6.3 Notes continued

4) Find a general solution to $yy' = 6\cos(\pi x)$. Then find the particular solution, $y = f(x)$, if the function passes through the point (1, 2).

$$yy' = 6\cos(\pi x)$$

$$y\left(\frac{dy}{dx}\right) = 6\cos(\pi x)$$

$$\frac{y\,dy}{dx} = \frac{6\cos(\pi x)}{1}$$

$$\int y\,dy = \int 6\cos(\pi x)\,dx$$

$u = \pi x$

$\frac{du}{dx} = \pi$

$\pi\,dx = du$

$dx = \frac{du}{\pi}$

$$\frac{y^2}{2} = \int 6\cos u \cdot \frac{du}{\pi}$$

$$\frac{y^2}{2} = 6 \cdot \frac{1}{\pi}\int \cos u\,du$$

$$\frac{y^2}{2} = \frac{6}{\pi}\sin u + c$$

$$2\left(\frac{y^2}{2} = \frac{6}{\pi}\sin(\pi x) + C\right)$$

$$y^2 = \frac{12}{\pi}\sin(\pi x) + C$$

use (1,2) solve for C

$$2^2 = \frac{12}{\pi}\sin(\pi(1)) + C$$

$$4 = \frac{12}{\pi}(0) + C$$

$$4 = C$$

point (1, 2)

$$\sqrt{y^2} = \pm\sqrt{\frac{12}{\pi}\sin(\pi x) + 4}$$

$$\boxed{y = +\sqrt{\frac{12}{\pi}\sin(\pi x) + 4}} \quad \text{or} \quad \cancel{y = -\sqrt{\frac{12}{\pi}\sin(\pi x) + 4}}$$

5) Solve $y' = (x + 1)y$

5) General equation for $y' = (x+1)y$

$$\frac{dy}{dx} = \frac{(x+1)y}{1}$$

$$dy = (x+1)y\,dx$$

$$\frac{dy}{y} = \frac{(x+1)y\,dx}{y}$$

$$\cancel{y\,dy = (x+1)dx}$$

$$\frac{dy}{y} = (x+1)dx$$

$$\int \frac{1}{y}\,dy = \int (x+1)\,dx$$

$$\int x+1\,dx$$

$$\ln|y| = \frac{x^2}{2} + x + C$$

$$* e^{\ln x} = x$$

$$e^{\ln|y|} = e^{\frac{x^2}{2} + x + C}$$

$$* a^{m+n} = a^m \cdot a^n$$

$$|y| = e^{x^2/2 + x} \cdot \boxed{e^C} \leftarrow C$$

$$|y| = Ce^{x^2/2 + x}$$

$$y = \pm Ce^{x^2/2 + x}$$

$$\boxed{y = Ce^{x^2/2 + x}}$$

Calculus AB **Chapter 6.2 Notes: Solving Differential Equation Word Problems**

1. Direct proportion equation : $y = kx$
2. Inverse (indirect) proportion equation: $y = \frac{k}{x}$
3. k is called the Constant of proportionality

Exponential Growth/Decay class examples

1) If the rate of change of y varies directly with the value of y, find the general equation:

$\boxed{y' = ky}$

$\frac{dy}{dt} = \frac{ky}{1}$

$\frac{dy}{y} = \frac{ky\,dt}{y}$

$\int \frac{dy}{y} = \int k\,dt$

$\int \frac{1}{y} dy = k \int 1\,dt$

$\ln|y| = kt + C$

$e \qquad e$

$|y| = e^{kt} \cdot e^{c}$

$\boxed{y = Ce^{kt}}$

2) The rate of increase of the population of a certain city is proportional to the population. If the population in 1930 was 50,000 and in 1960 it was 75,000, what was the expected population in 1990?

$P' = kP$

$P = Ce^{kt}$

$50{,}000 = Ce^{k(0)}$

$50{,}000 = C$

(time t, Population P)

(0, 50,000)

(30, 75,000)

(60, ___)

let $1930 \rightarrow t = 0$

$P = 50{,}000e^{kt}$

$\frac{75{,}000}{50{,}000} = \frac{50{,}000e^{k(30)}}{50{,}000}$

$1.5 = e^{30k}$

$\ln 1.5 = \ln e^{30k}$

$\frac{\ln 1.5}{30} = \frac{30k \cancel{\ln e}}{30}$

$\boxed{k = \frac{\ln 1.5}{30}}$

$P = 50{,}000e^{\left(\frac{\ln 1.5}{30}\right)t}$

$P = 50{,}000e^{\left(\frac{\ln 1.5}{30}\right)(60)}$

$\boxed{P = 112{,}500 \text{ is the population in 1990}}$

$r' = kr$

3) The rate of decay of radium is proportional to the amount present at any time. If 60 mg of radium are present now and its half-life is 1690 years, how much radium will be present 100 years from now?

(time, radium) let now be $t=0$

$r = Ce^{kt}$ (t, r)

$60 = Ce^{k(0)}$ (0, 60)

$60 = C$ (1690, 30)

$r = 60e^{kt}$ (100, ___)

$30 = 60e^{k(1690)}$ $\ln 0.5 = \ln e^{1690k}$

$\frac{30}{60} = \frac{60e}{60}$ $\ln 0.5 = 1690k \cancel{\ln e}$

$\boxed{\frac{\ln 0.5}{1690} = k}$

$r = 60e^{\left(\frac{\ln 0.5}{1690}\right)t}$

$r = 60e^{\left(\frac{\ln 0.5}{1690}\right)(100)}$

$\boxed{r = 57.589 \text{ mg}}$

4. In a certain culture where the rate of growth of bacteria is proportional to the amount present, the number triples in 3 hours.
A) If at the end of 12 hours there were 10 million bacteria, how many were present initially?
B) Find the specific exponential growth equation

Note for homework: Newton's law of cooling: the rate of change in the temperature of an object is proportional to the difference between the object's temperature and the temperature of the surrounding medium