

6.14 More Vector Review

Given: $X(-2, 8)$ and $Y(-5, 12)$

1. Find the component form of
- \overrightarrow{XY}
- .

$$\langle x_2 - x_1, y_2 - y_1 \rangle$$

$$\overrightarrow{XY} = \langle -5 - (-2), 12 - 8 \rangle = \boxed{\langle -3, 4 \rangle}$$

3. Write
- \overrightarrow{YX}
- as the sum of unit vectors.

$$Y(-5, 12) \quad X(-2, 8)$$

$$x_1 \quad y_1 \quad x_2 \quad y_2$$

$$\overrightarrow{YX} = \langle -2 - (-5), 8 - 12 \rangle = \langle 3, -4 \rangle$$

$$\boxed{3i - 4j}$$

Given: $\vec{u} = \langle -5, -1 \rangle$, $\vec{v} = \langle 4, -2 \rangle$

5. Find the angle between
- \vec{u}
- and
- \vec{v}
- .

$$\cos \theta = \frac{\vec{u} \cdot \vec{v}}{|\vec{u}| |\vec{v}|} = \frac{-5(4) + (-1)(-2)}{\sqrt{5^2 + 1^2} \cdot \sqrt{4^2 + 2^2}}$$

$$\cos \theta = \frac{-18}{\sqrt{26} \cdot \sqrt{20}} \quad \theta = \cos^{-1}\left(\frac{-18}{\sqrt{26} \cdot \sqrt{20}}\right)$$

$$\boxed{\theta = 142.125^\circ}$$

7. Find
- $\vec{m} = \vec{u} - \frac{1}{2}\vec{v}$
- . Show work algebraically.

$$\vec{m} = \langle -5, -1 \rangle - \frac{1}{2} \langle 4, -2 \rangle$$

$$= \langle -5, -1 \rangle - \langle 2, -1 \rangle$$

$$\boxed{\vec{m} = \langle -7, 0 \rangle}$$

9. Find the magnitude and direction of
- $\vec{m} = \vec{u} - \frac{1}{2}\vec{v}$
- .

$$|\vec{m}| = \sqrt{7^2 + 0^2} = \sqrt{49} = \boxed{7}$$

$$\theta = 180^\circ$$

use $\theta = \tan^{-1}\left(\frac{b}{a}\right)$ Date: _____

2. Find the direction, in standard position, of
- \overrightarrow{XY}
- .

$$\theta = \tan^{-1}\left(\frac{4}{-3}\right) = -53.130^\circ + 180^\circ$$

$$\boxed{\theta = 126.870^\circ}$$

4. Find the magnitude of
- \overrightarrow{YX}
- .

$$|\overrightarrow{YX}| = \sqrt{3^2 + 4^2} = \sqrt{9 + 16}$$

$$|\overrightarrow{YX}| = \boxed{5}$$

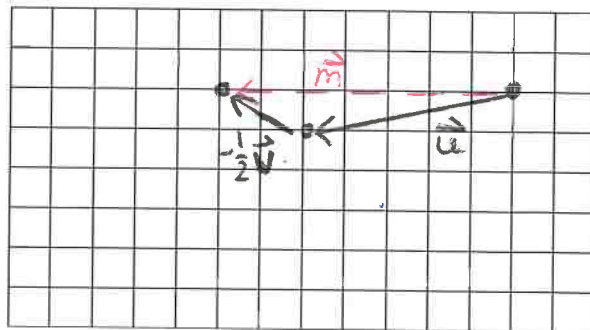
6. Find the magnitude and direction of
- \vec{u}
- .

$$|\vec{u}| = \sqrt{5^2 + 1^2} = \sqrt{26}$$

$$\theta = \tan^{-1}\left(\frac{-1}{-5}\right) = 11.310^\circ + 180^\circ$$

$$\boxed{\theta = 191.310^\circ}$$

8. Draw the vector diagram for
- $\vec{m} = \vec{u} - \frac{1}{2}\vec{v}$
- and label the resultant.



$$\vec{m} = \langle -7, 0 \rangle$$

10. Write
- \vec{m}
- as the sum of unit vectors.

$$-7i + 0j = \boxed{-7i}$$

$$\theta = 450 - 210$$

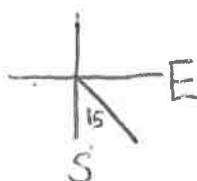
$$\theta = 240^\circ 30'$$

11. a) A pilot directs his plane to fly on a bearing of 210° at 460 mph. State the measurement of the angle for the direction in standard position. Find the component form of the velocity of the airplane.

$$\langle 460 \cos 240, 460 \sin 240 \rangle = \langle -230, -398.372 \rangle$$

- b) An 80 mph wind blowing in the direction of $S15^\circ E$ is pushing the plane off course. Find the ground speed and direction of the airplane (resultant path).

$$\vec{p} + \vec{w} = \vec{r}$$



$$\theta = 270 + 15$$

$$\theta = 285^\circ$$

$$\vec{p} \langle -230, -398.372 \rangle$$

$$+ \vec{w} \langle 20.706, -77.274 \rangle$$

$$\vec{w} = \langle 80 \cos 285, 80 \sin 285 \rangle$$

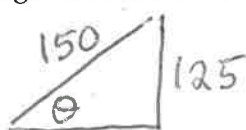
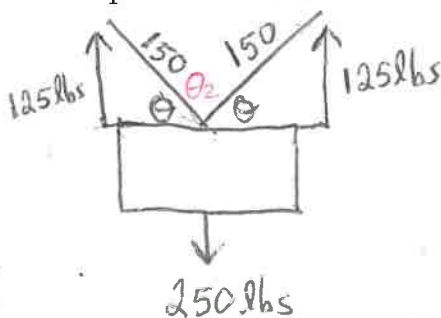
$$\vec{r} = \langle -209.294, -475.646 \rangle$$

$$\text{Speed} = |\vec{r}| = \sqrt{209.294^2 + 475.646^2} = 519.657 \text{ mph}$$

$$\theta = \tan^{-1}\left(\frac{-475.646}{-209.294}\right) = 66.249$$

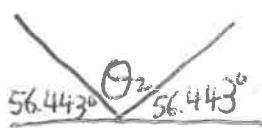
$$Q3 \rightarrow \theta = 66.249 + 180 = 246.249^\circ$$

12. A new score board is being placed in the gym. It will be supported from the ceiling by 2 cables. Each cable can withstand 150 pounds of tension. If the score board weighs 250 pounds, what is the largest possible measurement for the angle that the two cables make with each other?



$$\sin \theta = \frac{125}{150} \rightarrow \theta = \sin^{-1}\left(\frac{125}{150}\right)$$

$$\theta = 56.443^\circ$$



$$\theta_2 = 180 - 56.443 - 56.443$$

$$\theta_2 = 67.115^\circ$$

13. A pilot needs to plot a course that will result in a velocity of 520 miles per hour in a direction of $S32^\circ W$. If the wind is blowing 65 miles per hour in the direction of 12° , find the direction and the speed the pilot should set to achieve this resultant.

$$\vec{p} + \vec{w} = \vec{r}$$

$$\vec{p} = \vec{r} - \vec{w}$$



$$\theta = 270 - 32$$

$$\theta = 238^\circ$$

$$\vec{r} = \langle 520 \cos 238, 520 \sin 238 \rangle$$

$$- \vec{w} = \langle 65 \cos 12, 65 \sin 12 \rangle$$

$$\vec{p} = \langle -339.138, -454.499 \rangle$$

$$\text{Speed} = |\vec{p}| = \sqrt{339.138^2 + 454.499^2} = 567.084 \text{ mph}$$

$$\text{direction: } \theta = \tan^{-1}\left(\frac{-454.499}{-339.138}\right) = 53.270$$

Q3

+180

$$\theta = 233.270^\circ$$

Vectors

2D Vectors: $\vec{u} = \langle a_1, b_1 \rangle$ and $\vec{v} = \langle a_2, b_2 \rangle$

- Component form** shows the vector from the *initial point* to the *terminal point* based on the displacement of its dimensional values:
 - 2D vector, from (x_1, y_1) to (x_2, y_2) : $\vec{v} = \langle x_2 - x_1, y_2 - y_1 \rangle$
- Unit vector** is a vector of length 1. The standard unit vectors are $\vec{i} = \langle 1, 0 \rangle$ and $\vec{j} = \langle 0, 1 \rangle$. A vector can be written as the *sum of unit vectors* by using its components as scalars of standard unit vectors:
 - 2D vector: $\vec{v} = a\vec{i} + b\vec{j}$
- Magnitude** (length) of a vector:
 - 2D vector: $|\vec{v}| = \sqrt{a^2 + b^2}$
- Direction of a vector:**
 - 2D vector: $\theta = \tan^{-1}\left(\frac{b}{a}\right)$, add 180° if in quadrants 2 or 3.
- Given the **magnitude** and the **direction** of a vector, it is possible to determine its **components**:
 - 2D vector with magnitude $|\vec{v}|$ and direction θ , $\vec{v} = |\vec{v}| \langle \cos \theta, \sin \theta \rangle = \langle |\vec{v}| \cos \theta, |\vec{v}| \sin \theta \rangle$
- Resultant vector** is the sum of two or more vectors.
 - Geometrically, this is shown with the *tip-to-tail* method, also known as the *triangle* method. The *parallelogram* method also can determine the resultant vector.
 - Algebraically, this is calculated by finding the sum of the corresponding components.
 - 2D vectors: $\vec{u} + \vec{v} = \langle a_1 + a_2, b_1 + b_2 \rangle$
- Scalar multiplication:**
 - 2D vector: $k\vec{v} = \langle ka, kb \rangle$
- Dot product** (inner product) is used to determine if two vectors are perpendicular:
 - 2D vectors: $\vec{u} \cdot \vec{v} = a_1a_2 + b_1b_2$
 - For magnitude: $|\vec{v}| = \sqrt{\vec{v} \cdot \vec{v}}$
 - 2 vectors are orthogonal (perpendicular) if their dot product equals 0.
- Angle between two vectors** can be found with a dot product:
 - 2D vectors: $\cos \theta = \frac{\vec{u} \cdot \vec{v}}{|\vec{u}| |\vec{v}|}$ (will be provided on test)
- Angles** have different ways of being measured:
 - Standard Position** is measured from the positive x-axis, with positive angles opening counter-clockwise.
 - True Bearing** or **Compass Bearing** is measured from North, with positive angles opening clockwise.
 True bearing measurement = $450^\circ - \text{Standard position measurement}$
 Standard position measurement = $450^\circ - \text{True bearing measurement}$
 - Quadrant Bearing** is measured either from North or from South, opening toward East or toward West in such a way that the angle value is always acute.