6.07 Angle Between Vectors Notes

Dot Product: if u = <a₁, b₁> and v = <a₂, b₂> then $\vec{u}\cdot\vec{v}$ = $a_1 \cdot a_2 + b_1 \cdot b_2$

Ex: Find the dot product between the following pairs of vectors.

1. u = <3, 6>, v = <-4, 2>

$\vec{u}\cdot\vec{v} = 3(-4)+6(2)=0$

2. a = 2i + 5j, b = 8i – 4j

$\vec{a}\cdot\vec{b} = 2(8)+5(-4) = 16-20 = \boxed{-4}$

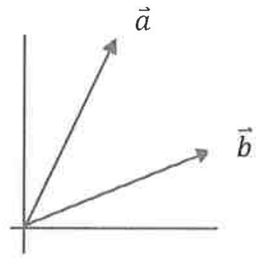
Angle Between 2 Vectors:

$\vec{a} =$ $\langle|\vec{a}|\cos$

$\vec{b} =$

$\vec{a}\cdot\vec{b} =$

$$\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}$$



$$\cos\theta = \frac{\vec{c}\cdot\vec{d}}{|\vec{c}|\cdot|\vec{d}|} \rightarrow \frac{(-5)(4)+8(-2)}{\sqrt{5^2+2^2}\cdot\sqrt{4^2+8^2}}$$

$$\cos\theta = \frac{-36}{\sqrt{29}\cdot\sqrt{80}} \rightarrow \cos\theta = \frac{-36}{\sqrt{2320}} = -0.7474$$

$\theta = \cos^{-1}(-0.7474)$

$\boxed{\theta = 138.366^\circ}$

$\boxed{\textbf{Ex:}}$ **Find the angle between vectors $\vec{c} = \langle -5, -2\rangle$ and $\vec{d} = \langle 4, 8\rangle$.**

Orthogonal Vectors: Two vectors are orthogonal (perpendicular) if and only if their dot product is zero.

Ex: Create a non-zero vector $\vec{n}$ that is orthogonal to $\vec{m} = \langle -4, 2\rangle$.

$-4\cdot(\underline{?})+2(\underline{?})=0$ $\quad \langle 2,4\rangle$ or $\langle 1,2\rangle$, or $\langle -3,-6\rangle$

(infinitely many possibilities)

Parallel Vectors: Vectors are parallel if the dot product is equal to the product of their magnitudes $(\vec{a}\cdot\vec{b} = |\vec{a}|\cdot|\vec{b}|)$

Ex: Determine the angle between the following pairs of vectors.

$<-2, 5>$ *and* $<-6, 15>$	$<8, 12>$ *and* $<6, 9>$	$<10, 11>$ *and* $<-5, -5.5>$
$\cos\theta = \frac{-2(-6)+5(15)}{\sqrt{4+25}\cdot\sqrt{36+225}}$	$\cos\theta = \frac{8(6)+12(9)}{\sqrt{64+144}\cdot\sqrt{36+81}}$	$\cos\theta = \frac{10(-5)+11(-5.5)}{\sqrt{10^2+11^2}\cdot\sqrt{25+5.5^2}}$

6.07 Practice: #1-9, 19-24

Determine if the following vectors are parallel, orthogonal, or neither.

1. $u = < 3, -5 >, v = < 6, 2 >$

2. $u = < -10, -16 >, v = < -8, 5 >$

3. $u = < 9, -3 >, v = < 1, 3 >$

4. $u = < 4, -4 >, v = < 7, 5 >$

5. $u = < 1, -4 >, v = < 2, -8 >$

6. $u = 11i + 7j;\ v = -7i + 11j$

7. $u = < -4, 6 >, v = < -5, -2 >$

8. $u = 8i + 6j; v = -i + 2j$

9. **SPORTING GOODS** The vector $\mathbf{u} = \langle 406, 297 \rangle$ gives the numbers of men's basketballs and women's basketballs, respectively, in stock at a sporting goods store. The vector $\mathbf{v} = \langle 27.5, 15 \rangle$ gives the prices in dollars of the two types of basketballs, respectively. (Example 1)

   a. Find the dot product $\mathbf{u} \cdot \mathbf{v}$.

   b. Interpret the result in the context of the problem.

1) $u \cdot v = 3(6) + 2(-5) = 18 - 10 = \boxed{8}$ (not orthogonal)

3) $u \cdot v = 9(1) + 3(-3) = \boxed{0}$ (orthogonal)

5) $u \cdot v = 1(2) + 4(8) = \boxed{34}$ (not orthogonal)

7) $u \cdot v = (-4)(-5) + 6(-2) = 20 - 12 = \boxed{8}$ (not orthogonal)

9) $u \cdot v = 406(27.5) + 297(15) = \$15{,}620$ (total cost of all basketballs in stock)

Find the angle θ between u and v to the nearest tenth of a degree. (Example 3)

16. $\mathbf{u} = \langle 0, -5 \rangle$, $\mathbf{v} = \langle 1, -4 \rangle$

17. $\mathbf{u} = \langle 7, 10 \rangle$, $\mathbf{v} = \langle 4, -4 \rangle$

18. $\mathbf{u} = \langle -2, 4 \rangle$, $\mathbf{v} = \langle 2, -10 \rangle$

19. $\mathbf{u} = -2\mathbf{i} + 3\mathbf{j}$, $\mathbf{v} = -4\mathbf{i} - 2\mathbf{j}$

20. $\mathbf{u} = \langle -9, 0 \rangle$, $\mathbf{v} = \langle -1, -1 \rangle$

21. $\mathbf{u} = -\mathbf{i} - 3\mathbf{j}$, $\mathbf{v} = -7\mathbf{i} - 3\mathbf{j}$

22. $\mathbf{u} = \langle 6, 0 \rangle$, $\mathbf{v} = \langle -10, 8 \rangle$

23. $\mathbf{u} = -10\mathbf{i} + \mathbf{j}$, $\mathbf{v} = 10\mathbf{i} - 5\mathbf{j}$

24. CAMPING Regina and Luis set off from their campsite to search for firewood. The path that Regina takes can be represented by $\mathbf{u} = \langle 3, -5 \rangle$. The path that Luis takes can be represented by $\mathbf{v} = \langle -7, 6 \rangle$. Find the angle between the pair of vectors. (Example 3)

24) $u = \langle 3, -5 \rangle \quad v = \langle -7, 6 \rangle$

$$\cos\theta = \frac{3(-7) + 6(-5)}{\sqrt{9+25} \cdot \sqrt{49+36}} = \frac{-51}{\sqrt{34} \cdot \sqrt{85}}$$

$$\cos\theta = \frac{-3}{\sqrt{10}} \qquad \theta = \cos^{-1}\left(\frac{-3}{\sqrt{10}}\right)$$

$$\boxed{\theta = 161.565^\circ}$$

20) $u = \langle -9, 0 \rangle \quad v = \langle -1, -1 \rangle$

$$\cos\theta = \frac{-9(-1) + 0(-1)}{\sqrt{81+0} \cdot \sqrt{1+1}} = \frac{+9}{9\sqrt{2}}$$

$$\cos\theta = \frac{+1}{\sqrt{2}}$$

$$\boxed{\theta = 45^\circ}$$

21) $u = \langle -1, -3 \rangle \quad V = \langle -7, -3 \rangle$

$$\cos\theta = \frac{(1)(7) + (-3)(-3)}{\sqrt{1^2+3^2} \cdot \sqrt{7^2+3^2}} = \frac{16}{\sqrt{10} \cdot \sqrt{58}}$$

$$\cos\theta = \frac{16}{\sqrt{580}} \qquad \theta = \cos^{-1}\left(\frac{16}{\sqrt{580}}\right)$$

$$\boxed{\theta = 48.366^\circ}$$

22) $u = \langle 6, 0 \rangle \quad V = \langle -10, 8 \rangle$

$$\cos\theta = \frac{6(-10) + 0(8)}{\sqrt{36+0} \cdot \sqrt{100+64}}$$

$$\cos\theta = \frac{-60}{(6)(\sqrt{164})}$$

$$\theta = \cos^{-1}\left(\frac{-5}{\sqrt{41}}\right)$$

$$\boxed{\theta = 141.340^\circ}$$

23) $u = \langle -10, 1 \rangle \quad V = \langle 10, -5 \rangle$

$$\cos\theta = \frac{-10(10) + 1(-5)}{\sqrt{100+1} \cdot \sqrt{100+25}}$$

$$\cos\theta = \frac{-105}{\sqrt{101} \cdot \sqrt{125}}$$

$$\theta = \cos^{-1}\left(\frac{-21}{\sqrt{505}}\right)$$

$$\boxed{\theta = 159.146^\circ}$$