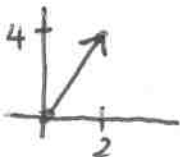
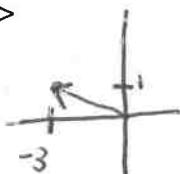
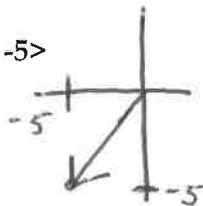


6.05 Algebraic Vectors Notes

Component form: $\langle a, b \rangle$

Ex: Draw the following vectors.

 $\langle 2, 4 \rangle$  $\langle -3, 1 \rangle$  $\langle -5, -5 \rangle$  $\langle 2, -10 \rangle$ Find component form of \overrightarrow{CD} given initial and terminal points: C (7,-3) and D (9,1)

$$\begin{aligned} \langle a, b \rangle &= \langle x_2 - x_1, y_2 - y_1 \rangle \\ &= \langle 9 - 7, 1 - (-3) \rangle \quad \left| \quad \overrightarrow{CD} = \langle 2, 4 \rangle \right. \end{aligned}$$

Operations with vectors:

If $\vec{u} = \langle a_1, b_1 \rangle$ and $\vec{v} = \langle a_2, b_2 \rangle$

$$\vec{u} + \vec{v} = \langle a_1 + a_2, b_1 + b_2 \rangle \quad \vec{u} - \vec{v} = \langle a_1 - a_2, b_1 - b_2 \rangle \quad k\vec{u} = \langle ka_1, kb_2 \rangle$$

Ex: Perform the indicated operations given $\vec{u} = \langle -4, 1 \rangle$ and $\vec{v} = \langle 2, 5 \rangle$.

$$\vec{u} + \vec{v} = \langle -4 + 2, 1 + 5 \rangle = \langle -2, 6 \rangle$$

$$\vec{u} - \vec{v} = \langle -4 - 2, 1 - 5 \rangle = \langle -6, -4 \rangle$$

 $2\vec{u} - \vec{v} =$

$$\langle -8, 2 \rangle - \langle 2, 5 \rangle = \langle -10, -3 \rangle$$

$$-3\vec{u} - 4\vec{v} = \langle +12, -3 \rangle - \langle 8, 20 \rangle$$

$$\langle 4, -23 \rangle$$

Unit Vector: vector with magnitude of 1

 $i =$ unit vector on the positive x-axis $\langle 1, 0 \rangle$ $j =$ unit vector on the positive y-axis $\langle 0, 1 \rangle$

Linear combination of unit vectors (or "sum of unit vectors"):

$$\langle a, b \rangle \rightarrow ai + bj$$

Ex: Write $\langle -3, 8 \rangle$ as a sum of unit vectors.

$$-3i + 8j$$

Ex: Write $\langle 0, -5 \rangle$ as a linear combination of unit vectors.

$$0i - 5j = -5j$$

Ex: Find the vector with initial point X (5,5) and terminal point Y (-2,6) as a sum of unit vectors.

$$\langle -2 - 5, 6 - 5 \rangle \rightarrow \langle -7, 1 \rangle$$

$$\overrightarrow{XY} = \langle -7, 1 \rangle = -7i + 1j$$

6.05 Practice: ODDS #1 - 17, 29-35

Find component form of \overrightarrow{AB} with the given initial and terminal points.

1. $A(-3, 1), B(4, 5)$ 2. $A(2, -7), B(-6, 9)$
 3. $A(10, -2), B(3, -5)$ 4. $A(-2, 7), B(-9, -1)$
 5. $A(-5, -4), B(8, -2)$ 6. $A(-2, 6), B(1, 10)$
 7. $A(2.5, -3), B(-4, 1.5)$ 8. $A(-4.3, 1.8), B(9.4, -6.2)$
 9. $A\left(\frac{1}{2}, -9\right), B\left(6, \frac{5}{2}\right)$ 10. $A\left(\frac{3}{5}, -\frac{2}{5}\right), B(-1, 7)$

Find each of the following for $\mathbf{f} = \langle 8, 0 \rangle$, $\mathbf{g} = \langle -3, -5 \rangle$, and $\mathbf{h} = \langle -6, 2 \rangle$. (Example 3)

11. $4\mathbf{h} - \mathbf{g}$ 12. $\mathbf{f} + 2\mathbf{h}$
 13. $3\mathbf{g} - 5\mathbf{f} + \mathbf{h}$ 14. $2\mathbf{f} + \mathbf{g} - 3\mathbf{h}$
 15. $\mathbf{f} - 2\mathbf{g} - 2\mathbf{h}$ 16. $\mathbf{h} - 4\mathbf{f} + 5\mathbf{g}$
 17. $4\mathbf{g} - 3\mathbf{f} + \mathbf{h}$ 18. $6\mathbf{h} + 5\mathbf{f} - 10\mathbf{g}$

Let \overrightarrow{DE} be the vector with the given initial and terminal points. Write \overrightarrow{DE} as a linear combination of the vectors \mathbf{i} and \mathbf{j} . (Example 5)

28. $D(4, -1), E(5, -7)$ 29. $D(9, -6), E(-7, 2)$
 30. $D(3, 11), E(-2, -8)$ 31. $D(9.5, 1), E(0, -7.3)$
 32. $D(-3, -5.7), E(6, -8.1)$ 33. $D(-4, -6), E(9, 5)$
 34. $D\left(\frac{1}{8}, 3\right), E\left(-4, \frac{2}{7}\right)$ 35. $D(-3, 1.5), E(-3, 1.5)$

$$\langle x_2 - x_1, y_2 - y_1 \rangle$$

$$1) \langle 4 - (-3), 5 - 1 \rangle \rightarrow \langle 7, 4 \rangle$$

$$3) \langle 3 - 10, -5 - (-2) \rangle \rightarrow \langle -7, -3 \rangle$$

$$11) \langle -24, 8 \rangle - \langle -3, -5 \rangle = \langle -21, 13 \rangle$$

$$13) \langle -9, -15 \rangle - \langle 40, 0 \rangle + \langle -6, 2 \rangle = \langle -55, -13 \rangle$$

$$29) \langle -7 - 9, 2 - (-6) \rangle \rightarrow \langle -16, 8 \rangle$$

$$\boxed{-16\mathbf{i} + 8\mathbf{j}}$$

$$31) \langle 0 - 9.5, -7.3 - 1 \rangle \rightarrow \langle -9.5, -8.3 \rangle$$

$$\boxed{-9.5\mathbf{i} - 8.3\mathbf{j}}$$

6.05 Practice: ODDS #1 - 17, 29-35

Find component form of \overline{AB} with the given initial and terminal points.

- | | |
|--|---|
| 1. $A(-3, 1), B(4, 5)$ | ① $\langle 7, 4 \rangle$ |
| 3. $A(10, -2), B(3, -5)$ | ③ $\langle -7, -3 \rangle$ |
| 5. $A(-5, -4), B(8, -2)$ | ⑤ $\langle 13, 2 \rangle$ |
| 7. $A(2.5, -3), B(-4, 1.5)$ | ⑦ $\langle -6.5, 4.5 \rangle$ |
| 9. $A\left(\frac{1}{2}, -9\right), B\left(6, \frac{5}{2}\right)$ | ⑨ $\langle \frac{11}{2}, \frac{23}{2} \rangle$ or $\langle 5.5, 11.5 \rangle$ |

Find each of the following for $f = \langle 8, 0 \rangle$, $g = \langle -3, -5 \rangle$, and $h = \langle -6, 2 \rangle$. (Example 3)

- | | | |
|-------------------|--|----------------------------|
| 11. $4h - g$ | $\langle -24, 8 \rangle - \langle -3, -5 \rangle$ | $\langle -21, 13 \rangle$ |
| 13. $3g - 5f + h$ | $\langle -9, -15 \rangle - \langle 40, 0 \rangle + \langle -6, 2 \rangle$ | $\langle -55, -13 \rangle$ |
| 15. $f - 2g - 2h$ | $\langle 8, 0 \rangle - \langle -6, -10 \rangle - \langle -12, 4 \rangle$ | $\langle 26, 6 \rangle$ |
| 17. $4g - 3f + h$ | $\langle -12, -20 \rangle - \langle 24, 0 \rangle + \langle -6, 2 \rangle$ | $\langle -42, -18 \rangle$ |

Let \overline{DE} be the vector with the given initial and terminal points. Write \overline{DE} as a linear combination of the vectors i and j . (Example 5)

- | | | |
|------------------------------|------------------------------|----------------|
| 29. $D(9, -6), E(-7, 2)$ | $\langle -16, 8 \rangle$ | $-16i + 8j$ |
| 31. $D(9.5, 1), E(0, -7.3)$ | $\langle -9.5, -8.3 \rangle$ | $-9.5i - 8.3j$ |
| 33. $D(-4, -6), E(9, 5)$ | $\langle 13, 11 \rangle$ | $13i + 11j$ |
| 35. $D(-3, 1.5), E(-3, 1.5)$ | $\langle 0, 0 \rangle$ | $0i + 0j$ |