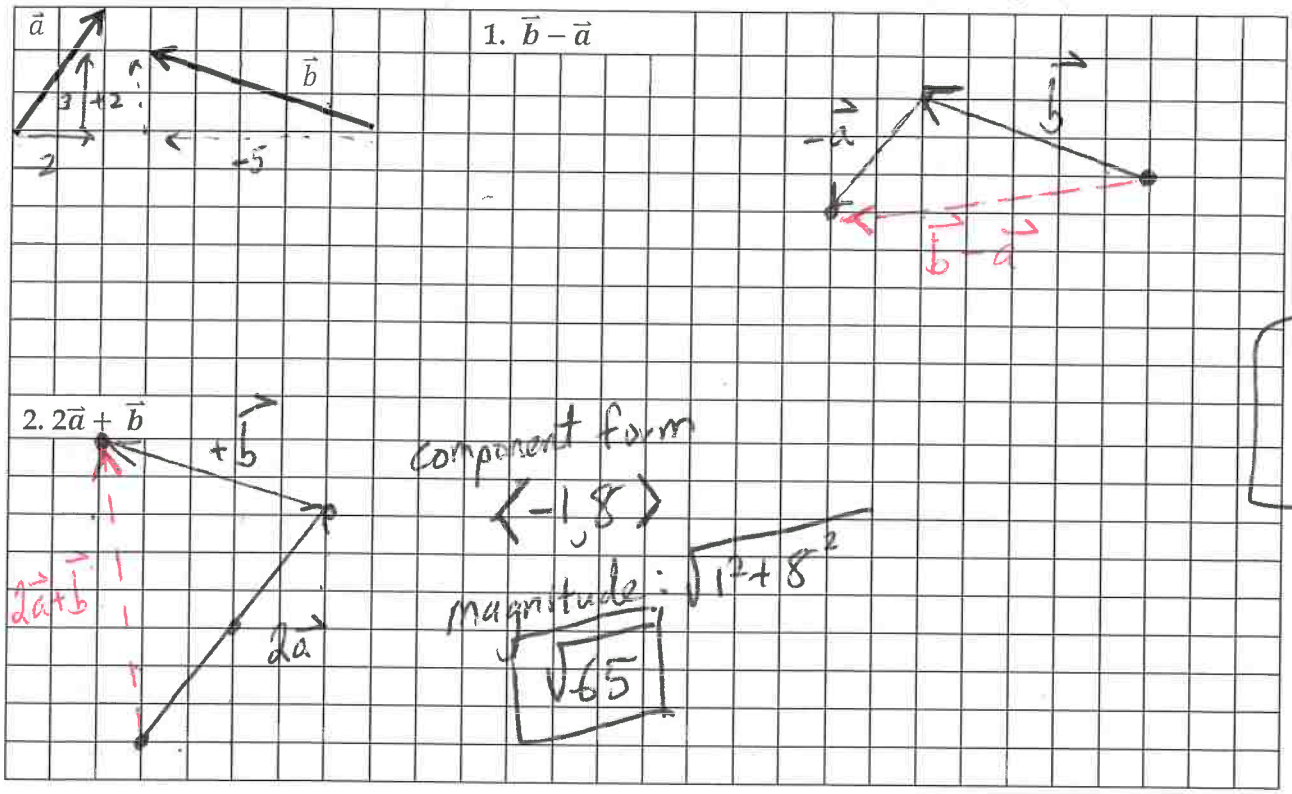
6.08 2D Vector Quiz Review

Draw each and label the resultant. Then find the component form and magnitude of each resultant.



component form $\langle -1, 8 \rangle$

magnitude: $\sqrt{1^2+8^2}$ $\sqrt{65}$

component form $\langle -7, -1 \rangle$

magnitude $\sqrt{7^2+1^2}$ $=\sqrt{50}=5\sqrt{2}$

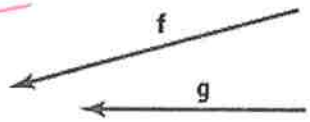
Draw the vector diagram and the resultant vector. Use either the triangle or parallelogram method.

skip

3. $\vec{h}+\vec{j}$

skip

4. $\vec{f}-2\vec{g}$



$\langle x_2-x_1, y_2-y_1 \rangle$

Use each set of points to create a vector with initial point & terminal point in *alphabetical order*. For each, find the component form, write as a sum of unit vectors, and find the magnitude and direction.

5. A (−3, 5) and B (5, −1)

$\langle 5--3, -1-5 \rangle$

$\langle 8, -6 \rangle \rightarrow 8i-6j$

Q4

$|\vec{AB}| = \sqrt{8^2+6^2} = \sqrt{100} = 10$

$\theta = \tan^{-1}\left(\frac{-6}{8}\right) = -36.87 + 360°$

$\theta = 323.13°$

6. C(10, −6) and D(−8,2)

$\langle -8-10, 2--6 \rangle$

$\langle -18, 8 \rangle \rightarrow -18i+8j$

$|\vec{CD}| = \sqrt{18^2+8^2} = 2\sqrt{97}$

$\theta = \tan^{-1}\left(\frac{8}{-18}\right) = -23.96$

Q2 $\rightarrow -23.96+180$

$\theta = 156.04°$

7. S(1,12) and T(−2, −9)

$\langle -2-1, -9-12 \rangle$

$\langle -3, -21 \rangle \rightarrow -3i-21j$

$|\vec{ST}| = \sqrt{3^2+21^2} = 15\sqrt{2}$

$\theta = \tan^{-1}\left(\frac{-21}{-3}\right) = 81.87°$

Q3 $\rightarrow 81.87+180$

$\theta = 261.87°$

$\vec{v} = \langle 1, -2 \rangle$

Given: $\vec{u} = \langle 5, -12 \rangle$, $\vec{v} = i - 2j$ and $\vec{w} = \langle 9, 3 \rangle$

8. a) Find: $-\vec{u} - \frac{1}{3}\vec{w}$ b) $2\vec{v} - \vec{u}$ $\langle 2, -4 \rangle - \langle 5, -12 \rangle$

$\langle -5, 12 \rangle - \langle 3, 1 \rangle = \boxed{\langle -8, 11 \rangle}$ $\boxed{\langle -3, 8 \rangle}$

c) Are any of the vectors orthogonal? Show work.

$u \cdot v = 5(1) + -2(-12) = 5 + 24 = 29 \neq 0$, not orthogonal

$u \cdot w = 5(9) + 3(-12) = 45 - 36 = 9 \neq 0$, not orthogonal

$\vec{v} \cdot \vec{w} = 1(9) + 3(-2) = 9 - 6 = 3 \neq 0$, not orthogonal

d) If the vectors are not orthogonal, use $\cos\theta = \frac{\vec{u} \cdot \vec{v}}{|\vec{u}||\vec{v}|}$ to find the angle between each pair of vectors.

$\cos\theta = \dfrac{5(1) + -2(-12)}{\sqrt{5^2+12^2} \cdot \sqrt{1^2+2^2}}$ $\cos\theta = 0.9976$ $\theta = \cos^{-1}(0.9976)$ $\boxed{\theta = 3.945°}$

$\cos\theta = \dfrac{29}{13 \cdot \sqrt{5}}$

Find the component form of $\vec{v}$ given the following. $\langle |\vec{v}|\cos\theta, |\vec{v}|\sin\theta \rangle$

9. $|\vec{v}| = 4, \theta = 135°$ $\langle 4\cos 135, 4\sin 135 \rangle$ $\langle -2\sqrt{2}, 2\sqrt{2} \rangle$

10. $|\vec{v}| = 6, \theta = 240°$ $\langle 6\cos 240, 6\sin 240 \rangle$ $\langle -3, -3\sqrt{3} \rangle$

11. $|\vec{v}| = 15, \theta = 330°$ $\langle 15\cos 330, 15\sin 330 \rangle$ $\left\langle \frac{15\sqrt{3}}{2}, -\frac{15}{2} \right\rangle$

12. A boat is traveling west at 25 mph. The current is moving south at 3 mph. What is the boat's resultant speed? What is the direction of the boat's movement?

$\vec{b} = \langle -25, 0 \rangle$ $\vec{c} = \langle 0, -3 \rangle$ $\vec{r} = \vec{b} + \vec{c}$ $\vec{r} = \langle -25, -3 \rangle$

Speed (magnitude) $\rightarrow |\vec{r}| = \sqrt{25^2+3^2} = 25.18$ mph

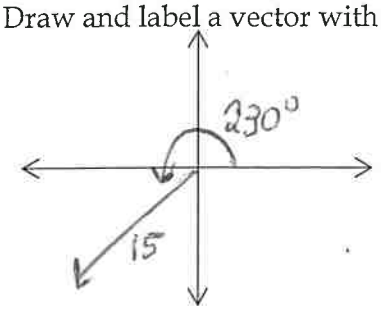
$\tan\theta = \frac{b}{a}$ $\theta = \tan^{-1}\left(\frac{-3}{-25}\right)$ $\theta = 6.843° + 180$ $\boxed{\theta = 186.84°}$

13. Alvin pulls a sled through the snow with a force of 50 newtons at an angle of 35° with the horizontal. Find the ~~magnitude of the force~~ horizontal and vertical components

$\langle 50\cos 35, 50\sin 35 \rangle$

$\boxed{\langle 40.958, 28.679 \rangle}$

14. Draw and label a vector with magnitude of 15 meters per second at a direction of 230°. (horizontal and vertical components)



$\langle 15\cos 230, 15\sin 230 \rangle$

$\boxed{\langle -9.642, -11.491 \rangle}$

15) Kaya is swimming due west at a rate of 1.5 meters/sec. A strong current flowing due north at rate of 1 meter per second. Find Kaya's resulting speed and direction.

$\vec{K} = \langle -1.5, 0 \rangle$

$\vec{c} = \langle 0, 1 \rangle$

resultant vector $\vec{r} = \vec{K} + \vec{c}$

$\vec{r} = \langle -1.5, 1 \rangle$

speed (magnitude) $= \sqrt{1.5^2 + 1^2} = 1.803$ meters/sec.

direction: $\theta = \tan^{-1}\left(\frac{1}{-1.5}\right) = -33.69°$

Q2 $\rightarrow -33.69 + 180 =$ **146.31°**

16) vector 1 is 15 meters per second squared at 60° angle to the horizontal and vector 2 is 9.8 meters per second squared downward. Determine the magnitude and direction of resultant of vector sum.

$\vec{v1} = \langle 15\cos 60, 15\sin 60 \rangle$

$\vec{v1} = \left\langle \frac{15}{2}, \frac{15\sqrt{3}}{2} \right\rangle$

$\vec{v2} = \langle 0, -9.8 \rangle$

resultant vector $\vec{r} = \left\langle \frac{15}{2}, 3.19 \right\rangle$ or $\langle 7.5, 3.19 \rangle$

$|\vec{r}| = \sqrt{7.5^2 + 3.19^2} = \boxed{8.15 \text{ m/s}^2}$

direction: $\theta = \tan^{-1}\left(\frac{3.19}{7.5}\right) = \boxed{23.04°}$

Q1