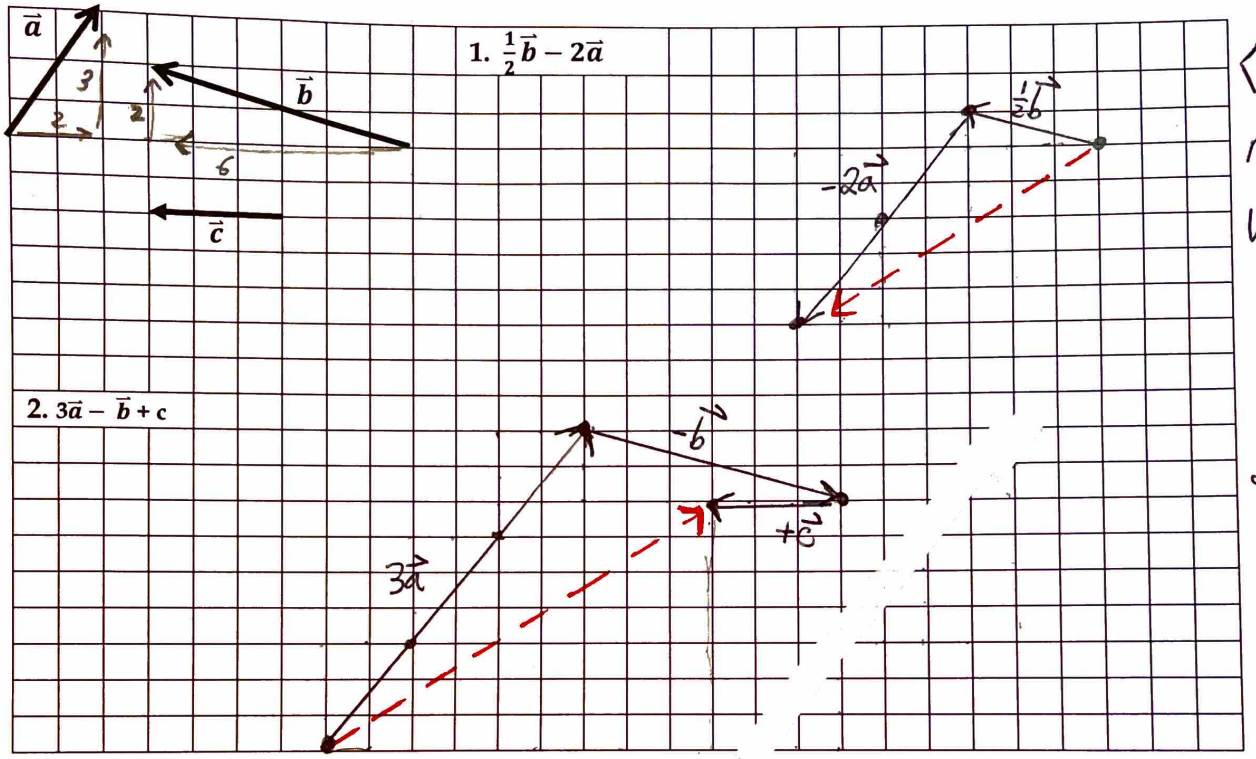
Accelerated Precalculus
6.08 Vector Quiz Review WS 2

Key

The angle between 2 vectors: $\cos\theta = \frac{\vec{v}\cdot\vec{w}}{|\vec{v}|\,|\vec{w}|}$

Draw each and label the resultant. Then find the component form and magnitude of each resultant.



$\langle -7, -5 \rangle$

magnitude

$\sqrt{7^2+5^2}$

$\boxed{\sqrt{74}}$

2) $\langle 9, 7 \rangle$

magnitude

$\sqrt{9^2+7^2}$

$\boxed{=\sqrt{130}}$

3. Given points A(-3, 6) and B(5, -4) with the initial& terminal point in alphabetical order, find the resulting vector $\vec{v}$ in component form. Find the magnitude and direction of vector.

$\vec{V} = \langle 5--3, -4-6 \rangle$

$\vec{V} = \langle 8, -10 \rangle \rightarrow 8i - 10j$

$|\vec{v}| = \sqrt{8^2+10^2} = \sqrt{164} = \boxed{2\sqrt{41}}$

$\theta = \tan^{-1}\left(\frac{-10}{8}\right) = -51.34°$

Q4 → $360 - 51.34°$

$\boxed{\theta = 308.66°}$

4. The terminal point of ***vector*** $\vec{k}$ is B(-3, 4). If $k = \langle 1, -6 \rangle$, find the initial point A.

$A(x_1, y_1)$ $B(-3, 4)$

$k = \langle -3 - x_1, 4 - y_1 \rangle$

$\langle 1, -6 \rangle = \langle -3 - x_1, 4 - y_1 \rangle$

$-3 - x_1 = 1$

$-4 = x_1$

$4 - y_1 = -6$

$10 = y_1$

$\boxed{\text{point A is } (-4, 10)}$

5. Determine the measure of the angle made between $\boldsymbol{a}$ = <-1, -3> and $\boldsymbol{b}$ = <5, -6>.

$$\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}|\cdot|\vec{b}|} = \frac{-1(5)+-3(-6)}{\sqrt{1^2+3^2}\cdot\sqrt{5^2+6^2}} \rightarrow \cos\theta = \frac{13}{\sqrt{10}\cdot\sqrt{61}}$$

$$\theta = \cos^{-1}\left(\frac{13}{\sqrt{610}}\right) \quad \boxed{\theta = 58.241^\circ}$$

6. Find $\boldsymbol{u}\cdot\boldsymbol{v}$ if $|\boldsymbol{u}| = 3$, $|\boldsymbol{v}| = 5$, and the angle between the vectors is $\theta = \frac{\pi}{6}$

$$\frac{\cos\theta}{1} = \frac{u\cdot v}{|u||v|} \quad \left|\quad \frac{\cos(\pi/6)}{1} = \frac{u\cdot v}{3\cdot 5} \quad \right| \quad \frac{\sqrt{3}}{2} = \frac{u\cdot v}{15} \quad \left|\quad 15\sqrt{3} = 2(u\cdot v)\right.$$

$$\boxed{\frac{15\sqrt{3}}{2} = u\cdot v}$$

Given: $\vec{u} = < 4, -8 >$, $\vec{v} = 2i - 3j$ and $\vec{w} = < 16, 4 >$

$\vec{v} = \langle 2, -3 \rangle$

7. a) Find: $-2\vec{u} - \frac{1}{4}\vec{w}$

$$-2\langle 4, -8\rangle - \frac{1}{4}\langle 16, 4\rangle$$

$$\langle -8, 16\rangle - \langle 4, 1\rangle$$

$$= \boxed{\langle -12, 15\rangle}$$

b) Are vectors $\vec{u}$ *and* $\vec{v}$ *orthogonal?* Justify with reason.

$u\cdot v = 4(2)+-8(-3) = 8+24 = 32 \neq 0$

Since $u\cdot v \neq 0$, vectors u and v are not orthogonal to each other.

Find the component form of $\vec{v}$ given the following.

8. $|\vec{v}| = 2, \theta = 120^\circ$

$$\langle 2\cos 120, 2\sin 120\rangle$$

$$\boxed{\langle -1, \sqrt{3}\rangle}$$

9. $|\vec{v}| = 2, \theta = 60^\circ$

$$\langle 2\cos 60, 2\sin 60\rangle$$

$$\boxed{\langle 1, \sqrt{3}\rangle}$$

10. A boat is traveling west at 35 mph. The current is moving 60 degrees at 2 mph. What is the boat's resultant speed? What is the direction of the boat's movement?

$\vec{b} = \langle -35, 0\rangle$

$\vec{c} = \langle 2\cos 60, 2\sin 60\rangle$

$\vec{c} = \langle 1, \sqrt{3}\rangle$

resultant $\vec{r} = \langle -34, \sqrt{3}\rangle$

speed $|\vec{r}| = \sqrt{34^2+\sqrt{3}^2} = \boxed{34.04 \text{ mph}}$

$\theta = \tan^{-1}\left(\frac{\sqrt{3}}{-34}\right) = -2.916^\circ + 180 = \boxed{177.08^\circ}$

Q2

11. Draw and label a vector with magnitude of 12 meters per second

At the direction of 330 degrees. Represent vector in component form.

$$\langle 12\cos 330, 12\sin 330\rangle = \boxed{\langle 6\sqrt{3}, -6\rangle}$$