6.11 Applications of Vectors: Notes Date: ___________

Formulas: $|\vec{v}| = \sqrt{a^2+b^2}$ | $\theta = \tan^{-1}\left(\frac{b}{a}\right)$ | $\cos\theta = \frac{u \cdot v}{|u| \cdot |v|}$

$\langle |\vec{v}|\cos\theta, |\vec{v}|\sin\theta \rangle$

Ex 1: Train A and Train B depart from the same station. The path that train A takes can be represented by $\langle 33, 12 \rangle$. If the path that train B takes can be represented by $\langle 55, 4 \rangle$, find the angle between the pair of vectors.

$\cos\theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| \cdot |\vec{b}|}$ | $\cos\theta = \frac{33(55)+12(4)}{\sqrt{33^2+12^2} \cdot \sqrt{55^2+4^2}}$ | $\cos\theta = \frac{1863}{\sqrt{1233} \cdot \sqrt{3041}}$

$\theta = \cos^{-1}\left(\frac{1863}{\sqrt{1233} \cdot \sqrt{3041}}\right)$ | $\boxed{\theta = 15.823^\circ}$

Ex 2: An airplane is flying at a direction of 115° at 530 mph. Find the component form of the velocity of the airplane.

$\theta = 115^\circ$

$|\vec{v}| = 530$

$\langle 530\cos 115, 530\sin 115 \rangle$

$\langle -223.988, 480.343 \rangle$

Ex 3: A captain sails a boat for 200 kilometers at a bearing of 150°. Find the component form of the velocity of the boat.

$450 - 150^\circ$

$\theta = 300^\circ$

$\langle 200\cos 300, 200\sin 300 \rangle$

$\langle 100, -173.205 \rangle$

Ex 4: Jordan is riding the bus to school. The bus travels north for 4.5 miles, east for 2 miles, and then N60°E for 1.5 miles. Find the component form of the resultant.

$\vec{v_1}\ \langle 0, 4.5 \rangle$

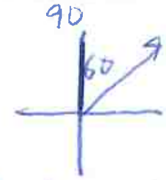
$\vec{v_2}\ \langle 2, 0 \rangle$

$v_3\ \langle 1.299, 0.75 \rangle$

$\vec{r} = \vec{v_1} + \vec{v_2} + \vec{v_3}$

$\boxed{\vec{r} = \langle 3.299, 5.25 \rangle}$

$90 - 60 = 30^\circ$



$\vec{V_3}\ \langle 1.5\cos 30, 1.5\sin 30 \rangle$

$\langle 1.299, 0.75 \rangle$

Ex 5: An airplane is flying with an airspeed of 500 miles per hour on a heading due north. If a 50-mile per hour wind is blowing at a bearing of 280°, determine the velocity and direction of the plane relative to the ground.

$\theta = 450 - 280 = 170^\circ \rightarrow \vec{w} = \langle 50\cos 170, 50\sin 170 \rangle$

$\vec{w} = \langle -49.240, 8.682 \rangle$

$\vec{a} = \langle 0, 500 \rangle$

$\vec{w} = \langle -49.240, 8.682 \rangle$

$\vec{r} = \vec{a} + \vec{w}$

$\vec{r} = \langle -49.240, 508.682 \rangle$

velocity (magnitude) → $|\vec{r}| = \sqrt{49.240^2 + 508.682^2}$

$\boxed{|\vec{r}| = 511.060 \text{ mph}}$

Direction: $\theta = \tan^{-1}\left(\frac{508.682}{-49.240}\right)$

$\theta = -84.529 + 180$

Q2 → $\boxed{\theta = 95.529^\circ}$

$*\cos\theta = \dfrac{\vec{p}\cdot\vec{c}}{|\vec{p}|\cdot|\vec{c}|}$

6.11 Applications of Vectors Practice Day 1

1. Two clay pigeons are thrown at the same time. If the path of the clay pigeons can be represented by the vectors $\mathbf{p} = \langle 42, 58\rangle$ and $\mathbf{c} = \langle 59, 73\rangle$, what is the measure of the angle between the clay pigeons?

$\cos\theta = \dfrac{42(59)+58(73)}{\sqrt{42^2+58^2}\cdot\sqrt{59^2+73^2}}$ | $\cos\theta = \dfrac{6712}{\sqrt{5128}\cdot\sqrt{8810}}$ | $\theta = \cos^{-1}\left(\dfrac{6712}{\sqrt{45177680}}\right)$

$\boxed{\theta = 3.036^\circ}$

2. A hiker is walking a trail at 2.5 miles per hour at a bearing of N50°W. Find the component form of the velocity of the hiker.

$\theta = 90+50$

$\theta = 140^\circ$

$\langle 2.5\cos 140, 2.5\sin 140\rangle$

$\boxed{\langle -1.915, 1.607\rangle}$

3. An airplane is traveling 300 kilometers per hour due east. A wind is blowing 35 kilometers per hour at an angle of 255°. A) What is the resulting speed of the airplane? B) What is the direction of the plane?

$*\ \vec{a} + \vec{w} = \vec{r}$

$\vec{w} = \langle 35\cos 255, 35\sin 255\rangle$

$\vec{a} = \langle 300, 0\rangle$

$+\ \vec{w} = \langle -9.059, -33.807\rangle$

$\vec{r} = \langle 290.941, -33.807\rangle$

A) $|\vec{r}| = \sqrt{290.941^2 + 33.807^2}$

$|\vec{r}| = 292.899$ kph

B) $\theta = \tan^{-1}\left(\dfrac{-33.807}{290.941}\right)$

Q4

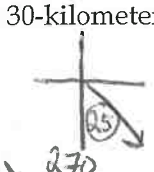
$\theta = -6.628 + 360$

$\boxed{\theta = 353.372^\circ}$

4. A helicopter is moving at a bearing of 105° with a velocity of 52 km/h. If a 30-kilometer per hour wind is blowing at S25°E, find the helicopter's resulting velocity and direction.

$*\ \vec{h} + \vec{w} = \vec{r}$

helicopter $\theta = 450 - 105 = 345^\circ$



wind $\theta = 270 + 25$

$\theta = 295^\circ$

$\vec{h} = \langle 52\cos 345, 52\sin 345\rangle \rightarrow \vec{h} = \langle 50.228, -13.459\rangle$

$\vec{w} = \langle 30\cos 295, 30\sin 295\rangle \rightarrow +\ \vec{w} = \langle 12.679, -27.189\rangle$

$\vec{r} = \langle 62.907, -40.648\rangle$

velocity $|\vec{r}| = \sqrt{62.907^2 + 40.648^2}$

$|\vec{r}| = 74.967$ km/hr

$\theta = \tan^{-1}\left(\dfrac{-40.648}{62.907}\right)$

$\theta = -32.869 + 360$

$\boxed{\theta = 327.131^\circ}$

5. Meredith is skateboarding along a path at a bearing of 70° for 35 meters. She then changes paths and travels for 45 meters along path at a bearing of 60°. A) Find the resulting distance, and B) the direction (bearing) of her path.

$\theta_1 = 20^\circ \quad \theta_2 = 30^\circ$

$*\ \vec{p1} + \vec{p2} = \vec{r}$

$\vec{p1} = \langle 35\cos 20, 35\sin 20\rangle \rightarrow \langle 32.889, 11.971\rangle$

$+\ \vec{p2} = \langle 45\cos 30, 45\sin 30\rangle \rightarrow \langle 38.971, 22.5\rangle$

$\vec{r} = \langle 71.860, 34.471\rangle$

B) $\theta = \tan^{-1}\left(\dfrac{34.471}{71.860}\right)$

$\theta = 25.627$

bearing $= 90 - 25.627$

$= \boxed{64.373^\circ}$

A) magnitude

$|\vec{r}| = \sqrt{71.860^2 + 34.471^2} = \boxed{79.700\text{m}}$