Ch. 6.2 Differential Equation Word Problem p. 412-414
#17-33 odd, 37

17) $\frac{dy}{dt} = \frac{-1}{2}y$ passes through point (0,10)

$\int \frac{dy}{y} = \int \frac{-1}{2}\,dt$

$\ln|y| = \frac{-1}{2}t + C$

$e^{\ln|y|} = e^{\frac{-1}{2}t + C}$

$|y| = e^{-\frac{1}{2}t} \cdot e^{C}$

$|y| = e^{-\frac{1}{2}t} \cdot C$

$|y| = Ce^{-\frac{1}{2}t}$

$y = Ce^{-\frac{1}{2}t}$

$10 = Ce^{-\frac{1}{2}(0)}$ $\quad 10 = C$

$\boxed{y = 10e^{-\frac{1}{2}t}}$

19) $N' = KN$

$\frac{dN}{dt} = KN$

$\int \frac{dN}{N} = \int k\,dt$

$N = Ce^{kt}$

(time, N)
(0, 250)
(1, 400)
(4, ___)

$250 = Ce^{k(0)}$ $\quad C = 250$

$N = 250e^{kt}$

$400 = 250e^{k(1)}$

$\frac{8}{5} = e^{k}$

$\ln(8/5) = \ln e^{k}$

$\ln(8/5) = k$

$N = 250e^{\ln(8/5)t}$

$N = 250e^{\ln(8/5)(4)}$

$\boxed{N \approx 1638.4 = \frac{8192}{5}}$

23) Find a specific equation in the form $y = Ce^{kt}$ passing through (1,5) and (5,2)

point (1,5) → $5 = Ce^{k(1)}$ $\quad 2 \rightarrow 10 = 2Ce^{k}$

point (5,2) → $2 = Ce^{k(5)}$ $\quad 5 \rightarrow 10 = 5Ce^{5k}$

$2\cancel{C}e^{k} = 5\cancel{C}e^{5k}$

$\frac{2}{5} = \frac{e^{5k}}{e^{k}}$

$\frac{2}{5} = e^{4k}$

$\ln(2/5) = \ln e^{4k}$

$\ln(2/5) = 4k$

$k = \frac{1}{4}\ln(2/5)$

$5 = Ce^{k}$

$5 = Ce^{\frac{1}{4}\ln(2/5)}$

$\frac{5}{e^{\frac{1}{4}\ln(2/5)}} = C$

$C \approx 6.287$

$\boxed{y = 6.287e^{\frac{1}{4}\ln(2/5)t}}$

29) $y = Ce^{kt}$ half-life: 1599 yrs. initial: 20g

t (time)	y (Amount)
0	20
1599	10
1000	—
10,000	—

$20 = Ce^{k(0)} \rightarrow C = 20$

$y = 20e^{kt}$

$10 = 20e^{k(1599)}$

$\frac{10}{20} = e^{1599k}$

$\frac{1}{2} = e^{1599k}$

$\ln(1/2) = \ln e^{1599k}$

$\ln(1/2) = 1599k$

$\frac{\ln(1/2)}{1599} = k$

$y = 20e^{\frac{\ln(1/2)}{1599}t}$

$y = 20e^{\frac{\ln(1/2)}{1599}(1{,}000)} \approx \boxed{12.96g}$ ← 1000 yrs.

$y = 20e^{\frac{\ln(1/2)}{1599}(10{,}000)} \approx \boxed{0.26g}$ ← 10,000 yrs.

37) $y = Ce^{kt}$ half-life: 1599 yrs.

t (time)	y (Amount)
0	$\underline{C}$
1599	$\frac{1}{2}C$
100	—

$y = Ce^{kt}$

$\frac{1}{2}\cancel{C} = \cancel{C}e^{1599t}$

$\frac{1}{2} = e^{1599t}$

$\ln(1/2) = \ln e^{1599t}$

$\ln(1/2) = 1599t$

$\frac{\ln(1/2)}{1599} = t$

$y = Ce^{\frac{\ln(1/2)}{1599}t}$

$y = Ce^{\frac{\ln(1/2)}{1599}(100)}$

$y = C(0.9575)$

$\boxed{\text{Therefore, 95.76\% remains after 100 yrs.}}$