{1-x^2}}\,dx$

Completing the Square In Exercises 33–42, find or evaluate the integral by completing the square.

33. $\int_0^2 \frac{dx}{x^2-2x+2}$

34. $\int_{-2}^{2} \frac{dx}{x^2+4x+13}$

35. $\int \frac{2x}{x^2+6x+13}\,dx$

36. $\int \frac{2x-5}{x^2+2x+2}\,dx$

37. $\int \frac{1}{\sqrt{-x^2-4x}}\,dx$

38. $\int \frac{2}{\sqrt{-x^2+4x}}\,dx$

39. $\int_2^3 \frac{2x-3}{\sqrt{4x-x^2}}\,dx$

40. $\int \frac{1}{(x-1)\sqrt{x^2-2x}}\,dx$

41. $\int \frac{x}{x^4+2x^2+2}\,dx$

42. $\int \frac{x}{\sqrt{9+8x^2-x^4}}\,dx$

Integration by Substitution In Exercises 43–46, use the specified substitution to find or evaluate the integral.

43. $\int \sqrt{e^t-3}\,dt$

$u = \sqrt{e^t-3}$

44. $\int \frac{\sqrt{x-2}}{x+1}\,dx$

$u = \sqrt{x-2}$

45. $\int_1^3 \frac{dx}{\sqrt{x}(1+x)}$

$u = \sqrt{x}$

46. $\int_0^1 \frac{dx}{2\sqrt{3-x}\sqrt{x+1}}$

$u = \sqrt{x+1}$

WRITING ABOUT CONCEPTS

Comparing Integration Problems In Exercises 47–50, determine which of the integrals can be found using the basic integration formulas you have studied so far in the text.

47. (a) $\int \frac{1}{\sqrt{1-x^2}}\,dx$
(b) $\int \frac{x}{\sqrt{1-x^2}}\,dx$
(c) $\int \frac{1}{x\sqrt{1-x^2}}\,dx$

48. (a) $\int e^{x^2}\,dx$
(b) $\int xe^{x^2}\,dx$
(c) $\int \frac{1}{x^2}e^{1/x}\,dx$

49. (a) $\int \sqrt{x-1}\,dx$
(b) $\int x\sqrt{x-1}\,dx$
(c) $\int \frac{x}{\sqrt{x-1}}\,dx$

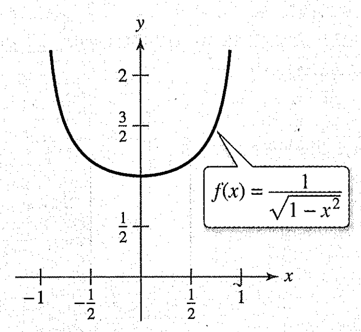
50. (a) $\int \frac{1}{1+x^4}\,dx$
(b) $\int \frac{x}{1+x^4}\,dx$
(c) $\int \frac{x^3}{1+x^4}\,dx$

51. **Finding an Integral** Decide whether you can find the integral

$$\int \frac{2\,dx}{\sqrt{x^2+4}}$$

using the formulas and techniques you have studied so far. Explain your reasoning.

52. HOW DO YOU SEE IT? Using the graph, which value best approximates the area of the region between the x-axis and the function over the interval $\left[-\frac{1}{2}, \frac{1}{2}\right]$? Explain.



(a) -3 (b) $\frac{1}{2}$ (c) 1 (d) 2 (e) 4

Differential Equation **In Exercises 53 and 54, use the differential equation and the specified initial condition to find y.**

53. $\dfrac{dy}{dx} = \dfrac{1}{\sqrt{4 - x^2}}$

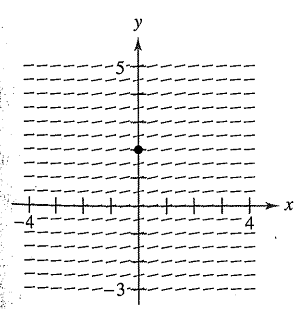
$y(0) = \pi$

54. $\dfrac{dy}{dx} = \dfrac{1}{4 + x^2}$

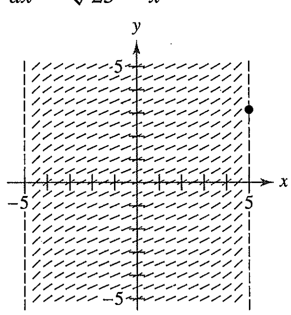
$y(2) = \pi$

Slope Field **In Exercises 55 and 56, a differential equation, a point, and a slope field are given. (a) Sketch two approximate solutions of the differential equation on the slope field, one of which passes through the given point. (b) Use integration to find the particular solution of the differential equation and use a graphing utility to graph the solution. Compare the result with the sketches in part (a). To print an enlarged copy of the graph, go to *MathGraphs.com*.**

55. $\dfrac{dy}{dx} = \dfrac{2}{9 + x^2}$, $(0, 2)$



56. $\dfrac{dy}{dx} = \dfrac{2}{\sqrt{25 - x^2}}$, $(5, \pi)$



Slope Field **In Exercises 57–60, use a computer algebra system to graph the slope field for the differential equation and graph the solution satisfying the specified initial condition.**

57. $\dfrac{dy}{dx} = \dfrac{10}{x\sqrt{x^2 - 1}}$

$y(3) = 0$

58. $\dfrac{dy}{dx} = \dfrac{1}{12 + x^2}$

$y(4) = 2$

59. $\dfrac{dy}{dx} = \dfrac{2y}{\sqrt{16 - x^2}}$

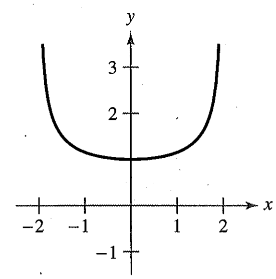
$y(0) = 2$

60. $\dfrac{dy}{dx} = \dfrac{\sqrt{y}}{1 + x^2}$

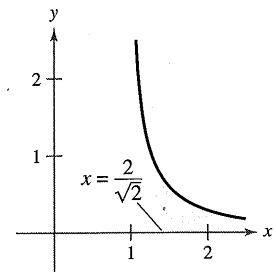
$y(0) = 4$

Area **In Exercises 61–66, find the area of the region.**

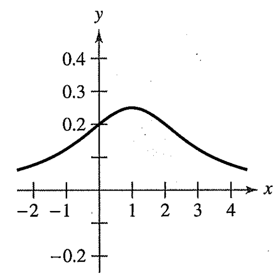
61. $y = \dfrac{2}{\sqrt{4 - x^2}}$



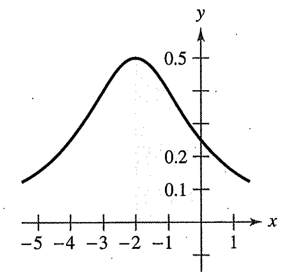
62. $y = \dfrac{1}{x\sqrt{x^2 - 1}}$



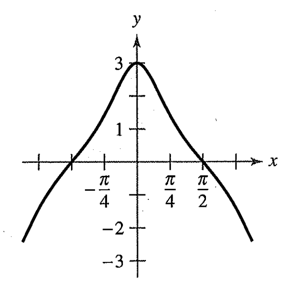
63. $y = \dfrac{1}{x^2 - 2x + 5}$



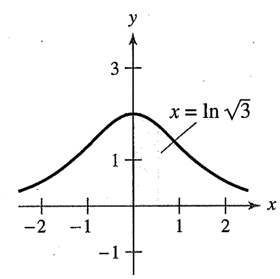
64. $y = \dfrac{2}{x^2 + 4x + 8}$



65. $y = \dfrac{3 \cos x}{1 + \sin^2 x}$



66. $y = \dfrac{4e^x}{1 + e^{2x}}$



67. Area

(a) Sketch the region whose area is represented by

$$\int_0^1 \arcsin x \, dx.$$

(b) Use the integration capabilities of a graphing utility to approximate the area.

(c) Find the exact area analytically.

68. Approximating Pi

(a) Show that

$$\int_0^1 \frac{4}{1 + x^2} \, dx = \pi.$$

(b) Approximate the number π using Simpson's Rule (with $n = 6$) and the integral in part (a).

(c) Approximate the number π by using the integration capabilities of a graphing utility.

69. Investigation Consider the function

$$F(x) = \frac{1}{2}\int_x^{x+2} \frac{2}{t^2+1}\,dt.$$

(a) Write a short paragraph giving a geometric interpretation of the function $F(x)$ relative to the function

$$f(x) = \frac{2}{x^2+1}.$$

Use what you have written to guess the value of x that will make F maximum.

(b) Perform the specified integration to find an alternative form of $F(x)$. Use calculus to locate the value of x that will make F maximum and compare the result with your guess in part (a).

70. Comparing Integrals Consider the integral

$$\int \frac{1}{\sqrt{6x - x^2}}\,dx.$$

(a) Find the integral by completing the square of the radicand.

(b) Find the integral by making the substitution $u = \sqrt{x}$.

(c) The antiderivatives in parts (a) and (b) appear to be significantly different. Use a graphing utility to graph each antiderivative in the same viewing window and determine the relationship between them. Find the domain of each.

True or False? **In Exercises 71–74, determine whether the statement is true or false. If it is false, explain why or give an example that shows it is false.**

71. $\displaystyle\int \frac{dx}{3x\sqrt{9x^2-16}} = \frac{1}{4}\operatorname{arcsec}\frac{3x}{4} + C$

72. $\displaystyle\int \frac{dx}{25+x^2} = \frac{1}{25}\arctan\frac{x}{25} + C$

73. $\displaystyle\int \frac{dx}{\sqrt{4-x^2}} = -\arccos\frac{x}{2} + C$

74. One way to find $\displaystyle\int \frac{2e^{2x}}{\sqrt{9-e^{2x}}}\,dx$ is to use the Arcsine Rule.

Verifying an Integration Rule **In Exercises 75–77, verify the rule by differentiating. Let $a > 0$.**

75. $\displaystyle\int \frac{du}{\sqrt{a^2-u^2}} = \arcsin\frac{u}{a} + C$

76. $\displaystyle\int \frac{du}{a^2+u^2} = \frac{1}{a}\arctan\frac{u}{a} + C$

77. $\displaystyle\int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a}\operatorname{arcsec}\frac{|u|}{a} + C$

78. Proof Graph

$$y_1 = \frac{x}{1+x^2}, \quad y_2 = \arctan x, \quad \text{and} \quad y_3 = x$$

on $[0, 10]$. Prove that

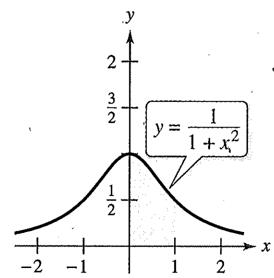
$$\frac{x}{1+x^2} < \arctan x < x \quad \text{for} \quad x > 0.$$

79. Numerical Integration

(a) Write an integral that represents the area of the region in the figure.

(b) Use the Trapezoidal Rule with $n = 8$ to estimate the area of the region.

(c) Explain how you can use the results of parts (a) and (b) to estimate π.



80. Vertical Motion An object is projected upward from ground level with an initial velocity of 500 feet per second. In this exercise, the goal is to analyze the motion of the object during its upward flight.

(a) If air resistance is neglected, find the velocity of the object as a function of time. Use a graphing utility to graph this function.

(b) Use the result of part (a) to find the position function and determine the maximum height attained by the object.

(c) If the air resistance is proportional to the square of the velocity, you obtain the equation

$$\frac{dv}{dt} = -(32 + kv^2)$$

where -32 feet per second per second is the acceleration due to gravity and k is a constant. Find the velocity as a function of time by solving the equation

$$\int \frac{dv}{32+kv^2} = -\int dt.$$

(d) Use a graphing utility to graph the velocity function $v(t)$ in part (c) for $k = 0.001$. Use the graph to approximate the time t_0 at which the object reaches its maximum height.

(e) Use the integration capabilities of a graphing utility to approximate the integral

$$\int_0^{t_0} v(t)\,dt$$

where $v(t)$ and t_0 are those found in part (d). This is the approximation of the maximum height of the object.

(f) Explain the difference between the results in parts (b) and (e).

■ **FOR FURTHER INFORMATION** For more information on this topic, see the article "What Goes Up Must Come Down; Will Air Resistance Make It Return Sooner, or Later?" by John Lekner in *Mathematics Magazine.* To view this article, go to *MathArticles.com.*