Vectors Help Session Test Review WS 3

Key

Given: X (-1, 9) and Y (-3,7)

1. Find the component form of $\overrightarrow{XY}$.

$\langle -3--1, 7-9 \rangle$

$\langle -2, -2 \rangle$

2. Find the direction, in standard position, of $\overrightarrow{XY}$.

$\theta = \tan^{-1}\left(\frac{-2}{-2}\right)$

$\theta = 45° + 180 = \boxed{225°}$

3. Write $\overrightarrow{YX}$ as the sum of unit vectors.

$\langle -1--3, 9-7 \rangle = \langle 2, 2 \rangle$

$\boxed{2i + 2j}$

4. Find the magnitude of $\overrightarrow{YX}$.

$|\vec{YX}| = \sqrt{2^2+2^2} = \sqrt{8} = \boxed{2\sqrt{2}}$

Given: $\vec{u}$ = <-2, -3>, $\vec{v}$ = <1, -3>

5. Find the angle between $\vec{u}$ and $\vec{v}$.

$\cos\theta = \frac{-2(1)+3(3)}{\sqrt{2^2+3^2}\cdot\sqrt{1^2+3^2}} = \frac{7}{\sqrt{13}\cdot\sqrt{10}}$

$\theta = \cos^{-1}\left(\frac{7}{\sqrt{130}}\right) \rightarrow \boxed{\theta = 52.125°}$

6. Find the magnitude and direction of $\vec{u}$.

$|\vec{u}| = \sqrt{2^2+3^2} = \boxed{\sqrt{13}}$

$\theta = \tan^{-1}\left(\frac{-3}{-2}\right) = 56.309 + 180 = \boxed{236.309°}$

Q3

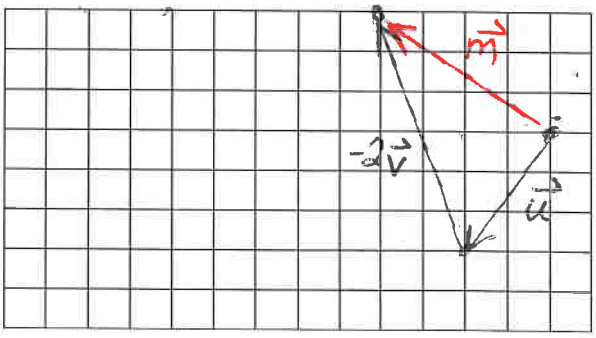
7. Find $\vec{m} = \vec{u} - 2\vec{v}$. Show work algebraically.

$\langle -2, -3 \rangle - 2\langle 1, -3 \rangle$

$\langle -2, -3 \rangle - \langle 2, -6 \rangle$

$\langle -4, 3 \rangle = \vec{m}$

8. Draw the vector diagram for $\vec{m} = \vec{u} - 2\vec{v}$ and label the resultant.



9. Find the magnitude and direction of $\vec{m} = \vec{u} - 2\vec{v}$.

$|\vec{m}| = \sqrt{(-4)^2+3^2} = \boxed{5}$ $\theta = \tan^{-1}\left(\frac{3}{-4}\right) = -36.869 + 180 = \boxed{143.130°}$

10. Create 2 vectors that are parallel to $\vec{v}$. Justify your answer.

*multiples of $\vec{v}\langle 1, -3 \rangle$

$\vec{a}\langle 2, -6 \rangle$

$\vec{b}\langle 3, -9 \rangle$

$\cos\theta = \frac{u \cdot v}{|u||v|}$ ← vectors are parallel if numerator is equal to denominator

$\vec{v}$ and $\vec{a} \rightarrow \frac{1(2)+3(6)}{\sqrt{1+3^2}\cdot\sqrt{2^2+6^2}} \rightarrow \frac{20}{\sqrt{400}} \rightarrow \frac{20}{20}$ ✓

$\vec{v}$ and $\vec{b}$

$\frac{1(3)+3(9)}{\sqrt{1+3^2}\cdot\sqrt{3^2+9^2}}$

$\downarrow$

$\frac{30}{\sqrt{10}\cdot\sqrt{90}} \rightarrow \frac{30}{30}$

11. Determine whether $\vec{a}$ and $\vec{b}$ are orthogonal. $\vec{a} = \langle 2, -3 \rangle$ and $\vec{b} = \langle 4, 5 \rangle$

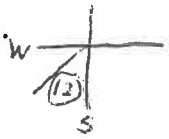
* $\vec{a}$ and $\vec{b}$ are orthogonal (perpendicular) if dot product = 0.

$\vec{a} \cdot \vec{v} = 2(4) + (-3)(5) = 8 - 15 = -7 \neq 0$, so vectors $\vec{a}$ and $\vec{b}$ are not orthogonal.

12. Convert bearing of 314° to standard position.

$450 - 314 = \boxed{136°}$

13. Convert S 12° W to standard position.



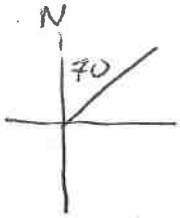
$270 - 12 = \boxed{258°}$

14. Convert the standard position of 197° into bearing.

$450 - 197 = 253$

$\boxed{\text{bearing of } 253°}$

15. A plane is flying at a speed of 320 mph on a bearing N 70° E. Its resultant speed is 370 mph and resultant direction is 60°. Find the speed and direction of the wind.



$90 - 70$

$\theta = 20°$

$\vec{p} + \vec{w} = \vec{r}$

$\vec{w} = \vec{r} - \vec{p}$

$\vec{r} = \langle 370\cos 60, 370\sin 60 \rangle$

$-\vec{p} = \langle 320\cos 20, 320\sin 20 \rangle$

$\vec{w} = \langle -115.702, 210.983 \rangle$

speed $= \sqrt{115.7^2 + 200.8^2}$

$= |\vec{w}| = 240.626 \text{ mph}$

$\theta = \tan^{-1}\left(\frac{210.983}{-115.702}\right)$

$-61.259 + 180$

$= \boxed{118.74°}$

16. A ship is sailing through the water in the English Channel with a velocity of 22 knots along a bearing of 157°. The current has a velocity of 5 knots along a bearing of 213°. The actual velocity of the ship is the vector sum of the ship's velocity and the water's velocity. Find the actual velocity.

$\theta_v = 450 - 157 = 293°$

$\theta_c = 450 - 213 = 237°$

$\vec{v} + \vec{c} = \vec{r}$

$\vec{v} \langle 22\cos 293, 22\sin 293 \rangle$

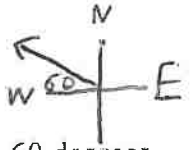
$+\vec{c} \langle 5\cos 237, 5\sin 237 \rangle$

$\vec{r} = \langle 5.873, -24.444 \rangle$

$|\vec{r}| = \sqrt{5.873^2 + 24.444^2}$

$\boxed{|\vec{r}| = 25.139 \text{ knots}}$

$\theta = 45°$



17. A bear travels 70 miles in a northeasterly direction from his den. It then travels 150 miles 60 degrees north of west. Determine how far and in what direction the bear is from his den.

$\vec{v1} + \vec{v2} = \vec{r}$

$\vec{v1} = \langle 70\cos 45, 70\sin 45 \rangle$

$+ \vec{v2} = \langle 150\cos 120, 150\sin 120 \rangle$

$\vec{r} = \langle -25.503, 179.401 \rangle$

$|\vec{r}| = \sqrt{25.503^2 + 179.401^2}$

$|\vec{r}| = 181.205 \text{ mi}$

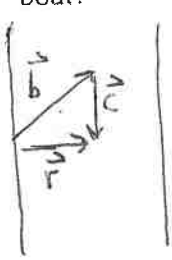
$\theta = 180 - 60$

$\theta = 120°$

$\theta = \tan^{-1}\left(\frac{179.401}{-25.503}\right) = -81.909 + 180$

$\boxed{\theta = 98.091°}$

18. A motorboat with a speed of 9 mph in still water must aim upstream at an angle of 25.5 degrees in order to travel directly across the stream. What is the speed of the current? What is the resultant speed of the boat?



$\vec{b} + \vec{c} = \vec{r}$

$\vec{c} = \vec{r} - \vec{b}$

$\vec{r} = \langle 9, 0 \rangle$

$-\vec{b} = \langle 9\cos 25.5, 9\sin 25.5 \rangle$

$\vec{c} = \langle 0.877, -3.875 \rangle$

$|\vec{c}| = \sqrt{0.877^2 + 3.875^2}$

$= 3.973 \text{ mph}$

$|\vec{b}| = \sqrt{8.123^2 + 3.875^2}$

$= 8.999 \text{ mph}$