6.2 Exercises

See CalcChat.com for tutorial help and worked-out solutions to odd-numbered exercises.

Solving a Differential Equation **In Exercises 1–10, solve the differential equation.**

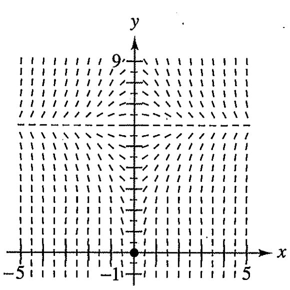
1. $\dfrac{dy}{dx} = x + 3$
2. $\dfrac{dy}{dx} = 5 - 8x$
3. $\dfrac{dy}{dx} = y + 3$
4. $\dfrac{dy}{dx} = 6 - y$
5. $y' = \dfrac{5x}{y}$
6. $y' = -\dfrac{\sqrt{x}}{4y}$
7. $y' = \sqrt{x}\,y$
8. $y' = x(1 + y)$
9. $(1 + x^2)y' - 2xy = 0$
10. $xy + y' = 100x$

Writing and Solving a Differential Equation **In Exercises 11 and 12, write and solve the differential equation that models the verbal statement.**

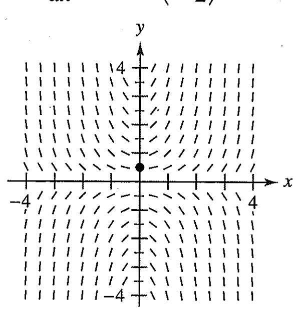
11. The rate of change of Q with respect to t is inversely proportional to the square of t.
12. The rate of change of P with respect to t is proportional to $25 - t$.

Slope Field **In Exercises 13 and 14, a differential equation, a point, and a slope field are given. (a) Sketch two approximate solutions of the differential equation on the slope field, one of which passes through the given point. (b) Use integration to find the particular solution of the differential equation and use a graphing utility to graph the solution. Compare the result with the sketch in part (a). To print an enlarged copy of the graph, go to *MathGraphs.com*.**

13. $\dfrac{dy}{dx} = x(6 - y), \quad (0, 0)$



14. $\dfrac{dy}{dx} = xy, \quad \left(0, \dfrac{1}{2}\right)$



Finding a Particular Solution **In Exercises 15–18, find the function $y = f(t)$ passing through the point (0, 10) with the given first derivative. Use a graphing utility to graph the solution.**

15. $\dfrac{dy}{dt} = \dfrac{1}{2}t$
16. $\dfrac{dy}{dt} = -9\sqrt{t}$
17. $\dfrac{dy}{dt} = -\dfrac{1}{2}y$
18. $\dfrac{dy}{dt} = \dfrac{3}{4}y$

Writing and Solving a Differential Equation **In Exercises 19 and 20, write and solve the differential equation that models the verbal statement. Evaluate the solution at the specified value of the independent variable.**

19. The rate of change of N is proportional to N. When $t = 0$, $N = 250$, and when $t = 1$, $N = 400$. What is the value of N when $t = 4$?
20. The rate of change of P is proportional to P. When $t = 0$, $P = 5000$, and when $t = 1$, $P = 4750$. What is the value of P when $t = 5$?

Finding an Exponential Function **In Exercises 21–24, find the exponential function $y = Ce^{kt}$ that passes through the two given points.**

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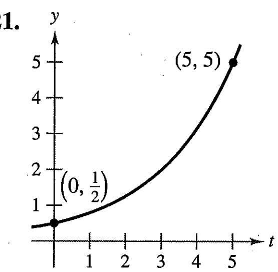

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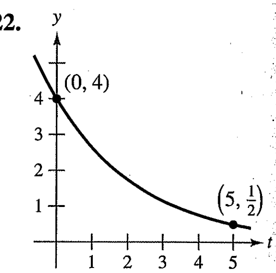

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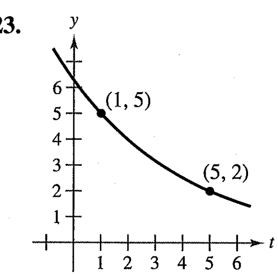

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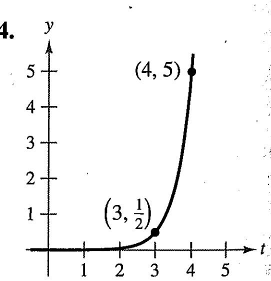


WRITING ABOUT CONCEPTS

25. **Describing Values** Describe what the values of C and k represent in the exponential growth and decay model, $y = Ce^{kt}$.
26. **Exponential Growth and Decay** Give the differential equation that models exponential growth and decay.

Increasing Function **In Exercises 27 and 28, determine the quadrants in which the solution of the differential equation is an increasing function. Explain. (Do not solve the differential equation.)**

27. $\dfrac{dy}{dx} = \dfrac{1}{2}xy$
28. $\dfrac{dy}{dx} = \dfrac{1}{2}x^2y$

Radioactive Decay In Exercises 29–36, complete the table for the radioactive isotope.

	Isotope	Half-life (in years)	Initial Quantity	Amount After 1000 Years	Amount After 10,000 Years
29.	^{226}Ra	1599	20 g		
30.	^{226}Ra	1599		1.5 g	
31.	^{226}Ra	1599			0.1 g
32.	^{14}C	5715			3 g
33.	^{14}C	5715	5 g		
34.	^{14}C	5715		1.6 g	
35.	^{239}Pu	24,100		2.1 g	
36.	^{239}Pu	24,100			0.4 g

37. **Radioactive Decay** Radioactive radium has a half-life of approximately 1599 years. What percent of a given amount remains after 100 years?

38. **Carbon Dating** Carbon-14 dating assumes that the carbon dioxide on Earth today has the same radioactive content as it did centuries ago. If this is true, the amount of ^{14}C absorbed by a tree that grew several centuries ago should be the same as the amount of ^{14}C absorbed by a tree growing today. A piece of ancient charcoal contains only 15% as much of the radioactive carbon as a piece of modern charcoal. How long ago was the tree burned to make the ancient charcoal? (The half-life of ^{14}C is 5715 years.)

Compound Interest In Exercises 39–44, complete the table for a savings account in which interest is compounded continuously.

	Initial Investment	Annual Rate	Time to Double	Amount After 10 Years
39.	\$4000	6%		
40.	\$18,000	$5\frac{1}{2}\%$		
41.	\$750		$7\frac{3}{4}$ yr	
42.	\$12,500		20 yr	
43.	\$500			\$1292.85
44.	\$6000			\$8950.95

Compound Interest In Exercises 45–48, find the principal P that must be invested at rate r, compounded monthly, so that \$1,000,000 will be available for retirement in t years.

45. $r = 7\frac{1}{2}\%, \quad t = 20$

46. $r = 6\%, \quad t = 40$

47. $r = 8\%, \quad t = 35$

48. $r = 9\%, \quad t = 25$

Compound Interest In Exercises 49 and 50, find the time necessary for \$1000 to double when it is invested at a rate of r compounded (a) annually, (b) monthly, (c) daily, and (d) continuously.

49. $r = 7\%$

50. $r = 5.5\%$

Population In Exercises 51–54, the population (in millions) of a country in 2011 and the expected continuous annual rate of change k of the population are given. (*Source: U.S. Census Bureau, International Data Base*)

(a) Find the exponential growth model

$$P = Ce^{kt}$$

for the population by letting $t = 0$ correspond to 2010.

(b) Use the model to predict the population of the country in 2020.

(c) Discuss the relationship between the sign of k and the change in population for the country.

	Country	2011 Population	k
51.	Latvia	2.2	-0.006
52.	Egypt	82.1	0.020
53.	Uganda	34.6	0.036
54.	Hungary	10.0	-0.002

55. **Modeling Data** One hundred bacteria are started in a culture and the number N of bacteria is counted each hour for 5 hours. The results are shown in the table, where t is the time in hours.

t	0	1	2	3	4	5
N	100	126	151	198	243	297

(a) Use the regression capabilities of a graphing utility to find an exponential model for the data.

(b) Use the model to estimate the time required for the population to quadruple in size.

56. **Bacteria Growth** The number of bacteria in a culture is increasing according to the law of exponential growth. There are 125 bacteria in the culture after 2 hours and 350 bacteria after 4 hours.

(a) Find the initial population.

(b) Write an exponential growth model for the bacteria population. Let t represent time in hours.

(c) Use the model to determine the number of bacteria after 8 hours.

(d) After how many hours will the bacteria count be 25,000?

57. **Learning Curve** The management at a certain factory has found that a worker can produce at most 30 units in a day. The learning curve for the number of units N produced per day after a new employee has worked t days is

$$N = 30(1 - e^{kt}).$$

After 20 days on the job, a particular worker produces 19 units.

(a) Find the learning curve for this worker.

(b) How many days should pass before this worker is producing 25 units per day?

58. Learning Curve Suppose the management in Exercise 57 requires a new employee to produce at least 20 units per day after 30 days on the job.

(a) Find the learning curve that describes this minimum requirement.

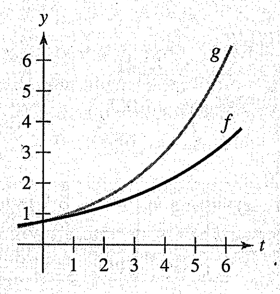
(b) Find the number of days before a minimal achiever is producing 25 units per day.

59. Insect Population

(a) Suppose an insect population increases by a constant number each month. Explain why the number of insects can be represented by a linear function.

(b) Suppose an insect population increases by a constant percentage each month. Explain why the number of insects can be represented by an exponential function.

60. HOW DO YOU SEE IT? The functions f and g are both of the form $y = Ce^{kt}$.



(a) Do the functions f and g represent exponential growth or exponential decay? Explain.

(b) Assume both functions have the same value of C. Which function has a greater value of k? Explain.

61. Modeling Data The table shows the resident populations P (in millions) of the United States from 1920 to 2010. (*Source: U.S. Census Bureau*)

Year	1920	1930	1940	1950	1960
Population, P	106	123	132	151	179

Year	1970	1980	1990	2000	2010
Population, P	203	227	249	281	309

(a) Use the 1920 and 1930 data to find an exponential model P_1 for the data. Let $t = 0$ represent 1920.

(b) Use a graphing utility to find an exponential model P_2 for all the data. Let $t = 0$ represent 1920.

(c) Use a graphing utility to plot the data and graph models P_1 and P_2 in the same viewing window. Compare the actual data with the predictions. Which model better fits the data?

(d) Use the model chosen in part (c) to estimate when the resident population will be 400 million.

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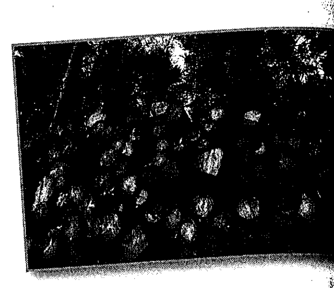
62. Forestry

The value of a tract of timber is

$$V(t) = 100{,}000e^{0.8\sqrt{t}}$$



where t is the time in years, with $t = 0$ corresponding to 2010. If money earns interest continuously at 10%, then the present value of the timber at any time t is

$$A(t) = V(t)e^{-0.10t}.$$

Find the year in which the timber should be harvested to maximize the present value function.

63. Sound Intensity The level of sound β (in decibels) with an intensity of I is

$$\beta(I) = 10 \log_{10}\left(\frac{I}{I_0}\right)$$

where I_0 is an intensity of 10^{-16} watt per square centimeter, corresponding roughly to the faintest sound that can be heard. Determine $\beta(I)$ for the following.

(a) $I = 10^{-14}$ watt per square centimeter (whisper)

(b) $I = 10^{-9}$ watt per square centimeter (busy street corner)

(c) $I = 10^{-6.5}$ watt per square centimeter (air hammer)

(d) $I = 10^{-4}$ watt per square centimeter (threshold of pain)

64. Noise Level With the installation of noise suppression materials, the noise level in an auditorium was reduced from 93 to 80 decibels. Use the function in Exercise 63 to find the percent decrease in the intensity level of the noise as a result of the installation of these materials.

65. Newton's Law of Cooling When an object is removed from a furnace and placed in an environment with a constant temperature of 80°F, its core temperature is 1500°F. One hour after it is removed, the core temperature is 1120°F. Find the core temperature 5 hours after the object is removed from the furnace.

66. Newton's Law of Cooling A container of hot liquid is placed in a freezer that is kept at a constant temperature of 20°F. The initial temperature of the liquid is 160°F. After 5 minutes, the liquid's temperature is 60°F. How much longer will it take for its temperature to decrease to 30°F?

True or False? **In Exercises 67–70, determine whether the statement is true or false. If it is false, explain why or give an example that shows it is false.**

67. In exponential growth, the rate of growth is constant.

68. In linear growth, the rate of growth is constant.

69. If prices are rising at a rate of 0.5% per month, then they are rising at a rate of 6% per year.

70. The differential equation modeling exponential growth is $dy/dx = ky$, where k is a constant.