6.06 Notes: More Algebraic Vectors

Magnitude: Length of a vector; can represent speed, distance, or force

$$|\vec{v}| = \sqrt{a^2+b^2}$$

Ex: Find $|\vec{v}|$ if $\vec{v} = \langle -3, 8\rangle$. $|\vec{v}| = \sqrt{(-3)^2+8^2} = \sqrt{9+64} = \sqrt{73}$

Ex: Find the magnitude of a vector with initial point A (-2,3) and terminal point B (4,5).

$\langle 4-(-2), 5-3\rangle$ $\quad \overrightarrow{AB} = \langle 6, 2\rangle$ $\quad |\overrightarrow{AB}| = \sqrt{6^2+2^2} = \sqrt{40} = 2\sqrt{10}$ $\quad (x_1, y_1)$ (x_2, y_2)

Direction: direction that vector is pointing to (in standard form)

$\tan\theta = \frac{b}{a}$ $\quad \theta = \tan^{-1}\left(\frac{b}{a}\right)$

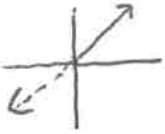
Ex: Find the direction of $\vec{e} = \langle 8, 5\rangle$ Q1 $\quad \theta = \tan^{-1}\left(\frac{5}{8}\right) = 32°$

Ex: Find the direction of $\vec{f} = -6i - 7j$

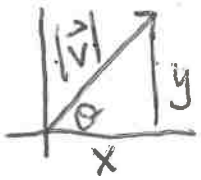
$\langle -6, -7\rangle$ $\quad \theta = \tan^{-1}\left(\frac{-7}{-6}\right) = 49.39° + 180 = 229.39°$

Q3

Think about it: If the direction of $\vec{x}$ is 50°, how would direction change for each of the following:

- $5\vec{x}$ no direction change
- $-3\vec{x}$ exact opposite direction  $50° + 180 = 230°$
- $\frac{1}{2}\vec{x}$ no direction change

Components from Magnitude and Direction:



$\cos\theta = \frac{x}{|\vec{v}|}$ $\quad x = |\vec{v}|\cos\theta$

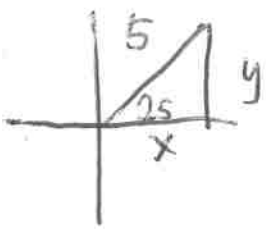
$\sin\theta = \frac{y}{|\vec{v}|}$ $\quad y = |\vec{v}|\sin\theta$

Ex: Find the components of $\vec{g}$ if it has magnitude of 5 and direction of 25°.

$\langle |\vec{v}|\cos\theta, |\vec{v}|\sin\theta\rangle$

$\langle 5\cos 25, 5\sin 25\rangle$

$\langle 4.532, 2.113\rangle$



6.06 Practice: ODDS #1–9, 39–51

Find the magnitude of $\overrightarrow{AB}$ with the given initial and terminal points (same 1-9 as yesterday).

1. $A(-3, 1), B(4, 5)$
2. $A(2, -7), B(-6, 9)$
3. $A(10, -2), B(3, -5)$
4. $A(-2, 7), B(-9, -1)$
5. $A(-5, -4), B(8, -2)$
6. $A(-2, 6), B(1, 10)$
7. $A(2.5, -3), B(-4, 1.5)$
8. $A(-4.3, 1.8), B(9.4, -6.2)$
9. $A\left(\frac{1}{2}, -9\right), B\left(6, \frac{5}{2}\right)$
10. $A\left(\frac{3}{5}, -\frac{2}{5}\right), B(-1, 7)$

1) $\langle 7, 4 \rangle$ $|\vec{AB}| = \sqrt{7^2+4^2} = \sqrt{65}$

3) $\langle -7, -3 \rangle$ $|\vec{AB}| = \sqrt{(-7)^2+(-3)^2} = \sqrt{58}$

Find the component form of v with the given magnitude and direction angle. (Example 6)

$\langle |\vec{v}|\cos\theta, |\vec{v}|\sin\theta \rangle$

38. $|\mathbf{v}| = 12, \theta = 60°$
39. $|\mathbf{v}| = 4, \theta = 135°$
40. $|\mathbf{v}| = 6, \theta = 240°$
41. $|\mathbf{v}| = 16, \theta = 330°$
42. $|\mathbf{v}| = 28, \theta = 273°$
43. $|\mathbf{v}| = 15, \theta = 125°$

39) $\langle 4\cos 135, 4\sin 135 \rangle = 4\left(\frac{-\sqrt{2}}{2}\right), 4\left(\frac{\sqrt{2}}{2}\right)$

$\langle -2\sqrt{2}, 2\sqrt{2} \rangle$

41) $\langle 16\cos 330, 16\sin 330 \rangle$

$16\left(\frac{\sqrt{3}}{2}\right), 16\left(-\frac{1}{2}\right)$

$\langle 8\sqrt{3}, -8 \rangle$

Find the direction angle of each vector to the nearest tenth of a degree. (Example 7)

44. $3\mathbf{i} + 6\mathbf{j}$
45. $-2\mathbf{i} + 5\mathbf{j}$
46. $8\mathbf{i} - 2\mathbf{j}$
47. $-4\mathbf{i} - 3\mathbf{j}$
48. $\langle -5, 9 \rangle$
49. $\langle 7, 7 \rangle$
50. $\langle -6, -4 \rangle$
51. $\langle 3, -8 \rangle$

45) $\langle -2, 5 \rangle$ Q2 $\theta = \tan^{-1}\left(\frac{5}{-2}\right) = -68.19° + 180 = 111.8°$

47) $\langle -4, -3 \rangle$ Q3 $\theta = \tan^{-1}\left(\frac{-3}{-4}\right) = 36.87° + 180 = 216.9°$

6.06 Practice: ODDS #1–9, 39–51

Find the magnitude of $\overline{AB}$ with the given initial and terminal points (same 1-9 as yesterday).

1. $A(-3, 1), B(4, 5)$ $\overrightarrow{AB} = \langle 7, 4 \rangle$ $|\overrightarrow{AB}| = \sqrt{49+16} = \boxed{\sqrt{65}}$
3. $A(10, -2), B(3, -5)$ $\overrightarrow{AB} = \langle -7, -3 \rangle$ $|\overrightarrow{AB}| = \sqrt{49+9} = \boxed{\sqrt{58}}$
5. $A(-5, -4), B(8, -2)$ $\overrightarrow{AB} = \langle 13, 2 \rangle$ $|\overrightarrow{AB}| = \sqrt{169+4} = \boxed{\sqrt{173}}$
7. $A(2.5, -3), B(-4, 1.5)$ $\overrightarrow{AB} = \langle -6.5, 4.5 \rangle$ $|\overrightarrow{AB}| = \sqrt{42.25+20.25} = \boxed{\sqrt{62.5}}$
9. $A\left(\frac{1}{2}, -9\right), B\left(6, \frac{5}{2}\right)$ $\overrightarrow{AB} = \langle 5.5, 11.5 \rangle$ $|\overrightarrow{AB}| = \sqrt{30.25+132.25} = \boxed{\sqrt{162.5}}$

Find the component form of v with the given magnitude and direction angle. (Example 6)

39. $|v| = 4, \theta = 135°$ $\langle 4\cos 135°, 4\sin 135° \rangle = \boxed{\langle -2\sqrt{2}, 2\sqrt{2} \rangle}$
41. $|v| = 16, \theta = 330°$ $\langle 16\cos 330°, 16\sin 330° \rangle = \boxed{\langle 8\sqrt{3}, -8 \rangle}$
43. $|v| = 15, \theta = 125°$ $\langle 15\cos 125°, 15\sin 125° \rangle = \boxed{\langle -8.604, 12.287 \rangle}$

Find the direction angle of each vector to the nearest tenth of a degree. (Example 7)

45. $-2i + 5j$ $\alpha = \tan^{-1}\left(\frac{5}{-2}\right) + 180 = \boxed{111.801°}$
47. $-4i - 3j$ $\alpha = \tan^{-1}\left(\frac{-3}{-4}\right) + 180 = \boxed{216.870°}$
49. $\langle 7, 7 \rangle$ $\alpha = \tan^{-1}\left(\frac{7}{7}\right) = \boxed{45°}$
51. $\langle 3, -8 \rangle$ $\alpha = \tan^{-1}\left(\frac{-8}{3}\right) + 360 = \boxed{290.556°}$